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Electron Trajectories, Pondermotive Force and Brightness of Undulator

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Abstract

In this article, numerical and analytical analysis of electron trajectories for brightness in THz region of linearly polarized undulator is studied. Pondermotive force and stimulated emission is derived in THz radiation. Spectral brightness of THz radiation in linearly polarized undulator is derived and investigated.

Keywords: Pondermotive Force, Spectral Brightness, THz and Electron Trajectories.

1. Introduction

Free Electron Lasers have widespread importance over quantum lasers in terms of wide tunability from microwave to coherent x-rays including THz radiations, high spectral brightness and precised monochromaticity [1]. Tunability is achieved either by controlling electron beam energy or undulator parameter K indirectly varying magnetic field of undulator. In quantum or conventional Lasers, evolution of high intensity and brightness may potentially damage the medium of Laser oscillator. When a high intensity light is propagated through Laser medium albeit a considerable amount of energy is dissipated by the medium leading to a rise in temperature causing thermal stress within the material. If the temperature of the medium reach beyond the threshold value of Laser materials, this effect can result in thermal expansion, even melting leads severe fracture of the medium. The heat generated amid light matter interaction can deteriorate the process to achieve necessary population inversion ceasing Laser operation. Thus, quantum lasers are vulnerable to produce high intensity for high power Laser applications [2-3].

However, Synchrotron Radiations (SRs), Backward Wave Oscillators (BWOs) and Free Electron Lasers (FELs) are cutting edge technologically advanced sources of wider tunable THz radiations. When a low energy electron beam bunchlet produced by linear accelerator with minimum energy spread and low beam emittance is passed through transverse spatially varying magnetic field produced by an undulator, as a result of interaction between undulator field and electron beam, a beam of coherent THz radiation due to stimulated emission in the presence of resonator is produced for scientific and industrial applications. In the presence of no laser medium in FELs, in principle there is no upper limit of average power production in FEL compared to quantum lasers [4-9]. A new technique based on slice energy spread modulation produced by periodic intensity envelope of pulsed Laser offers generation of strong sub picosecond density bunching due to high intensity relativistic electron beam leads the production of high-power THz radiations from the range 1-10 THz within narrow band [10].

THz radiations produced by lasers are being used for biological applications as biosensing, imaging, diagnostic and clinical treatments including stomach cancers, detection of genetic mutations, diabetes, wound healing treatments etc. In addition, THz radiations have applications in treatments of skin cancer, angina, lungs and mouth [11].

Advent of advanced Free Electron Laser (FEL) led the interaction of intense Laser beam with biological samples is well known systematically. The optimal wavelength required for the ablation of tissue is directly dependent on the relative differential absorption of tissue composition like water, mineral and specific proteins. This absorption spectrum of tissue confirms the potential wavelength far better for optimal ablation characterized by a high efficiency and collateral damage [12]. Being lower photon energy and non-ionizing characteristics, THz radiation is deemed suitable for non deststructive, nononvasive inspection of materials for security applications. Modern detecting systems including ground penetrating Radar, IR & X-ray imaging systems incapacitate standoff IEDs and fatal weapons. However, THz detection system is efficiently confirming the exact location of standoff objects and IEDs. THz radiation can penetrate plastics, polymer, cloths etc used to cover fatal weapons, IEDs and RDX leading the reduction of potential severe threats. THz spectra produced by imaging data based systems are employed as input to signal processing algorithms to determine the availability of any explosive spectra [13-15].

The present work describes electron trajectories studied analytically using Generalised Bessel Functions (GBFs) together simulations using 4^{th} order Runge Kutta method [16-17]. Longitudinal Pondermotive force responsible for microbunching is derived and investigated using relativistic Lorentz force equation [18-20]. Spectral brightness of THz radiation is studied analytically as a function of accelerator high quality electron beam parameters with minimum energy spread to achieve laser action in Free Electron Laser (FEL). The expression for on axis and off axis spectral brightness in case of linearly polarised undulator is investigated. Numerical calculations in this article confirms the production of THz radiation with high spectral brightness in the undulator [21-22].

Magnetic Field of Planar Undulator

We choose for THz radiation generation, M=4 which represents number of magnets required to complete one period, $L' = \lambda_u / 4$ and $\varepsilon = 1 (23)$, we obtain the peak value of undulator field $B_0 = 1.57(\text{Tesla}) \exp(-\pi g_u / \lambda_u)$ **(1)**

The expression of peak value of undulator parameter K is given by $K = 0.148 x \lambda_u (\text{mm}) x \exp(-\pi g_u / \lambda_u)$, where g_u is undulator gap in mm. **(2)**

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Electron Trajectories:

We use the Relativistic Lorentz Force (RLF) equation $d\vec{p}/dt = -ec(\vec{\beta} \times \vec{B})$, where B is the undulator field, for the calculations of velocity and electron trajectories using Generalised Bessel Function (GBF) and Relativistic Lorentz Force (RLF) equation,

$$\vec{r}(t) = -\frac{Kc}{\gamma\omega_u}\sin(\omega_u t).\hat{x} + 0.\hat{y} + \left[\beta^* ct - \frac{K^2 c}{8\gamma^2\omega_u}\sin(2\omega_u t)\right].\hat{z} \tag{4}$$

where $\beta^* = 1 - \frac{1}{2\gamma^2}(1 + K^2/2 + (\gamma\psi)^2)$ **(5)**

Pondermotive Force

Pondermotive or longitudinal force responsible for the evolution of microbunching process in the undulator subsequently produces coherent radiations as dictated by the laws of electrodynamics which leads the development of FEL. The pondermotive force is produced as a result of interaction between transverse velocity of accelerated electrons and magnetic field of seeded laser or Self Amplified Spontaneous Emission (SASE) of radiation. The Relativistic Lorentz Force (RLF) equation is given by

$\vec{F}_z = -ec(\vec{\beta}_\perp \times \vec{B}_L)$, **(6)** where B_L is magnetic field of radiation, $\beta_\perp$ is transverse velocity of electrons. $\vec{B}_L = B_{0L}\sin\{\psi\}.\hat{y}$,**(7)** where $\psi = (n_1 k_1 z - n_1\omega_1 t + \phi_L)$, and n_1=m+1 represent the emission harmonics. Hence the expression for pondermotive force using Generalised Bessel Function (GBF) is given by

$$\vec{F}_z = \frac{B_{0L} I_b N_u \lambda_u K}{2\gamma}[J_{\frac{m+1}{2}}(0,\eta_1)\sin\{(n_1(k_1 + k_u)z - n_1\omega_1 t + \phi_L\}].\hat{z} \tag{8}$$

$$\psi_s = [(n_1(k_1 + k_u)z - n_1\omega_1 t + \phi_L]$$

Pondermotive phase ψ_s plays a key role to understand interaction between electrons and radiation field [24]. If the resonance condition is achieved utterly making the pondermotive force phase constant, the energy exchange is taken place continuously between electron beam oscillating at slow frequency of undulator and faster oscillating electromagnetic field of radiation. Consequently, energy is lost due to faster moving electrons and the energy lost due to electron beam is gained by electromagnetic radiation thus confirming the amplification of laser beam which is underlying principle of FEL [25].

Brightness of Planar Undulator:

The expression of undulator brightness for N_e electrons in a bunch is given:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega_L^2}{16\pi^3\varepsilon_0 c}[N_e + N_e(N_e - 1)f(\omega)]\left|\int_{-\infty}^{+\infty}\left[\vec{n}\times(\vec{n}\times\vec{\beta})\right]\exp\{i\omega_L(t - \tfrac{\vec{n}.\vec{r}}{c})\}dt\right|^2 \tag{9}$$

where $f(\omega) = \left|\int \exp(i\omega r/c)S(\vec{r})d^3r\right|^2$ is the form factor responsible for coherent enhancement of radiation for N_e electrons, $S(\vec{r})$ is a continuous normalised particle distribution function of electron bunch such that factor $N_e S(\vec{r})d^3r$ determines probability of an electron in a volume d^3r around **r**. The spectral brightness in accelerator community is defined as

$$\tilde{\tilde{B}}(d\dot{N}/d\Omega, d\omega/\omega, 0.1\%BW) = \frac{7.53\text{x}10^6\gamma^2 N_u^2 I_b(A)[T_{xm}^2 + T_{ym}^2]\text{sinc}^2(\nu_m T/2)}{\sigma_x(mm)\sigma_y(mm)\sqrt{1+\frac{\sigma_x'^2}{\theta_{cen}^2}}\sqrt{1+\frac{\sigma_y'^2}{\theta_{cen}^2}}}\left[\frac{mK}{(1 + K^2/2 + (\gamma\psi)^2)}\right]^2\left[\frac{\text{Photons/sec}}{\text{mm}^2\text{mrad}^2(0.1\%BW)}\right] \tag{10}$$

Where,

$$T_{xm}=\frac{\cos(\phi)J_m(P_m,Q_m)}{\gamma}+\frac{K[J_{m+n}(P_m,Q_m)+J_{m-n}(P_m,Q_m)]}{2\gamma}$$

$$T_{ym}=\frac{\sin(\phi)J_m(P_m,Q_m)}{\gamma}$$

$$P_m=\frac{-2mK\gamma\psi\cos(\phi)}{\left(1+K^2/2+(\gamma\psi)^2\right)}$$

$$Q_m=\frac{-mK^2}{4\left(1+K^2/2+(\gamma\psi)^2\right)}$$

Where $\nu_m=(\omega/\omega_1-m)$ and $\omega_m=m\omega_1$ and the emission frequency is $\omega_1=\omega_L=\dfrac{2m\gamma^2\omega_u}{(1+K^2/2+(\gamma\psi)^2)}$

(11)

Results and Discussions:

In this article we have studied the electron trajectories, pondermotive force and brightness of planar undulator. Transverse and longitudinal electron trajectories are investigated analytically & numerically with 4th order Runge Kutta method. Fig. 1. (a) shows transverse electron trajectory for undulator gap g_u =2.5 cm with electron beam energy E_b =10 MeV. Both results confirm perfect overlap.

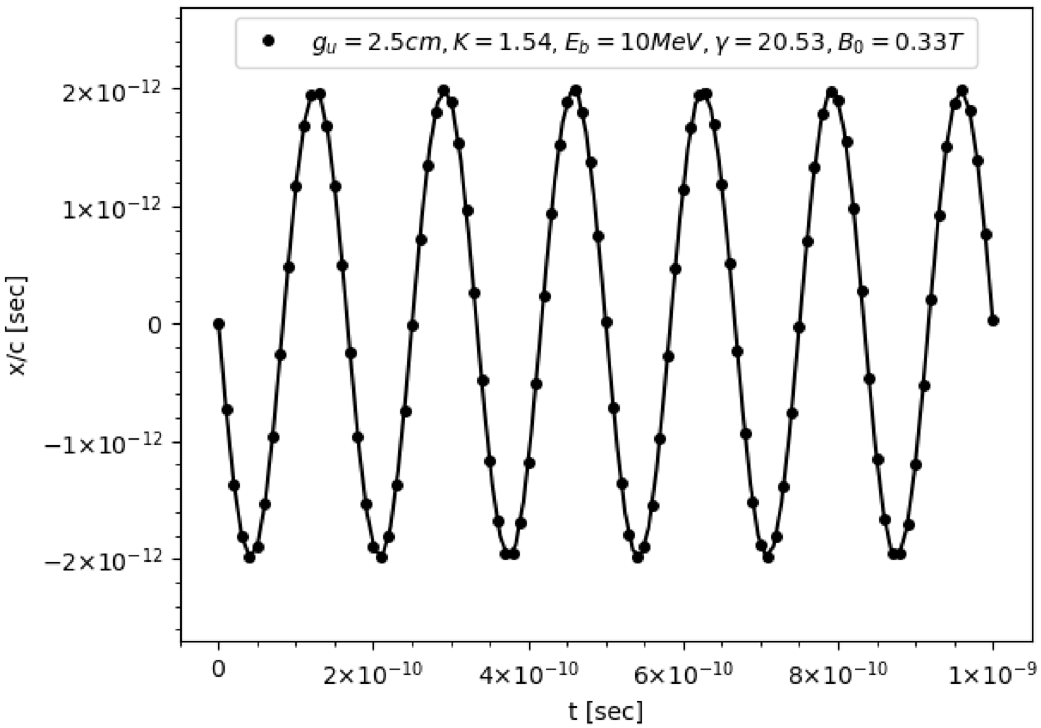

Fig.1. (a) shows reduction in amplitudes of transverse oscillations confined in x-axis through undulator due to decreasing value of K and B_0, both reduces on increasing undulator gap g_u=2.5 cms.

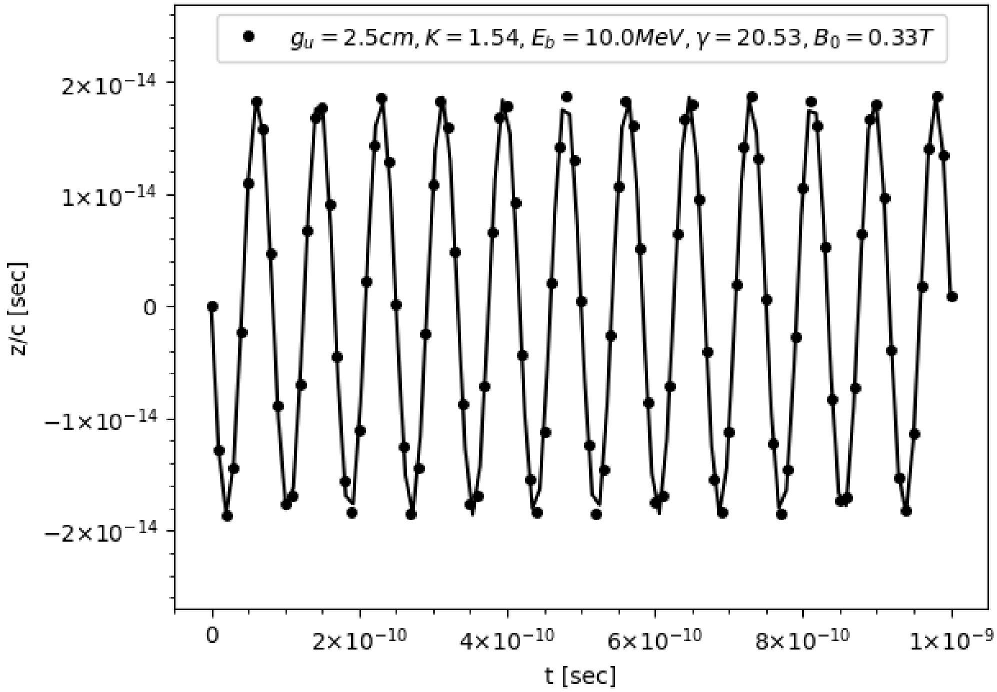

Fig. 1. (b) menisfets the effect of electron beam energy on the amplitudes of longitudinal oscillations in z-axis. It is evident that the apmplitude decrease with increasing energy electron beam energy E_b. Both results overlap analytically and numerically.

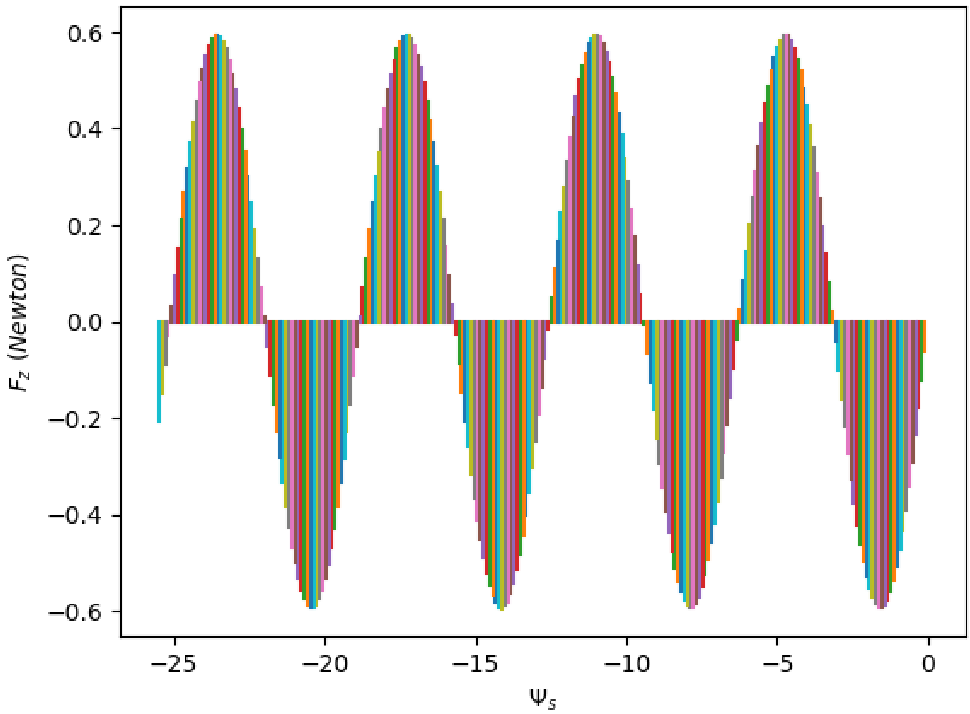

Fig. 2. manifests the pondermotive force vs pondermotive phase for electron beam having current 10 mA, electron beam energy E_b=10 MeV, electric field of radiation E_0= $6x10^{11}$(volts/m), number of undulator periods N_u=15, undulator gap g_u=20 mm and undulator period $\lambda_u = 50$mm.

Brightness of Planar Undulator

Off Axes Spectral Brightness:

When an electron beam with suitable amount of energy passes through transverse spatially varying magnetic field produced by undulator, electromagnetic radiation of THz region is generated. Production of THz radiation is cutting edge technology for many commercial and scientific applications. A 10 MeV energy of electron beam having 10 mA current produces off axis spectral brightness of 1.2×10^{11} (photons/sec/$mm^2$$mrad^2$(0.1%BW)) at 205.6 μm (n=1) wavelength with 1.46 THz fundamental frequency is shown in Fig.3. (a).

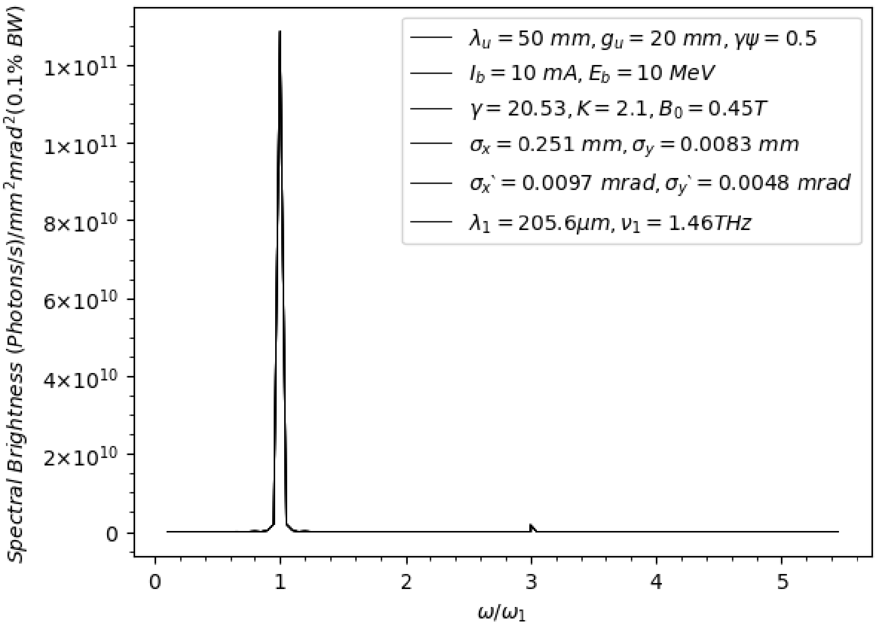

Fig.6. (a) reveals the generation of fundamental (n=1) THz frequency for an electron beam with energy E_b=10 MeV.

In the conclusion, this article confirms the generation of 1-5 THz radiation at wavelength from 66 μm -250 μm by Free Electron Laser (FEL) which is a powerful tool with broad applications in material characterization, imaging, spectroscopy, THz communication technologies and standoff detections. The ability to produce high-power, coherent, and widely tunable THz radiation presents FELs a valuable and advanced resource for both fundamental research and cutting edge practical technological advancements.

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Conflicts of interest

The corresponding authors declared that there are no conflicts of interest in this manuscript.

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