Study of Couple Mode Theory for Propagation of Electromagnetic Waves through Optical Fiber

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Abstract:
Coupled mode theory is a study of the phenomenon which is occurring during the propagation of electromagnetic waves through the optical fiber. During this phenomenon, transfer of power is between the modes of either the same fiber or of parallel neighboring fibers. We have reported in this article the causes of mode coupling and how the power transfer occurs between the modes. Couple mode theory of uniform coupler is also discussed. Simulation results described in the pictures are about the index profile of optical fiber, guided modes, the intensity of different modes with amplitude, the launching beam, how much power is transferred between modes and propagation of the intensity profile of the beam.

Keywords: Couple mode theory, Electromagnetic waves, Optical waveguide.

ARTICLE HIGHLIGHT

- It gives discussion over couple mode theory of electromagnetic waves in optical fiber.
- Highlight the causes of mode coupling.
- Describes how power transfer occurs between the modes.
- Discuss couple mode theory of uniform coupler.
- Index profile of optical fiber, guided modes, intensity of different modes with amplitude etc. have been described through simulation results.

I. INTRODUCTION

Optical fibers have become the backbone of high-speed communication networks, offering fast, reliable, and efficient data transmission capabilities. Several factors, including the geometry of the fiber, the refractive index profile, and the structure of the waveguide influence the behavior of electromagnetic waves as they propagate through these fibers. A comprehensive understanding of how light interacts with different modes within the fiber is necessary to improve system performance. Coupled mode theory is a powerful tool widely used in many disciplines of physics and engineering [1]. An important study has been about coupling modes between waveguides or within guided modes of a waveguide in optical fiber and integrated optics. It is essential to design many guided wave devices in recent photonic integrated circuits. Although mode coupling has been classified into two types, either the evanescent field coupling between modes of two adjacent waveguides/fibers or coupling between modes of the same waveguide/fiber due to periodic index perturbations [2]. Mutual coupling between optical modes is essential in the design of integrated optic devices. Coupled mode theory tells of this energy exchange, and it serves as the primary tool for designing optical couplers, switches, and filters. [3] consists of coupled waveguides in which the interaction of the temporary tails of the guided modes of neighbouring waveguides leads to the coupling of power from one waveguide to the other. Similar phenomena have been observed as periodic grating in waveguides/fibers like fiber Bragg gratings and long-period gratings result in coupling between co-propagating or counter-propagating guided modes of the waveguide itself [1] [4] [5-8]. So electromagnetic wave propagation through optical fibers is a topic of interest in the present case due to the possibility of using glass fiber as a waveguide at optical frequencies for communication purposes [9]. It is well known that electromagnetic wave propagation on fibers is in the form of surface modes that are guided as stable light patterns along the fiber [10]. Coupling of the energy or power of one mode to the others is due to any irregularity in the fiber (e.g. diameter variations, loss, and isolated particles). Mode coupling inside the optical fiber causes signal contamination, which is undesirable for optical communication systems. Coupled mode theory has been the most widely used analytical method for studying coupled optical waveguides and waveguide modes.

This paper will discuss the mechanism of energy transfer between the waves inside the optical fiber or neighbouring optical fiber [11] [12]. This coupling can sometimes be used to construct optical devices such as directional couplers, switches and filters [3]. In context to the most successful and versatile coupled waveguide device, i.e. the directional coupler, it works on evanescent field coupling between modes of two waveguides placed close to each other. While in its simplest form, it acts as a beam splitter. Its variants can be designed for more complicated devices such as switches and modulators. Optical waveguides have been divided into two categories; one can say one-dimensional waveguides and the other can say two-dimensional waveguides. The slab waveguide or planar waveguide is considered a one-dimensional waveguide. It has appeared in a wide range of applications, including photonic integrated circuits [13]. Except for all above devices, the microring resonator is also technically versatile and applicable as it has been developed by the coupled-mode theory of TE (Transverse Electric) and TM (Transverse Magnetic) modes [14]. In contemporary, mode coupling is also utilized to develop the coupled mode equations for the photonic slab waveguides [15]. This paper discuss the mode and mechanism of mode coupling, either inside the same waveguide/optical fiber or between neighbouring waveguides/optical fibers. In the simulation section, it has described how much energy has to be shared during the mode coupling.

II. MODE CONDITION IN CYLINDRICAL WAVE GUIDE

Consider an electromagnetic wave propagating in the hollow cylindrical waveguide along the z-direction. The electromagnetic field along the z-direction component can be derived with the help of two sets of modes called TE (Transverse Electric) and TM (Transverse Magnetic) modes. After the solution of Maxwell's equations

$$\vec{\nabla} \times \vec{E} = -j\mu_0 \omega \vec{H}$$

$$\vec{\nabla} \times \vec{H} = j\omega \varepsilon_0 \vec{E}$$

we get the component of the electric and magnetic field along the z-direction in terms of the Bessel function and further solution gives TM mode as T M01, T M02,T M03 etc. and TE mode is TE_{11},TE_{12},TE_{13}etc. respectively [17].

III. COUPLED MODE THEORY

The coupled mode theory describes the propagation of light along an optical fiber using the electromagnetic wave behaviour of light. The mode coupling occurs very often under the influence of additional effects, such as external disturbances or nonlinear interactions during the propagation of light in some waveguides or optical cavities. The basic idea of coupled-mode theory is to decompose all propagating light into the known modes of the undisturbed device, and then to calculate how these modes are coupled with each other by some additional influence. This approach is often technically and conceptually much more convenient than recalculating the propagation modes for the actual situation in which light propagates in the device [3], [16].

IV. MODE COUPLING IN SINGLE WAVEGUIDE / OPTICAL FIBER

If the material medium of the waveguide is either contaminated or structurally defective, like a narrowing change in radius, then the condition of mode coupling occurs in the same waveguide. This is also called coupling of spatial modes of different spatial distribution or different polarization or both. Consider an electromagnetic wave propagating along the waveguide/optical fiber in the z-direction. So, in normal mode characteristics, solutions of Maxwell's equations are

$$E_r = E_a(x,y)\exp^{j\beta_a z} \quad (1)$$

$$H_r = H_a(x,y)\exp^{j\beta_a z} \quad (2)$$

Where E_r and H_r are electric and magnetic fields at a point inside the waveguide/optical fiber. E_a and H_a are the amplitudes of the electric and magnetic field components of the electromagnetic wave of mode 'a' with propagation constant β_a. So, any optical field in the waveguide could be obtained by the combination of multiple fields here

$$E(r) = \sum_a A_a E_a(x,y)\exp^{j\beta_a z} \quad (3)$$

$$H_a = \sum_a H_a(x,y)\exp^{j\beta_a z} \quad (4)$$

Here, summation is nothing but putting all the Eigen solutions together and making a resultant field.Here normalized mode field satisfies ortho normalization and the intensity of the travelling mode is

$$I_a = (S_a + S_a^*)\hat{z} \quad (5)$$

Here (*) is a complex conjugate. So that energy is flowing along the z-direction and this is the intensity in waveguide mode. Hence the power of the waveguide in this mode is the integration of intensity, i.e

$$\int I_a(x,y)$$

Now, the power of the waveguide in TE and TM modes can be given as

$$P_{TE} = \frac{2\beta}{\omega\mu_0}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|E|^2 dxdy \quad (6)$$

$$P_{TM} = \frac{2\beta}{\omega}\int\int|H|^2 dxdy \quad (7)$$

We know that, for the satisfaction of orthogonality condition

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(E_a \times H_a^* + E_a^* \times H_a)\hat{z}dxdy = \pm P_a\delta_a \quad (8)$$

Equation (8) is called self-orthogonal equation. Hence the similar equation for orthogonality of two different modes 'a' and 'b' can be written as,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{+\infty}(E_a \times H_b^* + E_b \times H_a)\hat{z}dxdy = \pm P_a\delta_{ab} \quad (9)$$

Where delta is called the Kronecker delta function. Now for the orthonormality condition, equation (9) can be written as:

$$\int\int(E_a \times H_b^* + E_b^* \times H_a)\hat{z}dxdy = \pm\delta_{ab} \quad (10)$$

Here, + and – signs show the direction of propagation of that particular mode in the forward or backward direction respectively. For non-planer waveguide, "$a = mn$" and "$b = m'n'$", so, $\delta_{ab} = \delta_{mm'}\delta_{nn'}$. For planer waveguide "$a = m$"and $b = m'$, so, $\delta_{ab} = \delta_{mm'}$. Orthonormality in TE mode is,

$$\frac{2\beta_a}{\omega\mu_0}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_a \times E_b^* dxdy = \delta_{ab} \qquad (11)$$

Now, in TM Mode,

$$\frac{2\beta_a}{\omega}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H_a \times H_b^* dxdy = \pm\delta_{ab} \qquad (12)$$

These two orthogonal or orthonormal relation shows us that power cannot be transferred between two different modes in a linear and lossless waveguide. Mode coupling occurs, if the waveguide is perturbed by its manufacturing defects like narrowing or widening or a small part of the medium has been contaminated or a parallel waveguide coming near it. For the case of mode coupling in the single waveguide, only manufacturing defects or contaminant is responsible. When waves of different modes propagate through said contaminated waveguide, their propagation constants change and energy is transferred between them. So, Maxwell's equations are also perturbed or modified by a term ΔP, which can be represented as a polarization at a particular frequency. So, here Maxwell's equations become,

$$\vec{\nabla} \times \vec{E} = -j\omega\mu_0\vec{E} \qquad (13)$$

$$\vec{\nabla} \times \vec{H} = j\omega\vec{E} + j\omega\Delta P \qquad (14)$$

where, ΔP . Here the solutions of Maxwell's equations help find the mode coupling. Now relate two Maxwell's equations as [18]:

$$\vec{\nabla}(\vec{E}_a \times \vec{H}_b^* + \vec{E}_b^* \times \vec{H}_a) = -j\,\omega(\vec{E}_a \times \Delta\vec{P}_b^* + \vec{E}_b^* \times \Delta\vec{P}_a) \qquad (15)$$

Here the field of perturbed mode and field of unperturbed mode or normal mode fields are associated with perturbation as, (Ea, Ha) → ΔPa and (Eb, Hb) → ΔPb. Here, because of the unperturbed mode of fields. Now, total power entering and exiting along the z-direction with its cross-section of the waveguide is

$$\sum \frac{d}{dz}A_a(z)\exp(\beta_a\beta_b)z\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\vec{E}_a \times \vec{H}_b^* + \vec{E}_b^* \times \vec{H}_a)\hat{z}dxdy = \omega\exp^{-j\beta_b z}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\vec{E}_b\Delta P_a dxdy \qquad (16)$$

equation (16) is called the coupled mode equation. Here, for $\beta_b > 0$, modes are moving forward and for $\beta_b < 0$ modes are moving backwards. Here disturbance as a perturbation is polarization i.e. $\Delta P = (\Delta\varepsilon)E$; so $\Delta P_a = \varepsilon\sum_a A_a E_a \exp^{j\beta_a z}$ Here perturbation is the change in the dielectric constant of the waveguide medium. So propagation of coupled mode is $\pm\frac{dA_a}{dz} = \sum_b jk_{ab}A_b\exp^{j(\beta_b-\beta_a)z}$ .Here,

$$K_{ab} = \omega\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Delta\varepsilon E_a^*.E_b dxdy \qquad (17)$$

Equation (17) is the coupling coefficient between mode 'a' and 'b' [15]

V. MODE COUPLING BETWEEN MULTIPLE WAVEGUIDES/OPTICAL FIBERS

When two or more than two parallel waveguides/optical fibers are closer to each other and electromagnetic waves flow through in nearby wave guide/optical fiber , then one may make a perturbation for the other and here mode coupling may occur. The mechanism comes from the solution of Maxwell's curl equations, which when Fourier transformed in time, form

$$\vec{\nabla} \times \vec{E} = j\omega\mu_0\vec{H} \qquad (18)$$

$$\vec{\nabla} \times \vec{H} = -j\omega\varepsilon\vec{E} \qquad (19)$$

Here, $\vec{E}$ is the electric vector, $\vec{H}$ is the magnetic vector and $\vec{\varepsilon}=\vec{\varepsilon}(x,y,z)$ is the dielectric constant of the medium and μ0 is the magnetic permeability of a vacuum. The fields are known solutions of equations (18) and (19) for a uniform lossless system with $\varepsilon=\varepsilon(x,y)$. Using the identity for the divergence of a vector cross product, we find from equations (18) and (19) the ε and $\vec{\varepsilon}$ system:

$$\vec{\nabla}.\vec{F}=-j\omega(\vec{\varepsilon}^{*}-\vec{\varepsilon})\vec{E}.\vec{E}^{*} \quad (20)$$

Where $\vec{F}$ is defined as

$$\vec{F}=\vec{E}\times\vec{H}^{*}+\vec{E}^{*}\times\vec{H}$$

Equation (20) says that coupling is possible between two neighbouring waveguides/optical fiber when one is perturbed by the other [12], [13]. Hence it is considered that the total coupled field should be sum of the individual waveguide modes/optical fiber (it is assumed that each waveguide/optical fiber is to be of single mode). Suppose that wave flow through the waveguide is along the z-axis. Hence, the field at a point in the coupling region should be,

$$E(x,y,z)=A(z)E_1(x,y)\exp^{-j\beta_1 z}+B(z)E_2(x,y)\exp^{-j\beta_2 z} \quad (21)$$

$$H(x,y,z)=A(z)E_1(x,y)\exp^{j\beta_1 z}+B(z)E_2(x,y)\exp^{-j\beta_2 z} \quad (22)$$

$A(z)$ and $B(z)$ represent the amplitudes of the propagating mode in waveguide (1) and waveguide (2) respectively. $E_1(x,y)$, $H_1(x,y)$, $E_2(x,y)$ and $H_2(x,y)$ H2. The transverse mode field profiles of the individual waveguides/Optical fibers are assumed not to change in the presence of the second waveguide. β_1 and β_2 are the corresponding propagation constants. The propagation constants are frequently expressed in terms of an effective index, n_{eff} as $\beta=k_0 n_{eff}$, where, $k_0=\dfrac{2\pi}{\lambda}$. All the fields satisfy Maxwell's equations.

$$\nabla\times E=-j\omega\mu_0 H \quad (23)$$

$$\nabla\times H=j\omega\mu_0 n^2 E \quad (24)$$

Now substitute the above field equations (21) and (22) with equations (23) and (24) we obtain

$$(\hat{z}\times E_1)\frac{dA}{dz}+(\hat{z}\times E\ \ \frac{dB}{dz}=0 \quad (25)$$

$$F_1=(\hat{z}\times H_1)\frac{dA}{dz}-j\omega\varepsilon_0(n^2-n_1^2)AE_1+(\hat{z}\times H_2)\frac{dB}{dz}-j\omega\varepsilon_0(n^2-n_2^2)BE_2=0$$

$n(x,y)$ is the transverse refractive index profile of the mode region, whereas n_1 and n_2 are the refractive index profiles of waveguide (1) and waveguide (2) respectively. We substitute the above equation into the following equations,

$$F_2=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[\vec{H}_1(\hat{z}\times E_1)\frac{dA}{dz}-\vec{E}_1(\hat{z}\times H_1)\frac{dB}{dz}-j\omega\varepsilon_0(n^2-n_1^2)A\vec{E}+\vec{E}_2(\hat{z}\times\vec{H}_2)\frac{dB}{dz}-j\omega\varepsilon_0(n^2-n_2^2)BE_2]dxdy=0 \quad (27)$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{\infty}[\vec{H}_2(F_1)-\vec{E}_1(F_2)]=0 \quad (28)$$

Following coupled equations for the amplitudes are obtained after rigorous algebraic manipulations :

$$\frac{dA}{dz}+e_{12}\frac{dA}{dz}\exp^{-(\beta_2-\beta_1)z}+jk_{11}A+jk_{12}B\exp^{(\beta_2-\beta_1)z}=0 \quad (29)$$

$$\frac{dB}{dz}+e_{12}\frac{dB}{dz}\exp^{+(\beta_2-\beta_1)}+jk_{22}B+jk_{21}\exp^{+(\beta_2-\beta_1)z}=0 \quad (30)$$

Where $4k_{pq}$ and e_{pq},(p,q=1,2) are defined as [2][18]

$$k_{pq} = \frac{\omega\mu \int_{-\infty}^{+\infty} [n_p^2(x,y) - n_q^2(x,y)] E_p(x,y).E_q(x,y)dxdy}{2\int\int_{-\infty}^{+\infty} \hat{z}[E_p(x,y) \times H_p(x,y)]dxdy} \quad (31)$$

And

$$e_{pq} = -\, e_{pq} = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{z}[E_p(x,y) \times E_q(x,y)]dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{z}[E_p(x,y) \times H_p(x,y)]dxdy} \quad (32)$$

VI. SIMULATION

With the help of the RP Fiber calculator, we have calculated, then simulated and studied the fiber-based properties with the variation of some parameters. The RP Fiber calculator is a highly convenient software for doing various calculations on optical fibers with radially symmetric refractive index profiles. It has an inherent graphical user interface with tabs for the following purposes:

A. Index Profiles:

These define the radial refractive index profiles in either step index, linear, or spline shapes. Here are 10 types of optical fibers, each kind of fiber having an input core radius and several core segments (Fig.(1))

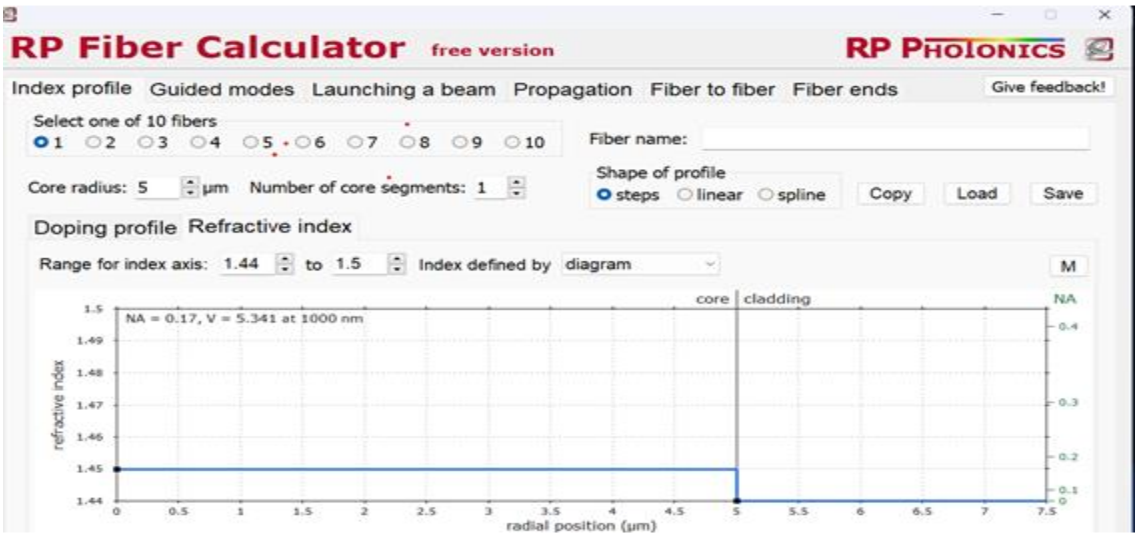

Fig. 1: Index profile

B. Guided modes:

It is a calculator to calculatee the properties of the guided modes of indices l and m [Linearly Polarized (LP), of l and m modes], at effective refractive index (n_{eff}), phase constant β, effective mode area (A_{eff}); when a fraction of power is propagating within the fiber core ($P_{in-core}$) at a particular cut-off wavelength ($\lambda_{cut-off}$). This mode solver is powerful, reliable and efficient. The following figures from Fig. (2)-(8) show possible guided modes of linearly polarized.

Parameter	value
l	0
m	1
$\beta = \frac{2\pi}{\lambda}$	11.380 μm
n_{eff}	1.449019
A_{eff}	49.8 μm^2
$P_{in-core}$	98.7%
λ	--

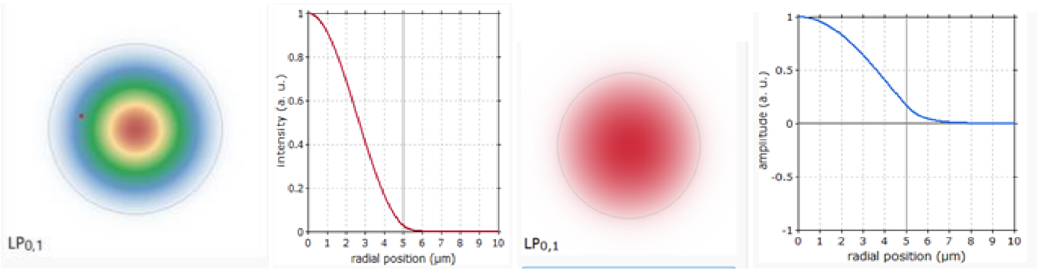

Fig. 2:(2a) $LP_{0,1}$ Intensity of 2D plot and radial, (2b.) $LP_{0,1}$ Amplitude of 2D and radial plot Mode Profile

Parameters	Value
l	1
m	1
$\beta = \frac{2\pi}{\lambda}$	11.3260 μm^{-1}
n_{eff}	1.447526
A_{eff}	46.0 μm^2
$p_{in-core}$	81.1%
$\lambda_{cutt-off}$	2211.6nm

Table II: Parameters and value for Figure (3)

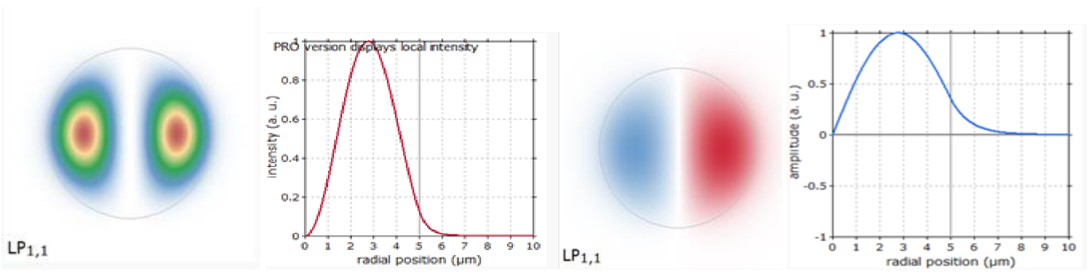

Fig.3:Fig(3a). LP_{11} intensity of 2D plot and radial, fig(3b). LP_{11} Amplitude of 2D and radial plotmode profile

Parameter	Value
l	1
m	2
$\beta = \frac{2\pi}{\lambda}$	11,3260 μm^{1}
n_{eff}	1.442070
A_{eff}	45.6 μm^{2}
$P_{in-core}$	81.1%
$\lambda_{cut-off}$	963.48nm

Table III: Parameters and value for Figure (4)

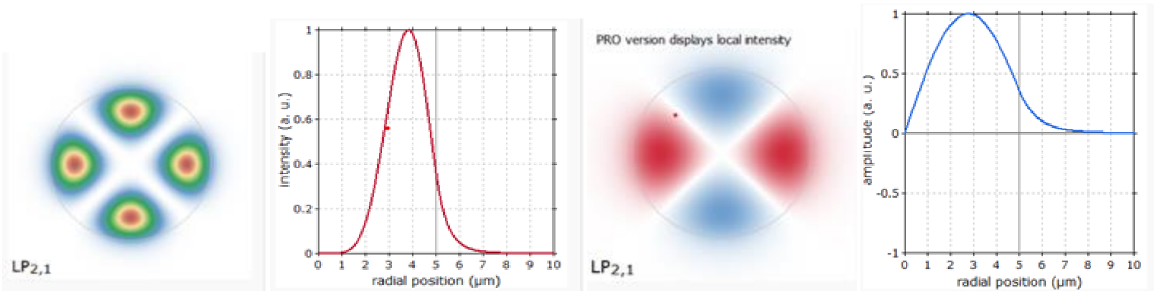

Fig.3:Fig(4a). LP_{21} Intensity of 2D plot and radial, fig(4b). $LP21$ Amplitude of 2D and radial plot mode profile.

Parameter	Value
l	3
m	1
$\beta = \frac{2\pi}{\lambda}$	11.13354 μm^{-1}
n_{eff}	1.440614
A_{eff}	49.8 μm^{-1}
$P_{in-core}$	88.2%
$\lambda cut-off$	1035.65nm

Table IV: Parameters and value for Figure (5)

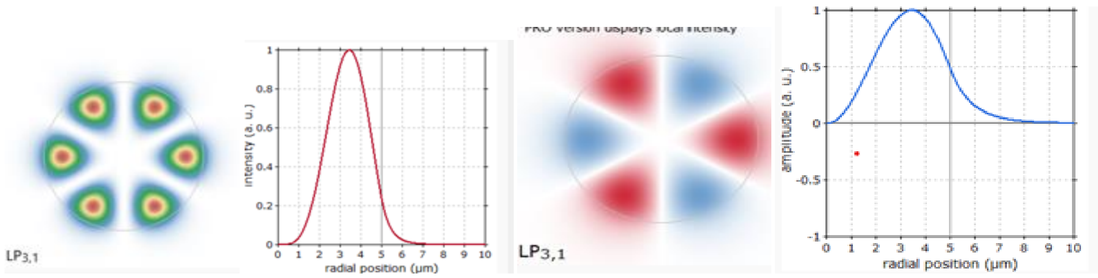

Fig. 5: (5a) LP3,1 Intensity of 2D plot and radial mode profile (5b) LP3,1 Amplitude of 2D and radial plot

Parameter	Value
l	4
m	1
$\beta = \frac{2\pi}{\lambda}$	11.3146 μm^{-1}
n_{eff}	1.440614
A_{eff}	53.7 μm^2
$P_{in-core}$	79.9%
$\lambda_{cut-off}$	833.6nm

Table V: Parameters and value for Figure (6)

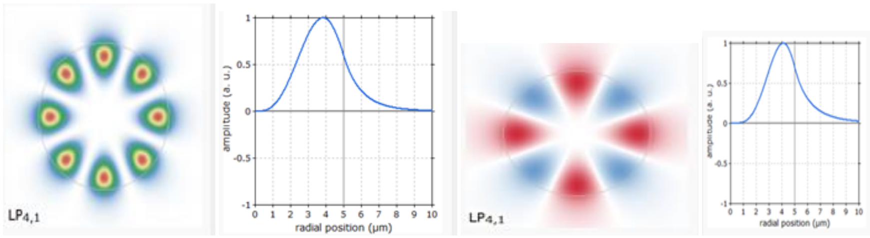

Fig. 6: (6a) LP3,1 Intensity of 2D plot and radial mode profile (6b) LP3,1 Amplitude of 2D and radial plot

Parameter	Value
l	0
m	2
$\beta = \frac{2\pi}{\lambda}$	11.3146 μm^{-1}
n_{eff}	1.444952
A_{eff}	41.8 μm^2
$P_{in-core}$	91.4%
$\lambda_{cut-off}$	1388.0nm

TABLE VI: Parameters and value for Figure (7)

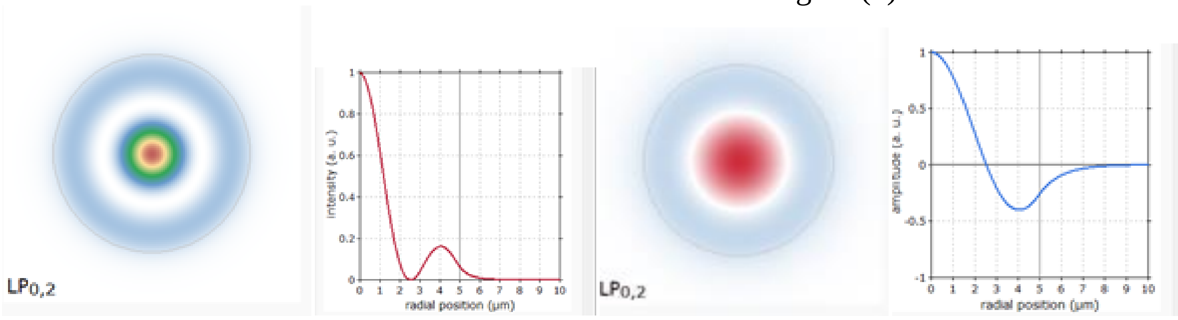

Fig. 7: (7a) LP0,2 Intensity of 2D plot and radial mode profile (7b) LP0,2 Amplitude of 2D and radial plot

Parameter	Value
l	0
m	2
$\beta = \frac{2\pi}{\lambda}$	11.3536 μm^{-1}
n_{eff}	1.445590
A_{eff}	48.4 μm^2
$P_{in-core}$	93.2%
$\lambda_{cut-off}$	1388.05

TABLE VII: Parameters and value for Figure (8)

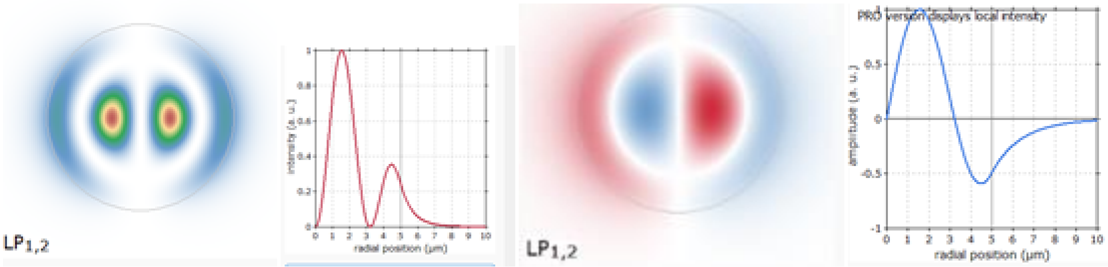

Fig. 8: (8a) LP2,1 Intensity of 2D plot and radial mode profile (8b) LP2,1 Amplitude of 2D and radial plot

C. Launch a beam

It defines a Gaussian laser beam and possibly misalignment of the fiber end and gets the powers launched into all the guided modes, as the total guided power (Fig. (9).

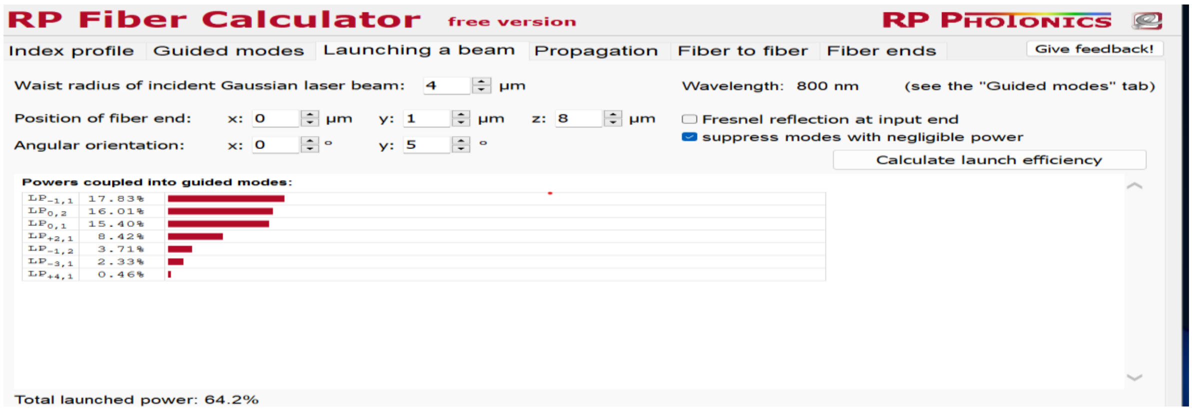

Fig. 9: Power coupled into guided modes

D. Propagation:

It shows how the beam profile evolves in the fiber, and what the corresponding near and far field profile of light exiting the fiber (in real space) and at the particular Z-position. The following figures from fig.(10)-(13) show the intensity plot of propagation of the launching beam in fiber at the near and far field with large display and amplitude plot of propagation of beam in fiber at the near and far field with large display.

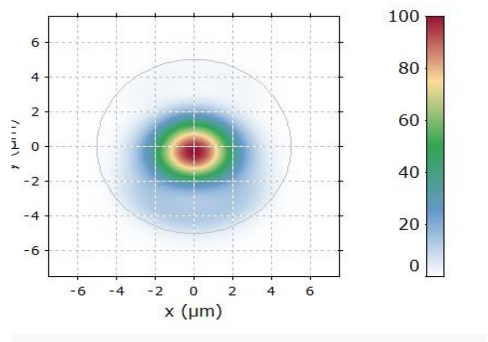

Fig. 10: Propagation of launching beam in the fiber of intensity plot at the near field with large display

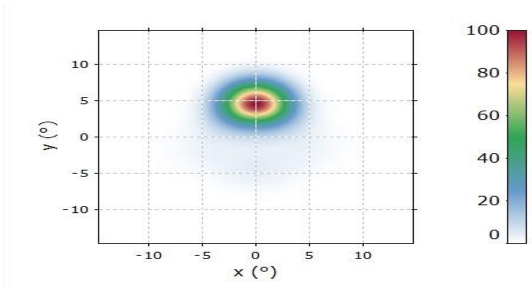

Fig. 11: Propagation of launching beam in the fiber, of intensity plot at the far field with large display

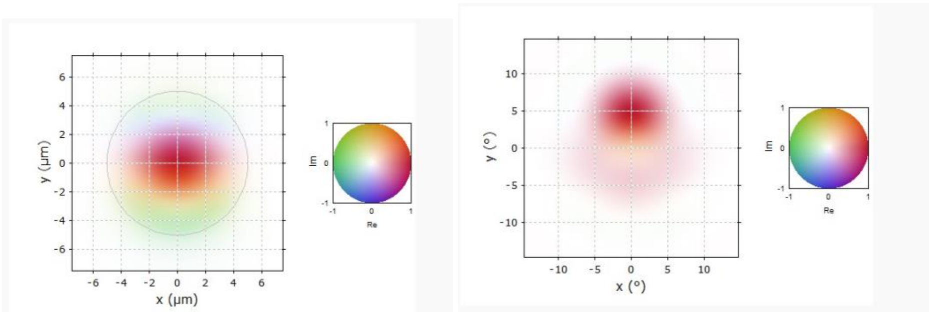

Fig. 12: Propagation of launching beam in the fiber of amplitude plot at the near field with large display

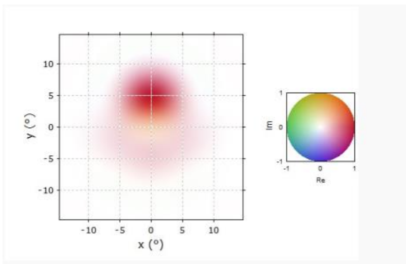

Fig. 13: Propagation of launching beam in the fiber of amplitude plot at the far field with large display

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