Transient Analysis of M/M/R Machine Repair Model with Mixed Spares and Additional Repairmen

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Abstract

In this paper markov model is considered for the analysis of machine repair problem consisting of M operating machines under the care of two types of repairmen and mixed spares. There is provision of Y cold standbys and S warm standbys to replace the failed units. When all spares are used, the failure of units occurs in degraded mode. To cope up with the increased load of failed units, there is facility of additional repairmen. The purpose of our study is to establish various performance measures in terms of transient probabilities. The expressions for the system reliability, availability and mean time to system failure are facilitated in terms of transient probabilities. Computation scheme based on matrix technique is facilitated to obtain the numerical results, which are displayed graphically and in tabular form.

KeyWords: Transient analysis, Threshold policy, Machine repair, Mixed spares, queue size, Additional repairmen, Matrix technique.

1. Introduction

In many fast growing industries, the operation of the machining system may be interrupted due to failure of machines involved in the system. The service facility therefore is to be so adjusted such that the failed machines are sent for repair instantly and the operation of the system is continued by using proper combination of spare part support, without any delay. The failed machines wait for repair until the repair of other failed machines ahead of them is completed. In case of several repairmen, if the machines fail, repairmen repair these failed machines and the excess number of failed machines beyond the number of repairmen, wait until at least one repairman is available. This affects the production and results into production loss.

In the present investigation we develop a model for the machine repair problem with mixed spares in which cold spares are first used to replace the failed units and when all cold spares are exhausted, the warm spares are used. Since the repair of failed units plays a central role in any machining system, the provision of repair facility consisting of permanent and additional removable repairmen has been made. The provision of additional repairmen may be helpful to ensure the desirable reliability with a limited number of spares at reasonable cost of the failed units in case of heavy work load. Sivazlian and Wang [14] gave economic analysis of the M/M/R machine repair problem with warm standby. Analysis of an M/M/R queue with servers vacations was performed by Kau and Naryana [8]. Wang [16] provided profit analysis of machine repair problem with a single service station subject to breakdown. Jain and Premlata [5] considered M/M/R machine repair problem with reneging and spares. Wang and Wu [18] discussed cost analysis of the M/M/R machine repair problem with spares and two modes of failure. M/M/R machine repair problem with spares and additional servers was analyzed by Jain [4]. The cost analysis of the M/M/R machine repair problem with balking, reneging and server breakdown was done by Ke and Wang [9]. Wang et al. [17] did profit analysis of M/M/R machine problem with balking, reneging and standby switching failures. Jain et al. [7] discussed M/M/R machine interference model with balking, reneging, spares and two modes of failure. Al-Seedy et al. [1] suggested transient solution of the M/M/c queue with balking and reneging. The non-perfect M/M/R machine repair problem with spares and two modes of failure was studied by Sharma [13]. Sundari and Srinivasan [15] considered time dependent solution of a Non-Markovian queue with triple stages of service having compulsory vacation and service interruptions. The unreliable multi-server queueing system with modified vacation policy was suggested by Bhargava and Jain [3] using matrix geometric approach.

A repairable system with spares, state dependent rates and additional repairman was explored by Jain et al. [6]. The analysis of R out of N systems with several repairmen, exponential life times and phase type repair times using an algorithmic approach was given by Barron et al. [2]. Rafael and Delia [12] considered a multiple warm standby system with operation and repair times following phase type distributions. Wang et al. [19] have done profit analysis of the M/M/R machine repair problem with balking, reneging and standby switching failures. Wu and Ke [20] used computational algorithm and parameter optimization for a multi-server system with unreliable servers and impatient customers. Lin and Ke [11] studied the discrete-time system with server breakdowns. Ke, et al. [10] suggested computational analysis of machine repair problem with unreliable multi-repairmen.

In this paper, we have performed transient analysis of M/M/R machine repair model with mixed spares. The organization of the rest of the chapter is as follows. In section 2, we develop model for M/M/R machine repair problem with spare part support and maintained by a pool of repairmen. The transient analysis using matrix method is given in section 3. Some performance indices are calculated in section 4. A sensitivity analysis is facilitated in section 5 to validate the analytical results. We conclude our investigations in final section 6 by highlighting the scope of the work done.

Model Descriptions

Consider a machine repair model consisting of K=M (operating) +Y (cold standby)+S (warm standby) units under the care of R permanent and r additional repairmen. The operating and warm standby units fail in Poisson fashion with failure rate λ and α ($0\leq \alpha\leq \lambda$), respectively. Here the failure of units refers to arrival of machines to get repair from the service facility on a FCFS basis. The model is developed by making the following assumptions:

- When a spare moves into an operating state its failure characteristics will be that of operating unit.
- For normal operation of system, M operating units are required but system may work in degraded mode (i.e. short mode) also with at least m units (m<M). If an operating unit fails, a spare (first cold, then warm) if available, is put into operation.
- The operating machine as well as spare units fails independently. Whenever an operating units or a warm spare unit fails it is immediately sent to service facility where it is repaired according to the first come first served (FCFS) discipline. Each repairman can repair only one failed unit at a time.
- The life time and repair time of operating and warm units are assumed to be exponentially distributed.

- The switchover from standby state to operating state or from repair to stand-by state is instantaneous and perfect.
- Once a unit is repaired, it is as good as new one. The repair unit goes to standby or operating state depending upon whether some standbys are left or all are exhausted.

The following notations are used to formulate the mathematical model:

λ	Failure rate of operating units.
S	Number of warm spare units in the system.
Y	Number of cold spare units in the system.
α	Failure rate of warm standby units.
λ_d	Degraded failure rate of operating unit when all spares are being used.
R	Number of permanent repairmen in the system.
r	Number of additional removable repairmen in the system.
T	Threshold increment value of the queue length, to turn on additional repairmen, one by one.
μ	Repair rate of permanent repairmen.
μ_j	Repair rate of j[th] additional repairman, j=1,2,……..,r.
$P_n(t)$	Probability that there are n failed units in the system at time t.

The Transient Analysis

The State Transition Rates

Let n denote the number of failed units in the system. The mean failure rate λ_n and the mean service rate μ_n for this model are given by

The mean failure rate is given by

$$\lambda_n = \begin{cases} M\lambda + S\alpha, & 0 \le n < Y \\ M\lambda + (Y+S-n)\alpha, & Y \le n < Y+S \\ (M+Y+S-n)\lambda_d, & Y+S \le n < K = M+Y+S+1. \\ 0, & Otherwise \end{cases}$$

The mean repair rate is given by

$$\mu_n = \begin{cases} n\mu, & 1 \le n \le R \\ R\mu, & R < n \le T \\ R\mu + \sum_{i=1}^{j} \mu_i, & jT < n \le (j+1)T, \quad \text{j = 1,2,.., r-1} \\ R\mu + \sum_{i=1}^{r} \mu_i, & rT < n \le K \end{cases}$$

Governing Equations

The differential-difference equations governing to the model are constructed by using appropriate transition rates and following flow conservation law which are as follows:

When all repairmen are on vacation i.e

Case-I: For R ≤ Y.

$$P_0'(t) = -[M\lambda + S\alpha]P_0(t) + \mu P_1(t) \quad (1)$$

$$P_n'(t) = -[M\lambda + S\alpha + n\mu]P_n(t) + [M\lambda + S\alpha]P_{n-1}(t) + [(n+1)\mu]P_{n+1}(t), \quad 1 \le n \le R \quad (2)$$

$$P_n'(t) = -[M\lambda + S\alpha + R\mu]P_n(t) + [M\lambda + S\alpha]P_{n-1}(t) + R\mu P_{n+1}(t), \quad R < n < Y \quad (3)$$

$$\begin{aligned} P_n'(t) = &-[M\lambda + (Y+S-n)\alpha + R\mu]P_n(t) \\ &+ [M\lambda + (Y+S-n+1)\alpha]P_{n-1}(t) + R\mu P_{n+1}(t), \end{aligned} \quad Y \le n < Y+S \quad (4)$$

$$\begin{aligned} P_n'(t) = &-[(M+Y+S-n)\lambda_d + R\mu]P_n(t) + (M+Y+S-n+1)\lambda_d P_{n-1}(t) \\ &+ R\mu P_{n+1}(t), \quad Y+S \le n \le \mathrm{T} \end{aligned} \quad (5)$$

$$\begin{aligned} P_n'(t) = &-\left[(M+Y+S-n)\lambda_d + R\mu + \sum_{i=1}^{j}\mu_i\right]P_n(t) + (M+Y+S-n+1)\lambda P_{n-1}(t) + \\ &\left[R\mu + \sum_{i=1}^{j}\mu_i\right]P_{n+1}(t), \quad jT < n \le (j+1)T \quad , j = 1,2\ldots..r-1 \end{aligned} \quad (6)$$

$$\begin{aligned} P_n'(t) = &-\left[(M+Y+S-m+1-n)\lambda_d + R\mu + \sum_{i=1}^{r}\mu_i\right]P_n(t) \\ &+ (M+Y+S-m+1-n+1)\lambda P_{n-1}(t) + \left[R\mu + \sum_{i=1}^{r}\mu_i\right]P_{n+1}(t), \end{aligned} \quad \text{r t< n <K} \quad (7)$$

$$\boldsymbol{P_K'(t) = -\left[R\mu + \sum_{i=1}^{r}\mu_i\right]P_K(t) + \lambda_d P_{K-1}(t)} \quad (8)$$

When some repairmen are on vacation i.e.

Case-II: For Y < R ≤ Y+S.

$$P_0'(t) = -[M\lambda + S\alpha]P_0(t) + \mu P_1(t) \quad (9)$$

$$\begin{aligned} P_n'(t) = &-[M\lambda + S\alpha + n\mu]P_n(t) + [M\lambda + S\alpha]P_{n-1}(t) \\ &+ [(n+1)\mu]P_{n+1}(t), \end{aligned} \quad 1 \le n \le Y \quad (10)$$

$$\begin{aligned} P_n'(t) = &-[M\lambda + (Y+S-n)\alpha + n\mu]P_n(t) \\ &+ [M\lambda + (Y+S-n+1)\alpha]P_{n-1}(t) + [(n+1)\mu]P_{n+1}(t), \end{aligned} \quad Y < n \le R \quad (11)$$

$$\begin{aligned} P_n'(t) = &-[M\lambda + (Y+S-n)\alpha + R\mu]P_n(t) \\ &+ [M\lambda + (Y+S-n+1)\alpha]P_{n-1}(t) + R\mu P_{n+1}(t), \end{aligned} \quad R < n \le Y+S \quad (12)$$

$$\begin{aligned} P_n'(t) = &-[(M+Y+S-n)\lambda_d + R\mu]P_n(t) + \\ &[(M+Y+S-n+1)\lambda]P_{n-1}(t) + R\mu P_{n+1}(t), \end{aligned} \quad Y+S < n \le \mathrm{T} \quad (13)$$

$$P_n'(t) = -\left[(M+Y+S-n)\lambda_d + R\mu + \sum_{i=1}^{j}\mu_i\right]P_n(t) + [(M+Y+S-n+1)\lambda]P_{n-1}(t) + \left[R\mu + \sum_{i=1}^{j}\mu_i\right]P_{n+1}(t), \quad jT < n \le (j+1)T \quad , \text{J} = 1,2\ldots, \text{r - 1} \tag{14}$$

$$P_n'(t) = -\left[(M+Y+S-n)\lambda_d + R\mu + \sum_{i=1}^{r}\mu_i\right]P_n(t) + [(M+Y+S-n+1)\lambda]P_{n-1}(t) + \left[R\mu + \sum_{i=1}^{r}\mu_i\right]P_{n+1}(t), \quad rT < n < M+Y+S-m+1 \tag{15}$$

$$P_K'(t) = -\left[R\mu + \sum_{i=1}^{r}\mu_i\right]P_K(t) + \lambda_d P_{K-1}(t) \tag{16}$$

When all repairmen are busy in providing repair, i.e

Case-III: For Y+S < R <T

$$P_0'(t) = -[M\lambda + S\alpha]P_0(t) + \mu P_1(t), \tag{17}$$

$$P_n'(t) = -[M\lambda + S\alpha + n\mu]P_n(t) + [M\lambda + S\alpha]P_{n-1}(t) + [(n+1)\mu]P_{n+1}(t), \ 1 \le n \le Y \tag{18}$$

$$P_n'(t) = -[M\lambda + (Y+S-n)\alpha + n\mu]P_n(t) + [M\lambda + (Y+S-n+1)\alpha]P_{n-1}(t) + [(n+1)\mu]P_{n+1}(t), \quad Y < n \le Y+S \tag{19}$$

$$P_n'(t) = -[(M+Y+S-n)\lambda_d + n\mu]P_n(t) + [(M+Y+S-n+1)\lambda_d]P_{n-1}(t) + [(n+1)\mu]P_{n+1}(t), \quad Y+S < n \le R \tag{20}$$

$$P_n'(t) = -[(M+Y+S-n)\lambda_d + R\mu]P_n(t) + [(M+Y+S-n+1)\lambda]P_{n-1}(t) + R\mu P_{n+1}(t), \quad R < n \le T \tag{21}$$

$$P_n'(t) = -\left[(M+Y+S-n)\lambda_d + R\mu + \sum_{i=1}^{j}\mu_i\right]P_n(t) + [(M+Y+S-n+1)\lambda]P_{n-1}(t) + \left[R\mu + \sum_{i=1}^{j}\mu_i\right]P_{n+1}(t), \quad jT < n \le (j+1)T, \quad \text{J} = 1,2,\ldots, \text{r - 1} \tag{22}$$

$$P_n'(t) = -\left[(M+Y+S-n)\lambda_d + R\mu + \sum_{i=1}^{r}\mu_i\right]P_n(t) + [(M+Y+S-n+1)\lambda_d]P_{n-1}(t) + \left[R\mu + \sum_{i=1}^{r}\mu_i\right]P_{n+1}(t), \quad rT < n < M+Y+S-m+1 \tag{23}$$

$$P_K'(t) = -\left[R\mu + \sum_{i=1}^{r}\mu_i\right]P_K(t) + \lambda_d P_{K-1}(t) \tag{24}$$

Matrix Method

Consider an irreducible Markov chain with transition probability matrix is as follows:

After taking Laplace transformation of above set of equations in each case, we find matrix equation in terms of Laplace transform of probabilities as

$$\underline{A}(s)\,\underline{\bar{P}}(s) = \underline{P}(0)$$

where

$$\underline{A}(s) = \begin{bmatrix} -(s+\lambda_0) & \lambda_0 & & & & & & & & & \\ \mu_1 & -(s+\lambda_1+\mu_1) & \lambda_1 & & & & & & & & \\ & \mu_2 & -(s+\lambda_2+\mu_2) & \lambda_2 & & & & & & & \\ & & & \ldots & & & & & & & \\ & & & & \ldots & & & & & & \\ & & & & \ldots & & & & & & \\ & & & & \mu_R & -(s+\lambda_R+\mu_R) & \lambda_R & & & & \\ & & & & & & \ldots & & & & \\ & & & & & & \ldots & & & & \\ & & & & & & \mu_{jT} & -(s+\lambda_{jT}+\mu_{jT}) & \lambda_{jT} & & \\ & & & & & & & & \ldots & & \\ & & & & & & & & \ldots & & \\ & & & & & & & & \mu & -(s+\lambda_{rT}+\mu_{rT}) & \lambda_{rT} \\ & & & & & & & & & \ldots & \\ & & & & & & & & & \ldots & \\ & & & & & & & & & \mu_K & -(s+\mu_K) \end{bmatrix}$$

(25)

$$\underline{\bar{P}}(s) = \left[\bar{P}_0(s),\ldots\bar{P}_Y(s),\ldots\bar{P}_{Y+S}(s),\ldots,\bar{P}_R(s),\ldots,\bar{P}_{jT}(s),\ldots,\bar{P}_{rT_r}(s),\ldots,\bar{P}_K(s)\right]^T$$

or $\underline{A}(s)\underline{P}(s) = I$, where $I=[1,0,0,\ldots,0]^T$.

Here A(s) is the cefficient matrix of order (K+1)$\times$ (K+1).

Using Cramer's rule $\bar{P}(s)$ can be determined as using

$$\underline{\bar{P}_n}(s) = \frac{|\underline{A_{n+1}}(s)|}{|\underline{A}(s)|}, \; n = 0,1,2,\ldots, K$$

where $|\underline{A}(s)|$ is the determinant of coefficient matrix A(s) and $|A_{n+1}(s)|$, (n =0,1,2,….,K) is the determinant of the matrix by replacing (n+1)th column of matrix A(s) with initial vector $I=[1,0,0,\ldots,0]^T$.

It is noted that s = 0 is a root of $|A(s)| = 0$. If we take $s = (-\phi)$, then we get $\underline{A}(-\phi) = \underline{A} - \phi\underline{I}$, where I is the identity matrix and $\underline{A}=\underline{A}(0)$ is (K+1)$\times$ (K+1) matrix. Putting the value of $\underline{A}(-\phi)$ in equation (25), we get

$$\underline{A}(-\phi)\,\underline{\bar{P}}(s) = (\underline{A} - \phi\underline{I})\,\underline{\bar{P}}(s) = \underline{P}(0)$$

The distinct eigen values ϕ_n ($\phi_n \neq 0$ where n = 0,1,2,…..,K) of the matrix $\underline{A} - \phi \underline{I}$, can be obtained by equating its determinant to zero. Now we assume that the other K real eigen values including 0, are denoted by ($\phi_1, \phi_2, .., \phi_K$). Then $\left|\underline{A}\ (s)\right|$ can be written as

$$\left|\underline{A}(s)\right| = s \sum_{j=1}^{K} (s + \phi_j)$$

and $\overline{\underline{P}}_n(s) = \dfrac{\left|\underline{A}_{n+1}(s)\right|}{s \sum_{j=1}^{K} (s+\phi_j)}$ j

$$= \frac{a_{0,n}}{s} + \sum_{j=1}^{K} \frac{a_{j,n}}{s+\phi_j}, \qquad n = 0,1,2,...,...,K \tag{26}$$

where $a_{0,n} = \dfrac{A_{n+1}(0)}{\left[\prod_{j=1}^{K} \phi_j\right]}$

$$a_{j,n} = \frac{\left|A_{n+1}(-\phi_j)\ \right|}{(-\phi_j)\left[\prod_{\substack{i=1 \\ i\neq j}}^{K} (\phi_i - \phi_j)\right]}; \quad j=1, 2…., K.$$

here $a_{0,n}$ and a $_{j,n}$ (j=1,2,…..,K) are all real numbers.

The inverse Laplace transform of equation (26) is given by

$$P_n(t) = a_{0,n} + \sum_{j=1}^{K} a_{j,n} e^{-\phi_j t}, \quad n=0,1,2,…,n \tag{27}$$

Some Performance Indices

In this section, we provide some measures of performance in terms of probabilities, which can be determined using matrix method discussed in previous section.

- Expected number of spare units in the system at time t is

$$E\{S(t)\} = S\sum_{n=1}^{Y} P_n(t) + \sum_{n=Y+1}^{Y+S} (Y+S-n) P_n(t) \tag{28}$$

- Expected number of operating units in the system at time t is

$$E\{O(t)\} = M - \sum_{n=Y+S+1}^{K} (n - \overline{Y+S}) P_n(t) \tag{29}$$

- Expected number of idle permanent repairmen in the system at time t is

$$E\{I(t)\} = \sum_{n=0}^{R-1} (R-n) P_n(t) \tag{30}$$

- Expected number of busy permanent servers in the system at time t is

$$E\{B(t)\} = R - E\{I(t)\} \tag{31}$$

➢ Machine availability i.e rate of production per machine at time t is

$$A(t) = 1 - \frac{E\{O(t)\}}{M + Y + S} \quad (32)$$

➢ Expected number of busy additional repairmen in the system at time t is

$$E\{BR(t)\} = \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} jP_n(t) + r \sum_{n=rT+1}^{K} P_n(t) \quad (33)$$

➢ Expected number of failed units in the system at time t is

$$E\{N(t)\} = \sum_{n=1}^{K} nP_n(t) \quad (34)$$

Sensitivity Analysis

t	E(S)			E(I)		
	λ=.5	λ=1	λ=1.5	λ=.5	λ=1	λ=1.5
0	0.00	0.00	0.00	0.00	0.00	0.00
1	2.84	1.95	1.16	1.02	0.51	0.20
2	2.81	1.76	0.86	1.00	0.44	0.12
3	2.80	1.71	0.78	1.00	0.42	0.10
4	2.80	1.69	0.76	1.00	0.40	0.09
5	2.80	1.68	0.76	1.00	0.40	0.09
t	α=.5	α=1	α=1.5	α=.5	α=1	α=1.5
0	0.00	0.00	0.00	0.00	0.00	0.00
1	2.21	1.95	1.70	0.65	0.51	0.40
2	2.07	1.76	1.48	0.59	0.44	0.32
3	2.03	1.71	1.41	0.57	0.42	0.29
4	2.02	1.69	1.39	0.56	0.40	0.28
5	2.01	1.68	1.38	0.55	0.40	0.28

Table 1: Expected number of spares and expected number of idle permanent repairman in the system for different values of λ and α.

t	E(O)			E(B)		
	λ=.5	λ=1	λ=1.5	λ=.5	λ=1	λ=1.5
0	10.00	10.00	10.00	2.00	2.00	2.00
1	9.98	9.59	8.25	0.98	1.49	1.80
2	9.96	9.15	6.98	1.00	1.55	1.87
3	9.95	8.99	6.63	1.00	1.57	1.89
4	9.95	8.94	6.54	1.00	1.58	1.89
50	9.95	8.92	6.52	1.00	1.58	1.89
t	α=.5	α=1	α=1.5	α=.5	α=1	α=1.5
0	10.00	10.00	10.00	2.00	2.00	2.00
1	9.76	9.59	9.35	1.35	1.49	1.60
2	9.50	9.15	8.72	1.41	1.55	1.67
3	9.40	8.99	8.50	1.42	1.57	1.69
4	9.36	8.94	8.43	1.43	1.58	1.70
5	9.35	8.92	8.40	1.43	1.58	1.70

Table 2: Expected numbers of operating units and expected number of busy servers in the system for different values of λ and α .

In this section, we obtain numerical results by taking an illustration for default parameters M=10, Y=5, S=3, T=8, R=2, r=2, λ=1, λ_d=1.8, μ=7, μ_1=1, μ_2=1. For computation purpose, we develop program in MATLAB software. The numerical results are summarized in tables 1-2. The graphical representation of numerical results has also been done in figures 1- 4. In table 1, we present numerical results for the expected number of spare units E{S(t)} and expected number of permanent idle repairmen E{I(t)} by varying the failure rates of operating units (λ) and warm standbys (α). We note that for a particular value of t, both E{S(t)} and E{I(t)} decrease as λ increases. Table 2 summarizes results for E {O(t)} and E{B(t)}. We notice that for fixed value of t, E{O(t)} decreases but E{B (t)} increases by increasing the failure rate of spare (α) and failure rate of operating units (λ). As expected E{S (t)}, E{I(t)} and E{O(t)} decrease but E{B(t)} increases as time grows.

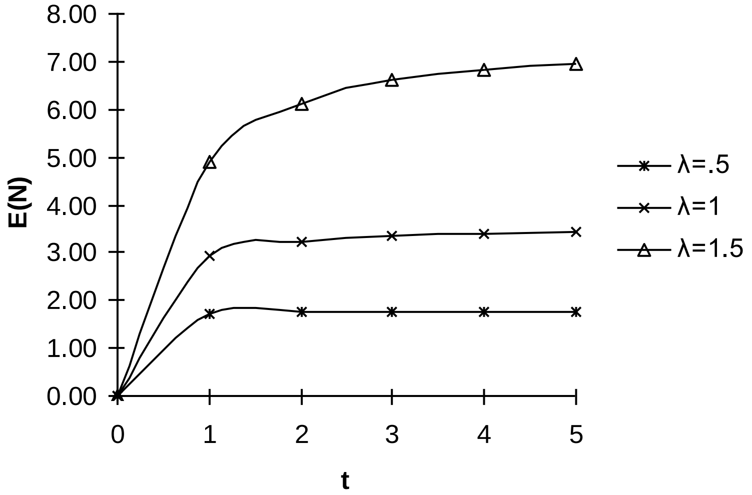

Fig. 1: Average number of failed units E(N) by varying time t for different values of λ. .

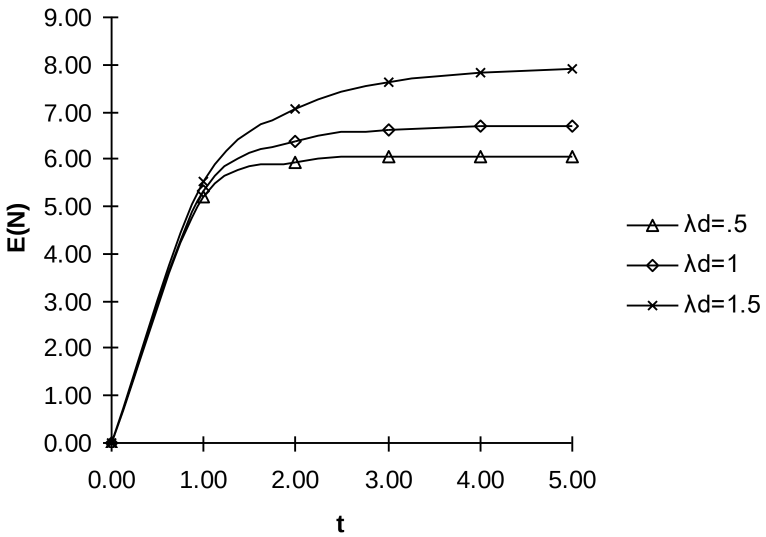

Fig. 2: Average number of failed units E(N) by varying time t for different values of λ_d .

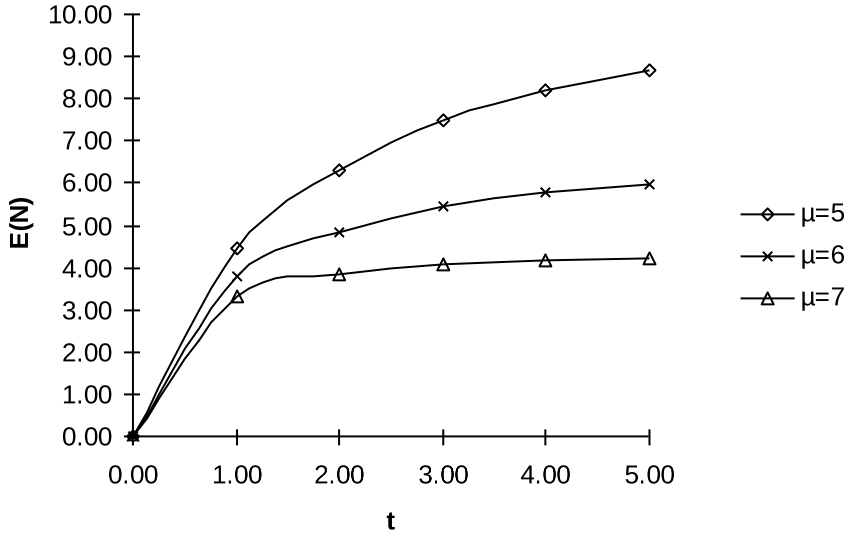

Fig. 3: Average number of failed units E(N) vs t for different values of μ.

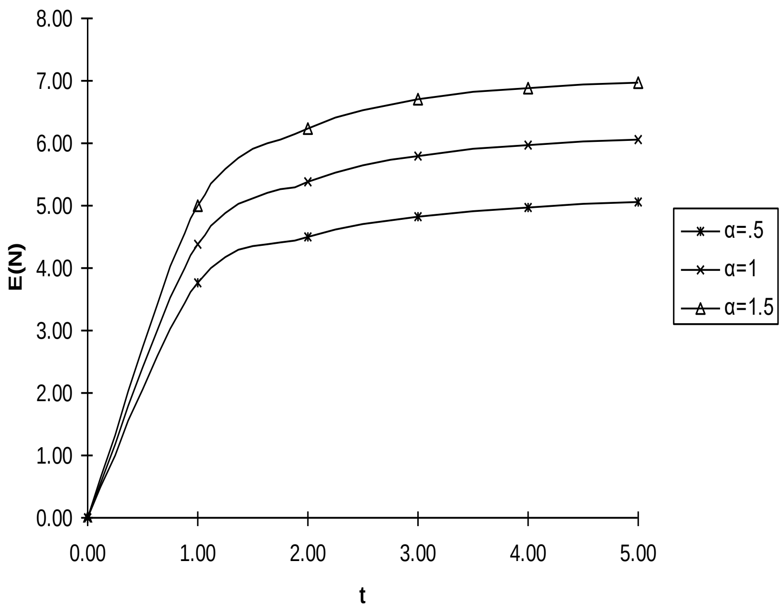

Fig. 4: Average number of failed units E(N) vs t for different values of α.

Figures 1 – 4 display the expected average number of failed unit E{(N(t)} vs time t. In figure 1, we illustrate the effect of failure rate λ of the operating units on the average queue length E{(N(t)}. We see from graph that the queue length increases initially sharply then after becomes almost constant when t increases for different value of λ. As expected, E{(N(t)} increases as failure rate λ increases. Figure 2 depicts the effect of degraded failure rate λ_d on E {(N(t)}; the queue length first increases sharply then gradually becomes constant, on increasing the time t. We also notice that E{(N(t)} increases on increasing λ_d.

The effect of repair rate (μ) of permanent repairmen on the average number of failed units E{(N(t)} is displayed in figure 3. We observed that the average number of failed units decreases as μ increases. The increasing trend of E{(N(t)} with respect to time t is also seen. It is clear from fig. 4 that E{(N(t)} increases on increasing the failure rate of warm standbys α.

Finally, we conclude that the expected number of spares, idle permanent repairmen and operating units decrease while that of busy servers increase on increasing either the failure rate of operating units or the failure rate of warm standby units. The expected number of spares, idle permanent repairmen, operating units, and busy servers decrease on increasing the time t for different values of λ and α. The average number of failed units increases with the increase in failure rate of operating units, degraded failure rate, failure rate of warm standby units and time while it decrease on increasing the service rate and with the increase in time t. The trend is as per our expectation.

Conclusions

In this paper, we have studied the transient analysis of M/M/R machine repair problem with mixed standbys. To cope up with heavy workload of failed machines, the repair crew consists of additional removable repairmen and permanent repairmen. The threshold policy developed may be advantageous for large complex systems wherein only spare part support is not sufficient to achieve desired efficiency and reliability/availability. The cost analysis may be helpful in determining the optimal combination of cold/warm standbys and the number of additional repairmen. The provision of mixed standbys in a machining system has additional advantages from the economic point of view.

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