Majority Dom-Chromatic Number of a Bipartite Graph

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Abstract

In this article, the majority dom-chromatic sets of a bipartite graphs are studied. The characterization theorems on the majority dom-chromatic number \( \gamma_{M\chi}(G) \) for bipartite graphs are determined. Also its relationship with other graph theoretic parameters and the majority dom-chromatic number for complement of a bipartite graphs are investigated.

Keywords: Majority dom-chromatic set, Majority dom-chromatic number.

1. Introduction

All the graphs \( G = (V, E) \) considered here are simple, finite and undirected. The concept of domination is early discussed by Ore and Berge in 1962. Then Haynes et.al [2] defined the domination number \( \gamma(G) \). The majority domination number \( \gamma_M(G) \) was introduced by Swaminathan and Joseline Manora [6] is the smallest cardinality of a minimal majority dominating set \( S \subseteq V(G) \) of vertices and satisfies \(|N[S]| \geq \left\lceil \frac{|V(G)|}{2} \right\rceil\).

Janakiraman and Poobalaranjani [3] defined the dom-chromatic set as a dominating set \( S \subseteq V(G) \) such that the induced subgraph \(<S>\) satisfies the property \( \chi(<S>) = \chi(G) \), where \( \chi(G) \) is the chromatic number of \( G \). The minimum cardinality of a dom-chromatic set \( S \) is called dom-chromatic number and is denoted by \( \gamma_{ch}(G) \).

Definition : 1.1 [4] The set \( S \subseteq V(G) \) is called the Majority Dominating Chromatic set (MDC- set) of a graph \( G \) if the set \( S \) is a majority dominating set and satisfies the property \( \chi(<S>) = \chi(G) \) where \(<S>\) is an induced subgraph of \( G \). It is also called a majority dom-chromatic set of a graph. A majority dom-chromatic number (MDC-number) \( \gamma_{M\chi}(G) \) is defined as the smallest cardinality of the majority dom-chromatic set of a graph \( G \).

Results on \( \gamma_G(G) \) and \( \gamma_{M\chi}(G) \) : 1.2 [4] and [6]

(i) For a path \( P_p \) and cycle \( C_p \), \( \gamma_M(G) = \left\lceil \frac{p}{6} \right\rceil, p \geq 3. \)

(ii) If a graph \( G = K_p \) then \( \gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil. \)
(iii) Let $G = mK_2, m \geq 1$. Then $\gamma_{M_X}(G) = \left\lceil \frac{p}{4} \right\rceil + 1, p \geq 2$.

(iv) Let $G$ be any graph of order $p$. Then $\gamma_{M_X}(G) = p$ if and only if $G$ is vertex color critical.

(v) For a graph $G = K_{m,n}, \gamma_{M_X}(G) = 2$.

(vi) For any cycle $C_p, \gamma_{M_X}(G) = \begin{cases} \left\lceil \frac{p}{6} \right\rceil, & \text{if } p \equiv 2 \pmod{6} \\ \left\lceil \frac{p}{6} \right\rceil + 1, & \text{if } p \equiv 0, 4 \pmod{6} \\ p, & \text{if } p \text{ is odd} \end{cases}$.

(vii) If $G$ is a path then $\gamma_{M_X}(G) = \begin{cases} \left\lceil \frac{p}{6} \right\rceil, & \text{if } p \equiv 1, 2 \pmod{6} \\ \left\lceil \frac{p}{6} \right\rceil + 1, & \text{if } p \equiv 0, 3, 4, 5 \pmod{6} \end{cases}$.

(viii) For a graph $G = D_{r,s}, \gamma_{M_X}(G) = 2$.

(ix) Let $G = K_{1,p-1}$ be a star graph. Then $\gamma_{M_X}(G) = 2$.

(x) [3] Let $G$ be a tree of diameter 3. Then $\gamma_X(G) \leq p - \Delta(G)$.

2. Characterisation Theorems for Bipartite Graph

**Theorem 2.1** Let $G$ be a connected bipartite graph with $p$ vertices. Then $\gamma_{M_X}(G) = 2$ if and only if $G_1 = K_{m,n}, m \leq n$, a path $G_1 = P_1, i \leq 8$ and $G_3 = B_{XY}$ such that $|N[u_1] \cup N[v_1]| \geq \frac{p}{2}$ and $d(u_1, v_1) = 1$, where $u_1 \in V_1(G)$ and $v_1 \in V_2(G)$.

**Proof:**

Let $\gamma_{M_X}(G) = 2$. Then $\chi(G) = 2 = \chi(<S>)$, where $S$ is a majority dom-chromatic set of $G$ with $|S| = 2$.

**Case:** (i) Suppose $diam(G) = 1$ then the graph $G = K_p$. Since $K_p$ is vertex color critical, $\gamma_{M_X}(G) = p$. By assumption (1), the only graph $G = K_2 = K_{1,1}$ is complete bipartite.

**Case:** (ii) Suppose $diam(G) = 2$ then the graph $G$ becomes $K_{m,n}, m \leq n$, $P_3$ and $K_{1,p-1}$, a star. Since $\gamma_{M_X}(G) = 2$, by the result(1.2), we obtain the graphs which have a structures as $G_1 = C_4 = K_{2,2}$ and $G_1 = K_{1,p-1}, G_2 = P_3$ and also $G_3 = B_{XY}$ includes the following structure with $diam(G) = 2$. 


For $G_3, S = \{u_2, v_2\} \subseteq V(G)$ such that $d(u_2, v_2) = 1, |N[S]| = |N[u_2] \cup N[v_2]| \geq \left\lceil \frac{p}{2} \right\rceil$ and $\chi(\leq S >) = 2 = \chi(G)$. It implies that $S$ is a majority dom-chromatic set of $G_3$. Hence $G_3 = B_{X,Y}$ with these properties.

**Case: (iii)** Suppose $\text{diam}(G) = 3$. The bipartite graph $G$ becomes $P_4$ and $D_{r,s}$, a double star. Since $\gamma'_{M\chi}(G) = 2$, by the result (1.2)(vii), $\gamma'_{M\chi}(P_4) = 2$. Hence $G_2 = P_4$. In $D_{r,s}$, $r \leq s$, by assumption (1), $S = \{u_1, v_1\}$ is the subset of $G$ such that $d(u_1) \leq \left\lfloor \frac{p}{2} \right\rfloor - 1$, and $d(v_1) \geq \left\lceil \frac{p}{2} \right\rceil - 1$ with $d(u_1, v_1) = 1$, where $u_1 \in V_1(G)$ and $v_1 \in V_2(G)$ and $|N[S]| = |N[u_2] \cup N[v_2]| \geq \left\lceil \frac{p}{2} \right\rceil$. Also $\chi(\leq S >) = 2 = \chi(G)$. Hence $S$ is a majority dom-chromatic set of $G$. It implies that $G_2 = B_{X,Y} = D_{r,s}, r \leq s$.

**Case: (iv)** Suppose $\text{diam}(G) \geq 4$. Then the bipartite graphs are $P_p, p \geq 5$ and any bipartite graph $B_{X,Y}$. By the result (1.2)(vii), $\gamma'_{M\chi}(P_p) = \left\lceil \frac{p}{6} \right\rceil = 2$, $p = 5, 6, 7, 8$ and $\gamma'_{M\chi}(P_p) > 2, if p \geq 9$. Since $\gamma'_{M\chi}(G) = 2$, the only bipartite graph $G_2 = P_5$ to $P_8$. For a bipartite graph $B_{X,Y}$, if $S = \{u_1, v_1\}u_1 \in V(G)$ such that $|N[u_1] \cup N[v_1]| \geq \left\lceil \frac{p}{2} \right\rceil$ and $d(u_1, v_1) = 1$ where $u_1 \in V_1(G)$ and $v_1 \in V_2(G)$ with $\text{diam}(G) = 4$, then $S$ is a majority dom-chromatic set of $B_{X,Y}$. Also clearly $\chi(\leq S >) = 2 = \chi(G)$ and satisfies the assumption (1). Hence the bipartite graph $G_3 = B_{X,Y}$ with the above said properties and also the only bipartite graphs are $G_2 = P_5$ to $P_8$.

Conversely, let $G = K_{m,n}$, $m \leq n$ which is complete bipartite with $p = m + n$. By the result (1.2)(v) and (vii), $\gamma'_{M\chi}(G_1) = 2$ and for a path $\gamma'_{M\chi}(P_i) = 2$, $i = 2, \ldots, 8$. Let $G_3 = B_{X,Y}$ be a graph with bipartition $V_1(G)$ and $V_2(G)$. Let $u_1 \in V_1(G)$ and $v_1 \in V_2(G)$ such
that $d(u_1, v_1) = 1$. Since $|N[u_1] \cup N[v_1]| \geq \frac{p}{2}$ and $\chi(< S >) = 2 = \chi(G)$. Hence $S = \{u_1, v_1\}$ is a majority dom-chromatic set of $G$ and $\gamma_{mX}(G_3) = 2$.

**Proposition: 2.2** Let $G$ be any bipartite graph $B_{X,Y}$ with $p$ vertices and without isolates. Then $\gamma_{mX}(G) \leq \left\lfloor \frac{p}{4} \right\rfloor + 1$ and $\gamma_{mX}(G) = \left\lfloor \frac{p}{4} \right\rfloor + 1$ if and only if $G = K_{1,j}$, $j = 1, 2, 3$, $K_{2,2}$, $P_4$ and $mK_2$, $m \geq 1$.

**Proof:** Let $G = B_{X,Y}$ be a bipartite graph with $\{u_1, u_2, \ldots, u_m\}$ and $\{v_1, v_2, \ldots, v_n\}$ and $|V(G)| = p = m + n$.

**Case:** (i) Suppose $G = K_{m,n}$, is a complete bipartite with $m \leq n$. Let $S = \{u_1, v_1\}$, where $u_1 \in V(X)$ and $v_1 \in V(Y)$. Then $|N[S]| = |N[u_1]| + |N[v_1]| = (n + 1) + (m + 1) \geq \left\lfloor \frac{p}{2} \right\rfloor$. Therefore $S$ is a majority dominating set of $G$. Since $G$ is complete bipartite, $\chi(G) = 2 = \chi(< S >)$. It implies that $S$ is a majority dom-chromatic set of $G$. Hence $\gamma_{mX}(G) \leq |S| = 2 = \left\lfloor \frac{p}{4} \right\rfloor + 1$, where $p = 2, 3, 4$. Thus the graph becomes $G = K_{1,1}$, $K_{1,2}$, $K_{1,3}$ and $K_{2,2}$. When $p \geq 5$, for $G = K_{m,n}$, $m \leq n$, by the result (1.2)(v), $\gamma_{mX}(G) = 2 < \left\lfloor \frac{p}{4} \right\rfloor + 1$. Hence, $\gamma_{mX}(G) \leq \left\lfloor \frac{p}{4} \right\rfloor + 1$, for $G = K_{m,n}$, $m \leq n$.

**Case:** (ii) The graph $G$ is not complete and connected bipartite.

Then the minimally connected bipartite graph is a path $P_p$, $p \geq 2$. By known result (1.2)(vii), $\gamma_{mX}(P_p) = \left\lfloor \frac{p}{6} \right\rfloor$ or $\left\lfloor \frac{p}{6} \right\rfloor + 1$. Hence in this structure, when $p = 2, 3, 4$,

$\gamma_{mX}(G) = 2 = \left\lfloor \frac{p}{6} \right\rfloor + 1 = \left\lfloor \frac{p}{4} \right\rfloor + 1$. When $p \geq 5$, $\gamma_{mX}(G) = \left\lfloor \frac{p}{6} \right\rfloor$ or $\left\lfloor \frac{p}{6} \right\rfloor + 1 < \left\lfloor \frac{p}{4} \right\rfloor + 1$.

Hence, $\gamma_{mX}(G) \leq \left\lfloor \frac{p}{4} \right\rfloor + 1$, if $p \geq 2$.

**Case:** (iii) The graph $G$ is not complete and disconnected bipartite.

Then the graph structure becomes $mK_2$, $mP_4$, $mC_4$ and $mP_6$. In such cases, by the result (1.2)(iii), $\gamma_{mX}(mK_2) = \left\lfloor \frac{p}{4} \right\rfloor + 1$ and all other graphs the majority dom-chromatic number is $\gamma_{mX}(G) < \left\lfloor \frac{p}{4} \right\rfloor + 1$. Hence $\gamma_{mX}(G) \leq \left\lfloor \frac{p}{4} \right\rfloor + 1$. From the above cases, we obtain $\gamma_{mX}(G) \leq \left\lfloor \frac{p}{4} \right\rfloor + 1$.

Conversely, let $\gamma_{mX}(G) = \left\lfloor \frac{p}{4} \right\rfloor + 1$. By case (i), if $G$ is a complete bipartite graph, we obtain the graphs $G = K_{1,j}$, $j = 1, 2, 3$ and $K_{2,2}$. By case (ii), if $G$ is not complete bipartite then the graphs are $G = P_2$, $P_3$, $P_4 = K_{1,1}, K_{1,2}, P_4$. Also by case (iii), if $G$ is not complete
and disconnected bipartite, the graph $G = mK_2$, $m \geq 1$. Hence $\gamma_{M\chi}(G) = \left\lceil \frac{p}{4} \right\rceil + 1$ if and only if $G = K_{1,j}, j = 1, 2, 3, K_{2,2}, P_4$ and $mK_2, m \geq 1$.

**Proposition: 2.3** Let $G$ be any connected bipartite graph with $p$ vertices. Then $\gamma_{M\chi}(G) = \left\lfloor \frac{p}{2} \right\rfloor$ if and only if $G = P_3, P_4, C_4$ and $K_{1,3}$.

**Proof:** Assume that $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$. Since $G$ is connected bipartite graph, $\chi(G) \geq 2$.

**Case:** (i) If $\text{diam}(G) = 1$, then $G = K_2$ and $\gamma_{M\chi}(G) = 2 = p$, which is a contradiction to the assumption (1). Hence $G \neq K_2$.

**Case:** (ii) If $\text{diam}(G) = 2$, then $G = P_3, C_4, K_{1,n}$. By the result (1.2)(vi), $\gamma_{M\chi}(P_3) = 2 = \left\lceil \frac{p}{2} \right\rceil$. By the result (1.2)(vi), $\gamma_{M\chi}(C_4) = \left\lfloor \frac{p}{2} \right\rfloor$. Suppose $G = K_{1,3}$, by the result(1.2)(ix), $\gamma_{M\chi}(G) = 2 = \left\lceil \frac{p}{2} \right\rceil$.

**Case:** (iii) If $\text{diam}(G) = 3$, then $G = P_4$ and $D_{r,s}$. By the result (1.2)(vii), $\gamma_{M\chi}(G) = 2 = \left\lceil \frac{p}{2} \right\rceil$. In $D_{r,s}$, by the result (1.2)(viii), $\gamma_{M\chi}(G) = 2$. The condition (1) holds when $r = s = 1$.

**Case:** (iv) If $\text{diam}(G) \geq 4$, then $G = P_p, C_p, p \geq 5$ and any other graphs. By the result (1.2)(vii), $\gamma_{M\chi}(G) = \left\lceil \frac{p}{6} \right\rceil + 1 = 2 < \left\lceil \frac{p}{2} \right\rceil$, which is a contradiction to the condition (1).

Thus from the above four cases, $G$ must be $P_3, P_4, C_4$ and $K_{1,3}$.

The converse is obvious.

**Proposition: 2.4** Suppose $G$ is a disconnected bipartite graph. If the graph structures are $G_1 = K_{1,3} \cup mK_2, m$ is even and $m \geq 2$, $G_2 = mP_p, m = 4, p = 3$ and $G_3 = mK_{1,3}, m = 3$ then $\gamma_{M\chi}(G) = \frac{p}{4}$.

**Corollary: 2.5** Let $G$ be a disconnected bipartite graph. If the graph structure is $K_{1,3} \cup mK_2, m$ is odd then $\gamma_{M\chi}(G) = \frac{p}{4} + 1$.

**Proposition: 2.6** Let $G$ be a disconnected bipartite graph without isolates. Then $\gamma_{M\chi}(G) = \frac{p}{2}$ if and only if $G = mK_2, 1 < m \leq 3$.

**Proof:** Let $\gamma_{M\chi}(G) = \frac{p}{2}$.

\[\begin{align*}
\gamma_{M\chi}(G) &= \frac{p}{2} \\
\text{Proof:} & \quad \text{Let} \quad \gamma_{M\chi}(G) = \frac{p}{2}
\end{align*}\]
Since $G$ be a disconnected bipartite graph, let $G_1, G_2, \ldots, G_k$ are the components of $G$ and $V(G) = V(G_1) \cup \cdots \cup V(G_k)$.

**Case (i)**: All components are of diameter 1. Then the graph $G = mK_2$. By the assumption \((1)\), when $G = mK_2$ if $m = 2$ and $3$ then $G = 2K_2$ and $3K_2$. It implies that $\gamma_{M\chi}(G) = 2$ and $3 = \frac{p}{2}$. Suppose $m \geq 4$, then by the result \((1.2)(iii)\), $\gamma_{M\chi}(G) = \left\lceil \frac{p}{4} \right\rceil + 1 < \frac{p}{2}$. It is a contradiction to the assumption \((1)\).

**Case (ii)**: Suppose $G$ contains the components which are of diameter 1 and 2.

Then $G = K_{1,t} \cup mK_2$, where $G_1 = K_{1,t}$, $G_2 = mK_2$ and $V(G) = \{u, u_1, \ldots, u_t, v_1, \ldots, v_{2m}\}$ with $p = 1 + t + 2m$.

**subcase**: (i) If $|t| \geq \left\lceil \frac{p}{2} \right\rceil - 1$ and $2m = p - \left(\left\lceil \frac{p}{2} \right\rceil - 1\right)$ then the majority dom-chromatic set $S = \{u, u_1\}$ where $u, u_1 \in V(G_1)$ such that $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$ and $\chi(G_1) = 2 = \chi(\langle S \rangle)$. It implies that $S$ is a majority dom-chromatic set of $G$ and $\gamma_{M\chi}(G) = 2 < \frac{p}{2}$, if $|t| \geq \left\lceil \frac{p}{2} \right\rceil - 1$, which is a contradiction to \((1)\). Therefore $G \neq K_{1,t} \cup mK_2$.

**subcase** : (ii) If $|t| \leq \left\lceil \frac{p}{2} \right\rceil - 2$ then the MDC-set $S = \{u, u_1, v_1, v_2, \ldots, v_k\}$, where $|k| = \left\lceil \frac{p}{2} \right\rceil - (1 + t)$ such that $|N[S]| = 1 + t + 2k \geq \left\lceil \frac{p}{2} \right\rceil$. Also $\chi(G) = 2 = \chi(\langle S \rangle)$. Hence $\gamma_{M\chi}(G) = |S| = (2 + k) < \frac{p}{2}$, it is a contradiction to \((1)\). Hence the graph $G \neq K_{1,t} \cup mK_2$.

**Case (iii)**: If the components $G_i$ of $G$ with $diam(G_i) \geq 2, i = 1, 2, \ldots, k$ then $\gamma_{M\chi}(G) < \frac{p}{2}$. From the above cases, we get the graph structures become $G = mK_2$, $1 < m \leq 3$.

Conversely, let $G = mK_2$, $m \leq 3$. Then by the result \((1.3)(iii)\), $\gamma_{M\chi}(G) = \left\lceil \frac{p}{4} \right\rceil + 1 = \frac{p}{2}$.

**Proposition : 2.7** Let $G$ be a disconnected graph which is not bipartite with isolates. Then $\gamma_{M\chi}(G) \leq \left\lceil \frac{p}{2} \right\rceil$ and $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$ if and only if $G = pK_1$.

**Proposition : 2.8** For a disconnected graph with $p$ vertices, $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$ if and only if $G_1 = mK_2, m = 2, 3$ and $G_2 = K_t \cup (p - t)K_1$, where $K_t$ is a complete graph of $t$ vertices with $|t| \leq \left\lceil \frac{p}{2} \right\rceil$. 


Proof: Let $G$ be a disconnected graph $p$ vertices. Suppose $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$, then $S$ is a majority dom-chromatic set with $\left\lceil \frac{p}{2} \right\rceil$ vertices. Also the chromatic number of the induced subgraph $\langle S \rangle$ and the graph $G$ are equal.

Case (i): The graph $G$ without isolates. Then $G = mK_2, m \geq 2$. By the result (1.2)(iii), $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil + 1$. It implies that when $G = 2K_2, 3K_2$, $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$. If $G = mK_3$ or $G = mP_3$ then $\gamma_{M\chi}(G) < \left\lceil \frac{p}{2} \right\rceil$. If each components of $G$ such as $mK_2, m \geq 4$, $mK_t, mP_t, t \geq 3$ then $\gamma_{M\chi}(G) < \left\lceil \frac{p}{2} \right\rceil$. Hence the graph $G_1 = mK_2, m = 2, 3$.

Case (ii): The graph $G$ has isolates. Let $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$. Then the majority dom-chromatic set $S$ contains $\left\lceil \frac{p}{2} \right\rceil$ vertices. It implies that, by the result (1.2) (iii), the graph $G = \overline{K_p} = K_1 \cup (p - 1)K_1$.

Subcase: (i) If $diam(G) = 1$ then the components of the given disconnected graph becomes a complete graph with isolates. i.e) $G = K_t \cup (p - t)K_1, t \geq 2$. Since $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$ and $|t| \leq \left\lceil \frac{p}{2} \right\rceil$, the graph structure is $G = K_t \cup (p - t)K_1$, where $K_t$ is the complete graph of $t$ vertices.

Subcase: (ii) If $diam(G) = 2$ then the components of the disconnected graph become $G_1 = P_3 \cup (p - 3)K_1$ or $G_2 = K_{1,t} \cup (p - (t + 1))K_1$ or $G_3 = C_4 \cup (p - 4)K_1$. Then $\gamma_{M\chi}(G_1) < \left\lceil \frac{p}{2} \right\rceil$ and $\gamma_{M\chi}(G_2) < \left\lceil \frac{p}{2} \right\rceil$. In particular, $G_2 = K_{1,1} \cup (p - 2)K_1 = K_2 \cup (p - 2)K_1$ and $\gamma_{M\chi}(G_2) = \left\lceil \frac{p}{2} \right\rceil$. Since $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$ and $|t| \leq \left\lceil \frac{p}{2} \right\rceil$, the majority dom-chromatic set $S$ must contain $\left\lceil \frac{p}{2} \right\rceil$ vertices. Since $G$ is disconnected graph with isolates, anyone component ’$g$’ of $G$ must be vertex color critical with $|V(S)| \neq t \leq \left\lceil \frac{p}{2} \right\rceil$ and other remaining vertices are isolates. Hence the graph $G$ takes the structure $G = K_t \cup (p - t)K_1$ where $K_t$ is a complete graph which is vertex color critical and $(p - t)$ isolates.

Subcase: (iii) Let $diam(G) \geq 3$. Then the disconnected graph becomes $G_1 = P_r \cup (p - r)K_1$ or $G_2 = D_{t_1,t_2} \cup (p - (t_1 + t_2))K_1$, where $P_r$ is a path on $r$ vertices and $D_{t_1,t_2}$ is a double star with $(t_1 + t_2)$ vertices. The majority dom-chromatic number of these graphs $G_1$ and $G_2$ is $\gamma_{M\chi}(G) < \left\lceil \frac{p}{2} \right\rceil$. Since $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$, $G$ must have a vertex color critical component ’$g$’ and isolates. Hence $|V(S)| = t \leq \left\lceil \frac{p}{2} \right\rceil$ and $(p - t)$ isolates. Hence the only graph structure $G = K_t \cup (p - t)K_1$, where $K_t$ is the complete graph of $t$ vertices and $|t| \leq \left\lceil \frac{p}{2} \right\rceil$. 


Conversely, let \( G = K_t \cup (p - t)K_1 \), where \(|t| \leq \left\lceil \frac{p}{2} \right\rceil\). Since \( K_t \) is the complete graph, it is a vertex color critical. Then by result (1.2) (iv), \( \gamma_{M\chi}(G) = p \). If \(|t| = \left\lceil \frac{p}{2} \right\rceil\), the graph \( G = K_{\left\lceil \frac{p}{2} \right\rceil} \cup \left( \left\lceil \frac{p}{2} \right\rceil K_1 \right) \) and \( \gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil\). If \(|t| < \left\lceil \frac{p}{2} \right\rceil\), then \(|t| = \left\lfloor \frac{p}{2} \right\rfloor\). The graph \( G \) becomes \( G = K_{\left\lfloor \frac{p}{2} \right\rfloor} \cup \left( p - \left\lfloor \frac{p}{4} \right\rfloor \right) K_1 \). The majority dom-chromatic number \( \gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil + \left( \frac{p}{2} - \left\lfloor \frac{p}{4} \right\rfloor \right) \). Suppose \(|t| > \left\lceil \frac{p}{2} \right\rceil\), then \( G = K_{t'} \cup (p - t')K_1 \), where \(|t'| > |t|\). Since \( K_{t'} \) is a complete graph with \( t' \) vertices, \( \gamma_{M\chi}(G) = t' > t = \left\lceil \frac{p}{2} \right\rceil\). Hence for a disconnected graph with isolates and \(|t| \leq \left\lceil \frac{p}{2} \right\rceil\), \( \gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil\).

3. \( \gamma_{M\chi} \) for complement of a graph \( G \)

**Proposition: 3.1** Let the bipartite graph \( G \) with \( diam(G) = 3 \). Then \( \gamma_{M\chi}(G) = \gamma_{M\chi}(\bar{G}) \) if and only if \( G = P_4 \), where \( \bar{G} \) is the complement of \( G \).

**Proof:** Let the equality holds and \( uv \) be the dominating edge of \( G \). Let \(|N[u]| = m, |N[v]| = n \) and \( p = m + n \). In the graph \( \bar{G} \), both \( N(u) \) and \( N(v) \) are of cardinality 2. The set \( \{N(u) \cup N(v)\} \) is a \( K_{m+n-2} \) graph, \( \chi(\bar{G}) = m + n - 2 \) and \( \{N(u) \cup N(v)\} \) be the majority dom-chromatic set for \( \bar{G} \Rightarrow \gamma_{M\chi}(\bar{G}) = m + n - 2 \). Since \( \gamma_{M\chi}(G) = \gamma_{M\chi}(\bar{G}) \), \( \frac{m+n}{2} = m + n - 2 \). It implies that \( m + n = 4 \). Hence the graph must be \( P_4 \) and \( C_4 \). The converse is obvious.

**Proposition: 3.2** If the graph \( G = K_p \) is the vertex color critical graph then \( 1 \leq \gamma_{M\chi}(\bar{G}) \leq \left\lceil \frac{p}{2} \right\rceil \).

**Proof:** Since the complete graph \( G = K_p \) is the vertex color critical graph, \( 1 \leq \gamma_{M\chi}(G) \leq p \). The complement of \( K_p \) is \( \bar{G} = K_p \). By the result (1.2)(ii), the majority dom-chromatic number is \( \gamma_{M\chi}(\bar{G}) = \left\lceil \frac{p}{2} \right\rceil \). And the lower bound attains for \( \bar{G} = K_2 \). Hence the result.

**Proposition: 3.3** Let \( G = K_{m,n}, m \leq n \) and \( m, n \geq 3 \) be a complete bipartite graph. Then majority dom-chromatic number of a complement \( \bar{G} \) is \( \gamma_{M\chi}(\bar{G}) \geq \left\lfloor \frac{p}{2} \right\rfloor \) and \( \gamma_{M\chi}(G) < \gamma_{M\chi}(\bar{G}) \).

**Proof:** Let \( \bar{G} = K_m \cup K_n \) be the complement of \( G \) where \( K_m \) and \( K_n \) both are complete graphs with \( m \) and \( n \) vertices.

**Case:** (i) Suppose \( m = n, n + 1, n + 2 \). Since \( K_m \) and \( K_n \) are vertex color critical and \( p = m + n, \gamma_{M\chi}(\bar{G}) = n \) or \( n + 1 \) and \( \gamma_{M\chi}(\bar{G}) = n + 2 \). Hence \( \gamma_{M\chi}(\bar{G}) = max\{m, n\} \).
Case: (ii) Let \( m < n \) and \( n \geq m + 3 \). Since \( K_m \) and \( K_n \) are vertex color critical and \( p = m + n, m < \left\lceil \frac{p}{2} \right\rceil \) and \( n > \left\lfloor \frac{p}{2} \right\rfloor \). Hence \( \gamma_{Mx}(\tilde{G}) = \max\{m, n\} \). If \( G = K_{m,n} \), \( m \leq n \), then by the result (1.2) (v), \( \gamma_{Mx}(G) = 2 \). By case (i), \( \gamma_{Mx}(\tilde{G}) = n \) or \( n + 1 = \left\lceil \frac{p}{2} \right\rceil \) and \( \gamma_{Mx}(\tilde{G}) = n + 2 > \left\lceil \frac{p}{2} \right\rceil \). By case (ii), \( \gamma_{Mx}(\tilde{G}) = n \), if \( m < n \). It implies that \( \gamma_{Mx}(\tilde{G}) > \left\lceil \frac{p}{2} \right\rceil \). Hence, \( \gamma_{Mx}(\tilde{G}) < \gamma_{Mx}(\tilde{G}) \), if \( m, n \geq 3 \).

**Proposition: 3.4** Let \( G \) be a bipartite graph with \( \text{diam}(G) \geq 6 \). Then \( \gamma_{Mx}(\tilde{G}) \geq \gamma_{M}(\tilde{G}) + 1 \), if \( \tilde{G} \) is the complement of \( G \) and \( \gamma_{M}(\tilde{G}) \) is the majority dominating number of \( \tilde{G} \).

**Proof:** If \( \text{diam}(G) \geq 6 \), then \( G = P_p, p \geq 7 \). The complement \( \tilde{G} \) contains two vertices with degree \( d( u_i) = p - 2, i = 1, p \) and \( d( v_i) = p - 3, i = 2, \ldots, p - 1 \). It gives that there are atleast two vertices with degree \( d( u_i) \geq \left\lceil \frac{p}{2} \right\rceil - 1 \) and the majority dominating number of \( \tilde{G} \) is \( \gamma_{M}(\tilde{G}) = 1 \). Since \( \tilde{G} \) contains a triangle, \( \chi(\tilde{G}) = 3 \) and \( \gamma_{Mx}(\tilde{G}) \geq 3 \). Hence, \( \gamma_{Mx}(\tilde{G}) \geq \gamma_{M}(\tilde{G}) + 1 \).

4. Bounds of \( \gamma_{Mx}(G) \)

**Proposition : 4.1** If \( G \) is a vertex color critical and a non-trivial connected graph with \( p \geq 2 \) then \( 2 \leq \gamma_{Mx}(G) \leq p \). These bounds are sharp.

**Proof:** Since \( G \) is connected and non-trivial graph with \( p \geq 2 \), \( \chi(G) \geq 2 \) and \( \gamma_{Mx}(G) \geq 2 \). Also since \( G \) is a vertex color critical graph, by known result (1.2)(iv), \( \gamma_{Mx}(G) = p \). Hence \( 2 \leq \gamma_{Mx}(G) \leq p, p \geq 2 \). When \( G = K_2 \) and \( G = K_p \), the lower and upper bounds are sharp.

**Proposition: 4.2** Let \( G \) be a connected bipartite graph with \( p \) vertices. Then \( \gamma_{Mx}(G) = p \) if and only if \( G = K_p \), \( p = 2 \).

**Proof:** Let \( G \) be a connected bipartite graph with \( p \) vertices. Since \( \gamma_{Mx}(G) = p \), then the graph must be a vertex color critical. The only connected bipartite vertex color critical graph is \( K_2 \). It implies that \( G = K_2 \). The converse is obvious.

**Proposition: 4.3** If \( G \) be a graph of \( \text{diam}(G) = 3 \) then \( \gamma_{Mx}(G) = 2 \) and \( \gamma_{Mx}(G) = \gamma_{M}(G) + 1 \).

**Proof:** Let \( G \) be a connected graph and \( \text{diam}(G) = 3 \). Then the graph \( G \) has the structure with two central vertices \( u \) and \( v \) which are adjacent with some pendants. Then \( G = P_4 \) and \( G = D_{r,s}, r \leq s \) where \( r \) and \( s \) number of pendants at \( u \) and \( v \) respectively. Then by result (i) 1.2), \( \gamma_{M}(G) = |\{v\}| = 1 \).
Case: (i) If \( s = r, r + 1, r + 2 \) then both \( u \) and \( v \) are adjacent to some number of pendant vertices. Since \( \chi(G) = 2 \), \( S = \{u, v\} \) be the majority dom-chromatic set of \( G \) and \( \gamma_{M\chi}(G) = |S| = 2 \). Hence \( \gamma_{M\chi}(G) = \gamma_M(G) + 1 \).

Case: (ii) If \( r < s \) and \( s \geq r + 3 \). Choose \( S = \{u, v\} \), where \( u \) and \( v \) are central vertices of \( G \). Then \( |N[S]| = d(u) + d(v) = r + s + 2 = p > \left\lceil \frac{p}{2} \right\rceil \).

Therefore, \( S \) is majority dominating set of \( G \). Also \( \chi(G) = 2 = \chi(<S>) \).

Hence \( S \) will be the majority dom-chromatic set of \( G \) and \( \gamma_{M\chi}(G) = |S| = 2 \). Since \( \gamma_M(G) = 1, \gamma_{M\chi}(G) = \gamma_M(G) + 1 \). This result is true for \( G = P_4 \).

**Proposition 4.4** Let \( G \) be a bipartite graph of \( diam(G) \leq 5 \). Then \( \gamma_{M\chi}(G) = 2 \) and \( \gamma_{M\chi}(G) = \gamma_M(G) + 1 \).

**Proof:** Since the graph \( G \) is bipartite, the graph structures are \( P_p, p \leq 6 \), \( K_{1,n} \), \( C_4 \) and \( K_2 \).

- **Case:** (i) Suppose \( diam(G) = 1 \), then the bipartite graph \( G \) becomes only \( K_2 \). By result [5], \( \gamma_M(G) = 1 \) and \( \chi(G) = 2 \) and by result (1.2)(iv), \( \gamma_{M\chi}(G) = 2 \). Hence \( \gamma_{M\chi}(G) = \gamma_M(G) + 1 \).

- **Case:** (ii) If \( diam(G) = 2 \), then the graph structures be \( G = P_3 \) or \( K_{1,n} \). By the result (1.2)(i), \( \gamma_M(G) = 1 \). Also by result (1.2)(vii), \( \gamma_{M\chi}(G) = 2 \). In both graphs, \( \gamma_{M\chi}(G) = \gamma_M(G) + 1 \).

- **Case:** (iii) Let \( diam(G) = 3 \). Then the graph becomes \( G = P_4 \) or and \( D_{r,s} \). By Proposition (4.3), the result is true.

- **Case:** (iv) when \( diam(G) = 4 \) and \( 5 \), the bipartite graph is \( P_p, p \leq 6 \). By the result (1.2)(i), \( \gamma_M(G) = 1 \). Since \( \chi(G) = 2 \), the set \( \{v_2, v_3\} \) be the majority dom-chromatic set of \( G \), where \( v_2, v_3 \in V(P_5) \). Hence \( \gamma_{M\chi}(G) = 2 = \gamma_M(G) + 1 \).

Hence, for all cases, \( \gamma_{M\chi}(G) = \gamma_M(G) + 1 \).

**Proposition 4.5** Let \( G \) be a bipartite graph with \( diam(G) \geq 6 \). Then

(i) \( \gamma_{M\chi}(G) = \gamma_M(G) \), if \( p = 1, 2 \) \((mod \ 6)\)

(ii) \( \gamma_{M\chi}(G) = \gamma_M(G) + 1 \), if \( p = 0, 3, 4, 5 \) \((mod \ 6)\).

**Proof:** If the bipartite graph \( G \) with \( diam(G) \geq 6 \), then \( G = P_p \), a path with \( p > 6 \). By the result (1.2)(i), \( \gamma_M(G) = \left\lfloor \frac{p}{6} \right\rfloor \), for all \( p \geq 7 \) and by the result (1.2)(vii),
\[
\gamma_{M_X}(G) = \begin{cases} 
\left\lceil \frac{p}{6} \right\rceil = \gamma_M(G) & , \text{if } p \equiv 1,2 \pmod{6} \\
\left\lceil \frac{p}{6} \right\rceil + 1 = \gamma_M(G) + 1 , \text{if } p \equiv 0,3,4,5 \pmod{6}.
\end{cases}
\]

Hence the result.

**Proposition 4.6** Let \( G \) be a 3-regular bipartite graph with \( p \) vertices. Then

\[
\gamma_{M_X}(G) = \begin{cases} 
\left\lfloor \frac{p}{8} \right\rfloor , \text{if } p \equiv 2,4 \pmod{8} \\
\left\lfloor \frac{p}{8} \right\rfloor + 1 , \text{if } p \equiv 0,6 \pmod{8}.
\end{cases}
\]

**Proof:** Let \( V_1(G) = \{v_1, v_2, \ldots, v_{\frac{p}{2}}\} \) and \( V_2(G) = \{u_1, u_2, \ldots, u_{\frac{p}{2}}\} \) with \( p = 2m \).

**Case: (i)** Let \( p \equiv 2,4 \pmod{8} \). Let \( S = \{v_1, u_1, v_j, v_{j+1}, \ldots, v_t\} \) be the subset of \( G \) with \( |S| = t = \gamma_{M_X}(G) \) such that \( d(v_1, u_1) = 1 \) and \( d(v_i, u_j) \geq 4 \). Then

\[
|N[S]| = |N[v_1] + N[u_1]| + \sum_{j=1}^{t-2} d(u_j) - (t-2) = 6 + 4(t-2) = 4t - 2
\]

\[
\geq \left\lfloor \frac{p}{2} \right\rfloor. \text{ Let } p = 8r + 2. \text{ Then } |N[S]| = 4t - 2 = 4 \left\lfloor \frac{p}{8} \right\rfloor - 2 = 4 \left( \frac{8r+2}{8} \right) - 2 = \frac{p}{2} - 2 + 2 > \left\lfloor \frac{p}{2} \right\rfloor. \text{ Let } p = 8r + 4. \text{ Then } |N[S]| = 4t - 2 = 4 \left\lfloor \frac{p}{8} \right\rfloor - 2 = 4 \left( \frac{8r+4}{8} \right) - 2 = \frac{p}{2} - 2 + 2 > \left\lfloor \frac{p}{2} \right\rfloor. \text{ Since } d(v_1, u_1) = 1, \text{ the induced subgraph } <S> \text{ contains } K_2 \text{ and } \chi(<S>) = 2 = \chi(G). \text{ Thus } S \text{ is a majority dom-chromatic set of } G \text{ and } \gamma_{M_X}(G) \leq |S| = \left\lfloor \frac{p}{8} \right\rfloor.
\]

(1)

Suppose that \( S = \{v_1, u_1, v_j, \ldots, v_t\} \) with \( |S| = t = \gamma_{M_X}(G) \) such that \( d(v_1, u_1) = 1 \), \( d(v_i, v_j) \geq 4 \) and \( |N[S]| \geq \left\lfloor \frac{p}{2} \right\rfloor \). Since \( S \) contains the induced subgraph \( K_2 \) and \( \chi(<S>) = 2 = \chi(G) \). Therefore \( |N[S]| \leq 4t = 4\gamma_{M_X}(G) \). Since \( |N[S]| \geq \left\lfloor \frac{p}{2} \right\rfloor \), \( \left\lfloor \frac{p}{2} \right\rfloor \leq 4\gamma_{M_X}(G) \). It implies that \( \gamma_{M_X}(G) \geq \frac{1}{4} \left\lfloor \frac{p}{2} \right\rfloor \).

Hence \( \gamma_{M_X}(G) \geq \left\lfloor \frac{p}{8} \right\rfloor \).

(2)

Combining (1) and (2), \( \gamma_{M_X}(G) = \left\lfloor \frac{p}{8} \right\rfloor \), if \( p \equiv 2,4 \pmod{8} \).

**Case: (ii)** Let \( p \equiv 0,6 \pmod{8} \). Let \( S_1 = \{v_1, u_1, v_j, \ldots, v_t\} \) be the subset of \( V(G) \) with \( |S_1| = t_1 = \left\lfloor \frac{p}{8} \right\rfloor + 1 = \gamma_{M_X}(G) \) and \( \chi(<S_1>) = 2 \). Let \( p = 8r \). Then \( |N[S_1]| = 4t - \)
2 = 4 \left(\frac{p}{8} + 1\right) - 2 = 4 \left(\frac{8r}{8} + 1\right) - 2 = 4 \left(\frac{p}{8}\right) + 2 > \frac{p}{2} + 2 > \left[\frac{p}{2}\right]. \text{ Let } p = 8r + 6. \text{ Then } |N[S_1]| = 4t_1 - 2 = 4 \left(\frac{p}{8}\right) + 1 - 2 = 4 \left(\frac{8r + 6}{8} + 1\right) - 2 = 4 \left(\frac{p}{8}\right) + 2 > \frac{p}{2} + 2 > \left[\frac{p}{2}\right]. \text{ Hence } |N[S_1]| \geq \frac{p}{2}. \text{ Therefore } S_1 \text{ is a majority dom-chromatic set of } G \text{ and } \gamma_{M_{\chi}}(G) \leq |S_1| = t_1 = \frac{p}{8} + 1. \text{ Applying the same argument as in case (i), } \gamma_{M_{\chi}}(G) \geq \frac{p}{8} + 1. \text{ Hence } \gamma_{M_{\chi}}(G) = \frac{p}{8} + 1, \text{ if } p \equiv 0, 6 \pmod{8}.

**Proposition: 4.6** If the graph $G$ is a bipartite with $diam(G) \leq 2$ then $\gamma_{M_{\chi}}(G) \leq p - \Delta(G) + 1$ and $\gamma_{M_{\chi}}(G) = p - \Delta(G) + 1$ if and only if $G = K_2, P_3$ and $K_{1,p-1}, p \geq 2$.

**Proof:** Let $G$ be a bipartite graph with $diam(G) \leq 2$. If $\Delta(G) = 1$, the graph $G$ becomes $K_2$. By the result (i), $\gamma_{M_{\chi}}(G) = 2 = p - \Delta(G) + 1$, if $G = K_2$. If $\Delta(G) = 2$, the graph structures becomes $P_p$, a path and $K_{2,2}$. Since $diam(G) \leq 2$, if $G = P_3$, by the result (1.2)(vii), $\gamma_{M_{\chi}}(G) = 2 = p - \Delta(G) + 1$ and $\gamma_{M_{\chi}}(K_{2,2}) = 2 < p - \Delta(G) + 1$. Suppose $\Delta(G) = 3$. Then $G = K_{3,3}$. By the result (1.2)(v), $\gamma_{M_{\chi}}(K_{3,3}) = 2 < p - \Delta(G) + 1$. If $\Delta(G) \geq 4$ then the graph $G$ becomes $K_{m,n}, m = n \geq 4$. By the result (1.2)(v), $\gamma_{M_{\chi}}(G) = 2 < p - \Delta(G) + 1$. This is true for $\Delta(G) = 1, 2, 3, \ldots, (p - 2)$. Suppose $\Delta(G) = p - 1$. Then the only bipartite graph $G = K_{1,p-1}$. By the result (1.2)(ix), $\gamma_{M_{\chi}}(G) = 2 = p - \Delta(G) + 1$. Hence from the above cases, $\gamma_{M_{\chi}}(G) \leq p - \Delta(G) + 1$. Also from the above cases, $\gamma_{M_{\chi}}(G) = p - \Delta(G) + 1$ is true if and only if $G = K_2, P_3$ and $K_{1,p-1}, p \geq 2$.

**Proposition: 4.7** Let $G$ be a bipartite graph with $diam(G) = 3$. Then $\gamma_{M_{\chi}}(G) \leq p - \Delta(G)$. Also $\gamma_{M_{\chi}}(G) = p - \Delta(G)$ if and only if $G = P_4$ and $D_{r,s}, r = 1$ and $s = p - 3$.

**Proof:** Let $G$ be a bipartite graph with $diam(G) = 3$. By the result (1.2)(x), $\chi(G) \leq p - \Delta(G)$. Since $\gamma_{M_{\chi}}(G) \leq \chi(G)$, $\gamma_{M_{\chi}}(G) \leq \chi(G) \leq p - \Delta(G)$. Hence $\gamma_{M_{\chi}}(G) \leq p - \Delta(G)$.

Let $\gamma_{M_{\chi}}(G) = p - \Delta(G)$. \hfill (1)

**Case:** (i) Since $diam(G) = 3$, the graph $G$ has a dominating edge $uv$ with some pendants at $u$ and $v$. Let $V(G) = \{u, v, u_1, \ldots, u_r, v_1, v_2, \ldots, v_s\}$ where $u_i, i = 1, \ldots, r$ and $v_j, j = 1, \ldots, s$ are pendants with $r \leq p - 3$ and $s \geq 1$. Clearly, since $G$ is bipartite, $\chi(G) = 2$. By the assumption (1), $S = \{u, v, u_1, \ldots, u_r\}$ is a majority dom-chromatic set with $|S| = p - \Delta(G) = t$.

**Subcase:** (i) Let $d(u) = p - 2$ and $d(v) = 2$. Since $G$ has a dominating edge $e = uv$, $\gamma_{M_{\chi}}(G) = |S| = 2$. By the assumption (1), $\gamma_{M_{\chi}}(G) = p - \Delta(G)$. It implies that $2 = p - \Delta(G)$.
\[d(u) \implies 2 = p - (p - 2).\] It gives the structure of the graph \(G\) with \(d(u) = p - 2, d(v) = 2,\) and the graph is \(G = D_{r,s}, r < s\) with \(r = 1\) and \(s = p - 3.\)

**Subcase: (ii)** Let \(d(u) \leq p - 3\) and \(d(v) \geq 3.\) The majority dom-chromatic set for the graph \(G\) is \(S = \{u, v\}.\) It implies that \(\gamma_{M}(G) = |S| = 2.\) By the assumption (1), \(\gamma_{M}(G) = p - \Delta(G) = p - d(u) = p - (p - 3) = 3.\) Hence, \(\gamma_{M}(G) < p - \Delta(G)\).

**Subcase: (iii)** If \(d(u) = p - 2\) and \(d(v) = p - 2\) then the majority dom-chromatic set becomes \(S = \{u, v\}.\) It implies that \(\gamma_{M}(G) = |S| = 2.\) By the assumption (1), \(\gamma_{M}(G) = p - \Delta(G) = p - d(u) \implies 2 = p - (p - 2).\) Since \(d(u) = p - 2\) and \(d(v) = p - 2, r = s = 1 \implies p = r + s + 2 = 4.\)

Hence the graph \(G\) with \(p = 4\) vertices and \(diam(G) = 3\) is \(P_{4}.\)

**Case: (ii)** Suppose \(G\) has no dominating edge \(e = uv.\) Then the graph \(G\) is a wounded spider with \(diam(G) = 3\) and the graph contains a vertex \(u\) with \(d(u) = \frac{p}{2}\) and \(d(u_{1}) \leq 2, u_{1} \in (V(G) - \{u\}).\) Hence \(S = \{u, u_{1}\}\) be the majority dom-chromatic set of \(G\) with \(d(u_{1}) = 2,\) where \(d(u, u_{1}) = 1\) and \(\gamma_{M}(G) = |S| = 2.\) By the assumption (1), \(\gamma_{M}(G) = p - \Delta(G) = p - \frac{p}{2} = \frac{p}{2}.\) Hence \(\gamma_{M}(G) < p - \Delta(G)\).

Thus, \(\gamma_{M}(G) = p - \Delta(G)\) if and only if \(G = P_{4}\) and \(D_{r,s}, r = 1\) and \(s = p - 3.\)

5. Conclusion

In this paper, we studied majority dom-chromatic number for a bipartite graph. The characterisation theorems on \(\gamma_{M}(G)\) for bipartite graphs are established and its relationship with other domination parameters are discussed. Some results of a disconnected graph and the majority dom-chromatic number for the complement \(\tilde{G}\) of the graph \(G\) are investigated.

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