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From School Buses to Carpooling: A Multifaceted Approach to Solving Vehicle Routing Problems in People Transportation

Aditya Kumar Sharma

University of Washington

ABSTRACT

This paper delves into the complex realm of vehicle routing problems (VRPs) with a specific focus on the transportation of people, highlighting the nuanced challenges and optimization opportunities distinct from those associated with goods transportation. Among these, the Dial-a-Ride Problem (DARP) is emphasized as a pivotal area of study. DARP caters to providing personalized, accessible transportation services, particularly for individuals unable to use standard transit options, such as the elderly and handicapped. This research meticulously explores the mathematical formulation of DARP, and operational challenges and extends the discourse to encompass related VRPs in people transportation, thereby broadening the scope of investigation. Operational intricacies of DARP include managing time windows for pickups and deliveries, adhering to vehicle capacity limits, and ensuring tailored transportation solutions that optimally satisfy both operational costs and customer convenience. The paper reviews a wide spectrum of heuristic and metaheuristic solution methods developed over the years, tracing the evolution of these algorithms from their inception to the sophisticated techniques currently employed to address DARP's dynamic, static, and stochastic variants. Additionally, the research extends to related problems such as the School Bus Routing Problem (SBRP), which emphasizes the efficient and timely transportation of students, and the Carpooling Problem, focusing on organizing shared rides to reduce vehicle kilometers and enhance mobility. Demand Responsive Transportation (DRT) problems are also discussed, highlighting their importance in areas where fixed-route services are inefficient. Conclusively, the paper discusses the future scope of research in this domain, underscoring the necessity for novel solution concepts to address the growing complexity and scale of transportation demands. The integration of intermodal aspects, combining public transportation with demand-responsive services, and the advancement of hybrid algorithms are identified as key areas for future exploration. This comprehensive study not only contributes to the academic discourse on optimizing vehicle routing for people transportation but also aims to inform the development of more efficient, accessible, and customer-oriented transportation systems.

Keywords: Dial-a-Ride Problem (DARP), vehicle routing problems (VRPs), heuristic and metaheuristic solutions, School Bus Routing Problem (SBRP), Carpooling Problem, Demand Responsive Transportation (DRT)

I. INTRODUCTION

In the realm of modern transportation, the quest for systems that not only enhance efficiency and comfort but also minimize noise, pollution, cost, and delays, is paramount. This pursuit is equally crucial in the transportation of goods and people. While previous discussions have centered on vehicle routing problems (VRPs) for goods, this paper transitions focus towards the nuanced VRPs inherent in the transportation of people. The latter introduces a plethora of optimization challenges, spawned by the diverse nature of applications in people transportation. Among these challenges, the dial-a-ride problem (DARP) emerges as a significant focal point. DARP is a specialized service aimed at providing accessible transportation for those unable to use standard transit options, notably the elderly and handicapped. This chapter not only aims to dissect the mathematical underpinnings of DARP but also to survey the landscape of existing research, thereby laying a comprehensive foundation for understanding this complex issue.

The narrative begins with an introduction to DARP, positioning it as a vital link between the rigid structures of bus systems and the flexible nature of taxi services. By offering door-to-door, multi-occupancy transport, DARP endeavors to fill a critical gap in the transportation ecosystem, ensuring personalized service for its users. Unlike goods transportation, where efficiency and cost minimization are often the primary goals, DARP places a premium on service quality and customer satisfaction. This distinction underscores the necessity of including customer satisfaction metrics in planning and optimization models for DARP. Furthermore, the paper delves into the operational intricacies of DARP, which involve intricate planning to address various customer requests under stringent constraints. These include managing time windows for pickups and deliveries, adhering to vehicle capacity limits, and ensuring that each route optimally satisfies both operational costs and customer convenience. The complexity of these problems is amplified by the requirement to serve each customer with a tailored transportation solution that adheres to their specific needs and constraints. Additionally, this research paper expands upon the DARP by exploring related VRPs in people transportation, thus broadening the scope of investigation beyond the initial focus. This includes a detailed analysis of the mathematical formulation of DARP, a review of prior research endeavors, and a discussion on emerging challenges and solutions within this domain. By examining the broader spectrum of VRPs related to people transportation, the paper aims to shed light on the multifaceted challenges and opportunities that define this critical field. In essence, this paper sets the stage for a deep dive into the complexities of optimizing vehicle routing for people transportation, with a keen eye on improving service quality, efficiency, and accessibility. Through a meticulous exploration of DARP and related VRPs, the paper contributes to the evolving discourse on making transportation systems more responsive to the needs of all users, particularly those with limited access to conventional transit options.

II. PROBLEM STATEMENT

The formulation of the Dial-a-Ride Problem (DARP) intricately intertwines the complexities of vehicle routing and scheduling challenges. At its core, vehicle routing involves the strategic design of routes, assignment of customer requests to specific vehicles, and the order in which various locations are visited. Scheduling, on the other hand, focuses on determining the precise timing for each vehicle's service commencement at the respective locations. Although the optimal solution for DARP necessitates a unified approach to both problems, certain heuristic methods have been developed that address the scheduling aspect independently once vehicle routes have been established. These methods are particularly aimed at reducing the overall duration of each route.

To adeptly navigate through the mathematical landscape of DARP, a set of notations, largely aligned with conventions established by Cordeau and others, is adopted. The customer base is denoted by (n), and for each customer (i), numbered (1) through (n), the pickup and delivery points are symbolized as (i) and (i + n), respectively. The sets (P) and (D) represent all origins and destinations collectively, while the numerical identifiers (0) and (2n + 1) signify the starting and ending points of routes, referred to as depots. These elements constitute the vertices (V) of a directed graph (G = (V, A)), where (A) represents the arcs, indicating possible movements between different locations. Specific restrictions apply to the composition of (A), excluding direct transitions from the initial depot to the final depot, from the initial depot to any destination without an associated pickup, and from any pickup location directly to the final depot without a corresponding delivery. Within this framework, subsets of vertices (S) are considered, with (A(S)) encompassing arcs connecting vertices within (S) and δ+(S) including arcs leading out of (S) to the rest of the graph. In scenarios where (S) is a subset of P∪D, (S') denotes the complementary set of locations within P∪D that are not included in (S).

Additionally, time constraints are critical to the model, with each location (i) in P∪D having an associated time window [ai,bi]. Customer preferences further define the maximum allowable ride time Ri and waiting time Wi, which are considered known quantities. It is presumed that all customer requests uniformly consume vehicle capacity, denoted by (Q), which is a fixed parameter. This mathematical formulation serves as a foundational structure for addressing DARP, highlighting the necessity for sophisticated optimization techniques to balance the intricate demands of routing and scheduling within the context of personalized transportation services.

Here is a basic structure that outlines the problem setup, including the creation of nodes (customers, pickups, and deliveries), edges (possible routes), and constraints (time windows, maximum ride and waiting times). This Python code example will solve DARP and set up the problem's data structure for further optimization through heuristic or exact methods. This code sets the groundwork for a DARP problem, allowing for the input of customer numbers, vehicle capacity, and specific constraints like time windows, ride times, and waiting times. Adjustments and extensions can be made to incorporate specific heuristic or optimization algorithms for solving the DARP.

```python
class DARPProblem:
    def __init__(self, n, Q):
        self.n = n  # Number of customers
        self.Q = Q  # Vehicle capacity
        self.P = list(range(1, n+1))  # Pickup points
        self.D = list(range(n+1, 2*n+1))  # Delivery points
        self.V = [0] + self.P + self.D + [2*n+1]  # Vertices including depots
        self.A = self._generate_arcs()  # Arcs representing possible movements
        self.time_windows = {i: (ai, bi) for i in self.P + self.D}  # Time windows for each location
        self.maximum_ride_time = {i: Ri for i in self.P}  # Maximum ride time for each customer
        self.maximum_waiting_time = {i: Wi for i in self.P}  # Maximum waiting time for each customer

    def _generate_arcs(self):
        """Generate valid arcs based on the constraints specified."""
        arcs = []
        for i in self.V:
            for j in self.V:
                if i == j or (i == 0 and j == 2*self.n+1):
                    continue  # Skip same node or direct route from start to end depot
                if i == 0 and j in self.D:
                    continue  # Skip routes from start depot to delivery without pickup
                if i in self.P and j == 2*self.n+1:
                    continue  # Skip routes from pickup directly to end depot without delivery
                arcs.append((i, j))
        return arcs
```

```python
    def add_time_window(self, i, ai, bi):
        """Add or update time window for location i."""
        self.time_windows[i] = (ai, bi)

    def add_maximum_ride_time(self, i, Ri):
        """Add or update maximum ride time for customer i."""
        self.maximum_ride_time[i] = Ri

    def add_maximum_waiting_time(self, i, Wi):
        """Add or update maximum waiting time for customer i."""
        self.maximum_waiting_time[i] = Wi

# Example of instantiation and setting up a DARP problem with placeholder values
n = 5  # Example: 5 customers
Q = 3  # Example: Vehicle capacity
problem = DARPProblem(n, Q)
# Example: Adding time windows, maximum ride and waiting times (ai, bi, Ri, Wi would be actual values)
for i in problem.P + problem.D:
    problem.add_time_window(i, ai, bi)
    if i in problem.maximum_ride_time:
        problem.add_maximum_ride_time(i, Ri)
    if i in problem.maximum_waiting_time:
        problem.add_maximum_waiting_time(i, Wi)
```

Figure 1: Python code for DARP Problem

III. ADDRESSING THE SCHEDULING CHALLENGE

Consider a specific route that initiates at point 0, concludes at 2n+1, and encompasses q distinct locations from the combined sets P and D. The task is to ascertain if these locations accurately reflect the sequence of customer pickups and deliveries for a given subset of requests, ensuring that each pickup precedes its corresponding delivery and is executable by a vehicle with finite capacity. This verification process can be efficiently performed in O(q) time, with a positive outcome indicating route feasibility.

The core of the scheduling challenge lies in pinpointing the optimal departure time from the depot and the precise timing for initiating service at each pickup point. Integral to this task are constraints related to time windows, maximum waiting periods, and the maximum duration a customer spends on the vehicle, all aimed at curtailing the overall route time. This issue falls into the category of timing problems, as documented in various scholarly works.

For ease of representation, let the notation i+1 denote the subsequent location after i in the route, and i-1 denote the preceding one. Thus, the route sequence is represented as 0, 1, ..., h, 2n+1. Let Ti symbolize the service commencement time at each of the h locations, with T0 marking the vehicle's departure from the initial point and Th+1 the arrival time at the final point, 2n+1. The mathematical goal is to minimize Th+1, ensuring that the service start time at each location adheres to the constraints:

- $T_i + t_{i,i+1} \leq T_{i+1}$ for every i within $P \cup D$,
- $a_i \leq T_i \leq b_i$ for every i within $P \cup D$,
- $T_i - a_i \leq W_i$ for every i within P,
- $(T_{(i+n)-1} + t_{(i+n)-1,i+n}) - T_i \leq R_i$ for every i to i+n,

where Wi represents the maximum waiting time allowed for customer i, and Ri denotes the maximum ride time for customer i. Research by Firat and Woeginger introduces an algorithm that operates in O(q) time, evaluating the feasibility of addressing this scheduling issue and providing solutions when feasible. This approach reinterprets the feasibility assessment as a shortest-path dilemma within a vertex-weighted interval graph. This methodology, alongside contributions from seminal works and subsequent studies, offers significant insights into resolving the scheduling complexities inherent in DARP solutions.

Element	Description
Route Sequence	Starts at 0, ends at 2n+1, traversing through q distinct locations from sets P (pickups) and D (deliveries).
Verification Process	Ensures that pickups precede corresponding deliveries within a feasible route, verifiable in O(q) time.
Capacity Constraint	Vehicle must have the capacity to execute the subset of requests.
Timing Constraints	• Ti + ti,i+1 ≤ Ti+1: Service at i must start so the vehicle can reach i+1 in time. • ai ≤ Ti ≤ bi: Service at each location must start within a specific time window. • Ti - ai ≤ Wi: Maximum waiting time at pickups. • (T(i+n)-1 + t(i+n)-1,i+n) - Ti ≤ Ri: Maximum ride time from pickup to delivery.
Mathematical Goal	Minimize Th+1, the arrival time at the final point, ensuring adherence to all timing constraints.
Algorithm by Firat & Woeginger	Evaluates route feasibility and scheduling issue in O(q) time through a shortest-path approach in a vertex-weighted interval graph.

Figure 2 : Scheduling challenges faced in DARP

IV. OVERVIEW OF HEURISTIC METHODS

The development of heuristic and metaheuristic algorithms for solving the Dial-a-Ride Problem (DARP) has seen significant progress over the past several decades, addressing both its static, dynamic, and stochastic variants. This overview traces the evolution of heuristic solution methods for DARP, presenting the milestones achieved over the years in chronological order. The journey began in the late 1970s with the introduction of heuristic algorithms designed specifically for the dynamic version of DARP. These early attempts introduced three insertion algorithms focusing on route optimization through various strategies, such as prioritizing the nearest stops, alternating between the nearest origins and destinations, and inserting deliveries based on passenger count. This foundational work laid the groundwork for future explorations in routing efficiency and strategy development.

The early 1980s saw the advent of algorithms aimed at the static multi-vehicle version of DARP, employing an interactive approach that separated clustering and routing phases. This methodology, though innovative, mainly concentrated on partitioning and routing without a detailed focus on user-specific needs or constraints. Midway through the 1980s, the focus shifted to applying decomposition techniques to the single-vehicle DARP, distinguishing the scheduling problems that could be optimally solved from those where routing required heuristic approaches. Subsequently, a sequential insertion method that organized customers based on their earliest pickup times was introduced, enhancing the algorithm's applicability to scenarios involving multiple vehicles. This approach marked a significant step forward by incorporating active vehicle periods into the problem-solving framework. The late 1980s brought a novel concept to the fore with the use of dynamic programming in a column generation approach for the multi-vehicle DARP, introducing "mini-clusters". This innovative strategy grouped customers with similar geographical and temporal characteristics, facilitating more efficient route construction. Advancements continued into the mid-1990s with the refinement of the mini-clustering approach through enhanced column generation techniques. This period also saw the introduction of algorithms that balanced traditional cost objectives against client-centered goals, including minimizing ride times and service time deviations, reflecting the multi-objective nature of DARP.

The dynamic aspect of DARP, particularly for multi-vehicle scenarios, received attention with algorithms focusing on the dynamic insertion of new transportation requests into existing clusters. This approach highlighted the adaptability of heuristic methods to real-life instances and larger problem sizes. The early 2000s witnessed the development of the regret insertion algorithm for static DARP, revolutionizing the request insertion process by giving priority to potentially difficult requests, thus improving the effectiveness and adaptability of the heuristic approach. Over the years, the evolution from initial heuristic algorithms to more sophisticated metaheuristic techniques have demonstrated a continuous enhancement in solving the complex challenges presented by DARP. Each milestone reflects an amalgamation of innovation, optimization, and practical application, contributing to the rich landscape of solution strategies for this multifaceted problem.

V. OVERVIEW OF METAHEURISTIC SOLUTION METHODS

Metaheuristic algorithms have played a pivotal role in advancing the solutions for the Dial-a-Ride Problem (DARP), encompassing static, dynamic, and stochastic variants. These methods, emerging from the mid-1990s onward, have introduced sophisticated techniques to tackle the complexities of DARP, highlighting a trend towards more complex and real-world applications, especially in patient transportation. The inception of metaheuristic methods for DARP can be traced back to the mid-1990s with the development of a local search-based metaheuristic, initiating the exploration into efficient solution methods for the static multi-vehicle DARP. This period marked the beginning of employing sophisticated algorithms that constructed initial solutions through parallel insertion and leveraged detailed neighborhood descriptions for optimization.

Entering the 21st century, the focus expanded to include tabu search algorithms, notable for their capability to dynamically adjust routes by exploring the movement of requests between routes. These algorithms incorporated unique features like reverse move restrictions and aspiration criteria, enhancing their ability to find superior solutions by navigating through complex solution spaces. The subsequent years witnessed a significant shift towards addressing DARP in healthcare and patient transportation contexts, acknowledging the unique challenges these scenarios present. Researchers developed models catering to double request DARPs with soft time windows, aiming to minimize both vehicle transportation costs and clients' inconvenience times, including excess ride time and deviations in pickup and delivery timings. Real-world applications, particularly in large hospital settings, inspired further innovations. Studies extended classical DARP to accommodate the intricate constraints of hospital environments, integrating dynamic requests and developing heuristic procedures tailored to the operational complexities of patient transportation. These efforts underscored the adaptability of metaheuristic methods to real-life challenges, contributing to the development of decision support systems for healthcare logistics. The exploration into patient transportation problems continued, with solutions extending to cover the dynamic and stochastic nature of real-world transportation requests. Techniques such as variable neighborhood search and stochastic variable neighborhood search emerged, designed to optimally configure vehicle routes under partially dynamic and stochastic conditions. These methodologies aimed to leverage historical data and

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probabilistic information to enhance solution quality, addressing both outbound and inbound transportation requests within healthcare settings.

Recent advancements have further broadened the scope of DARP solutions, incorporating transfer points into pickup-and-delivery models and exploring the impact of different transportation modes and driver-related constraints on solution effectiveness. Innovative approaches, including adaptive large neighborhood search and hybrid methods combining exact and heuristic techniques, have demonstrated significant improvements in objective function values, showcasing the potential of metaheuristic methods to revolutionize DARP solutions. Moreover, the integration of scheduling and routing aspects in patient transportation, particularly in hospital environments, has been a focal point of recent studies. By combining scheduling of treatments with the transportation of patients, researchers have unveiled cooperative hybrid metaheuristics that outperform traditional methods, emphasizing the benefits of integrative approaches in optimizing healthcare logistics. Through these developments, metaheuristic methods for DARP have evolved from foundational algorithms to sophisticated techniques capable of addressing the nuanced demands of modern transportation challenges, particularly in healthcare. This progression underscores the ongoing innovation and adaptation of metaheuristic solutions, paving the way for more efficient and effective transportation systems.

VI. OVERVIEW OF SCHOOL BUS ROUTING PROBLEM

The School Bus Routing Problem (SBRP) is intricately linked to the Dial-a-Ride Problem (DARP), focusing on the transportation of students from their homes to schools. This involves routing a fleet of buses to ensure timely and efficient pickup and delivery of students, with the unique aspect that most passengers share the same destination. Unlike the traditional DARP, SBRP places a significant emphasis on hard time-window constraints at schools and considers maximum ride time limits for students, which are often managed through the strategic setting of time windows at bus stops. SBRP encompasses multiple objectives, including minimizing operating costs for the bus company and ensuring students arrive at school on time but not excessively early. Balancing these objectives with the goal of minimizing students' time on the bus presents a complex challenge, as these goals often conflict. Cost minimization is typically associated with reducing total driving time and route length, while ensuring timely arrivals at school may require longer routes or earlier pickups, affecting ride times and overall operational efficiency.

Addressing SBRP involves tackling various subproblems such as bus stop selection, route generation, adjusting school bell times, and scheduling buses efficiently. Early efforts and concepts in solving SBRP introduced solution techniques ranging from integer programming models aimed at coordinating school start times with public bus services to algorithms designed for optimizing home-to-work bus services in metropolitan areas. These approaches often required balancing efficiency, effectiveness, and equity, with some models incorporating multi-objective formulations to navigate the trade-offs between conflicting criteria.

The development of solution techniques for SBRP has also seen the application of tabu search algorithms, simulated annealing, and other heuristic methods aimed at generating and improving feasible solutions. Innovations have extended to the integration of school starting times with public transportation schedules, leveraging preprocessing techniques, model reformulations, and cutting planes to enhance solution quality and efficiency. In more specific applications, studies have focused on designing bus services for metropolitan areas, taking into account the equilibrium between various service criteria, including the synchronization with other modes of transportation. This "cluster-first route-second" approach has facilitated the modeling of both bus stop locations and routing in complex urban road networks, employing multi-objective models solved through advanced search algorithms. Further explorations have introduced integrated procedures for identifying optimal student concentration points and computing routes to serve those stops, applied to real-world data from cities like Lisbon. The scheduling aspect has also been explored, with algorithms developed to optimize bus schedules while considering fixed time windows for schools, thereby improving the efficiency of bus utilization and service quality.

Recent studies have delved into the complexities of SBRP with transfers, examining scenarios where students may need to switch buses, thereby introducing additional layers of complexity in terms of service level implications, cost considerations, and operational impacts. These investigations have developed heuristic frameworks and compared various solution techniques, focusing on cost minimization while also considering service level impacts like ride times and the number of transfers. Through these developments, the solution space for SBRP has evolved significantly, incorporating a range of sophisticated methodologies to address the multifaceted challenges of school bus routing and scheduling. The ongoing innovation in this field reflects a deepening understanding of the operational, cost, and service level considerations fundamental to efficiently transporting students in diverse urban and rural contexts.

VII. OVERVIEW OF CARPOOLING PROBLEM

Carpooling represents a class of problems closely related to the Dial-a-Ride Problem (DARP), focusing on organizing groups of employees to share rides to and from work, thereby determining the optimal routes and designating drivers. Unlike DARP, carpooling typically involves common origins or destinations for all participants, based on the direction of the commute. The problem can be approached as either a unified model for both directions or as separate models for journeys to work and from work.

Key considerations in carpooling include balancing client-centered objectives, such as limiting maximum travel time and detours, against economic goals. Various carpooling variants have been explored, including daily carpooling, where drivers and their vehicles are pre-specified and the challenge lies in passenger assignment; long-term carpooling, which requires selecting drivers and passengers from a pool of users; and more flexible models where groups decide on their drivers. Carpooling offers significant benefits, notably reducing total vehicle kilometers, which aligns with efforts to make transport more environmentally sustainable. In places like the United States, carpool lanes incentivize shared rides by allowing vehicles with multiple passengers to bypass traffic, further promoting carpooling as a solution to enhance mobility, especially in rural areas where public transport options may dwindle due to decreased demand.

Despite its advantages, practical implementation of carpooling schemes often faces challenges, primarily due to the complexities involved in planning. Most studies have focused on scenarios where either the starting point or destination is shared by all users, involving only private vehicles in the solution processes. Solutions to the carpooling problem have included both exact and heuristic methods, with real-life applications demonstrating their viability. Techniques such as ant colony optimization have also been explored, particularly for addressing long-term carpooling challenges. The scope of carpooling problems has been expanded to encompass many-origins-to-many-destinations scenarios, accommodating multiple vehicles and diverse passenger groups. This variant considers personal preferences and characteristics, such as smoking habits and gender, to improve passenger compatibility. The problem has been modeled as an integer multiple commodity network flow problem, with solutions derived from Lagrangian relaxation and heuristic methods aimed at providing optimal or near-optimal arrangements.

A closely associated issue is dynamic ridesharing, which differs mainly in its planning horizon. Facilitated by the advent of smartphones, dynamic ridesharing caters to individuals with similar routes and schedules, allowing them to arrange shared rides with little notice. This innovation represents a response to modern technological capabilities, offering a flexible, real-time solution to commuting challenges. Various strategies for effective dynamic ridesharing have been explored through simulation, indicating a growing interest in leveraging technology to enhance the efficiency and appeal of carpooling as a sustainable commuting option. Through these developments, carpooling continues to evolve as a multifaceted problem, offering potential solutions to reduce traffic congestion, lower transportation costs, and contribute to environmental sustainability, all while navigating the inherent challenges of coordinating shared transportation in diverse commuting landscapes.

Aspect	Description
SBRP vs. DARP	SBRP specifically targets the transportation of students to schools, emphasizing hard time-window constraints and maximum ride times, unlike the broader scope of DARP.
Objectives	Includes minimizing operating costs, ensuring timely student arrivals without excessive earliness, and minimizing time on the bus. These objectives often conflict, necessitating a balance between efficiency, cost, and student welfare.
Challenges	Involves bus stop selection, route generation, adjusting school bell times, and efficient scheduling, with early solutions ranging from integer programming to optimizing bus services in metropolitan areas.
Solution Techniques	Early efforts included integer programming and algorithms for home-to-work services. Recent innovations employ tabu search, simulated annealing, and heuristic methods for generating and improving solutions, integrating school starting times with public transportation schedules, and optimizing bus schedules with fixed time windows.
Recent Studies	Explored SBRP with transfers, adding complexity in terms of service levels, costs, and operational impacts. Heuristic frameworks and comparison of solution techniques have focused on cost minimization while considering service levels.
Carpooling Relation	Carpooling, often related to DARP, involves arranging shared rides among groups with common origins or destinations. It addresses environmental sustainability by reducing total vehicle kilometers and is incentivized in some regions through car pool lanes. Challenges include balancing travel time, detours, and economic goals, with solutions ranging from exact to heuristic methods.
Dynamic Ridesharing	A variant of carpooling facilitated by modern technology, allowing for real-time, flexible arrangements among individuals with similar routes and schedules. It represents a modern response to commuting challenges, leveraging smartphones and technology to enhance efficiency and sustainability in transportation.

Figure 3 : Core aspects of SBRP

VIII. OVERVIEW OF DEMAND RESPONSIVE TRANSPORTATION

Demand Responsive Transportation (DRT) problems, closely linked to the Dial-a-Ride Problem (DARP), focus on adapting public transportation services to meet fluctuating demands, especially in areas where traditional transit systems may not offer efficient or high-quality service due to their rigid structures. This need for flexibility has led to the evolution of public transportation toward more adaptable systems, merging traditional fixed-route services with the flexible features of DARP systems to create semi-flexible transportation solutions. These semi-flexible systems require intricate planning and a formalized decision-making process to effectively address their unique characteristics and complexities. A notable application of DRT is found in the transportation of personnel in the offshore petroleum industry, where helicopters are commonly used. The helicopter routing problem within this context aims to enhance transportation safety by minimizing the expected number of fatalities, incorporating multiple objectives such as travel time and risk to passengers and pilots. This demonstrates the critical balance between reducing passenger transportation risk and potentially increasing pilot risk through extended travel times.

IX. CONCLUSION AND FUTURE SCOPE

In the realm of air transportation, on-demand air services represent another facet of demand-responsive solutions, allowing travelers to schedule flights a few days in advance. Effective scheduling systems for these services are crucial for creating minimum-cost itineraries for pilots and jets, addressing accepted transportation requests. The development of integer multicommodity network flow models with side constraints, coupled with innovative optimization technologies embedded in large neighborhood search schemes, exemplifies the advanced solutions designed to manage these complex logistical challenges efficiently, even on a large scale. The broader field of vehicle routing problems related to people transportation, with DARP as its most extensively studied variant, encompasses a wide range of both exact and heuristic approaches. Despite methodological similarities with goods transportation problems, transporting people introduces a multi-objective dimension due to the difficulty in numerically expressing customer dissatisfaction. Additionally, these problems often exhibit a high degree of stochasticity, with customer demand becoming known even as vehicles are enroute, further complicating the optimization challenge.

Special features, such as the integration of transit points, add layers of complexity to these problems, necessitating a diverse array of exact and heuristic methods to generate effective solutions. The advancement of hybrid algorithms, supported by current telecommunications technology, has significantly improved results for standard DARP instances, highlighting the dynamic interplay between technological innovation and solution development for these problems. Looking forward, the integration of intermodal aspects, combining public transportation with demand-responsive services and car or bike-sharing initiatives, presents new challenges and opportunities for transportation planners, especially in rural areas. This evolving landscape of vehicle routing problems underscores the growing need for novel solution concepts that can accommodate the increasing complexity and scale of transportation demands in various contexts.

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