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Application of Graph Theory in Real Life to Develop Routes

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Abstract

Graph theory is a special part of discrete mathematics, in which we describe the relationship between points and lines. It plays a significant role in the field of science, engineering and technology. With the help of Graph theory, we plan to develop bus routes to pick up students and take them to school. We denote each stop as a vertex, and the route as an edge. In this research paper, we use the Hamiltonian path to represent the efficiency of including each vertex within the route. Whether visiting a water park, theme park, or zoo, we plan an efficient route to visit a selected attraction or all of the attractions. Each vertex within the graph is represented by a Hamiltonian path or circuit. The obtained result is efficient to elucidate the methodology.

Keywords: Develop, Graph theory, Hamiltonian circuit, Hamiltonian path, Route.

1. Introduction

The basic idea of graphs were first introduced in the 18th century by the most eminent Swiss mathematician Leonhard Euler. He worked on the famous problem known as “Seven Bridges of Königsberg problem” (1735), which are quoted as the origin of the graph theory. In discrete mathematics, the study of Graphs is one of the principal objects. A Graph consists of the relationship between edges or links or lines and vertices or nodes or points. And the study of graphs is known as Graph theory. Simply, Graph Theory is the study of lines and points. It is a sub-field of mathematics that deals with graphs and diagrams (involving points and lines). Hence, it pictorially represent mathematical truths. Therefore, for given a set of nodes and connections, graph theory provides a helpful tool to simplify and quantify the many moving parts of dynamic systems.

Hamiltonian paths and cycles are named after William Rowan Hamilton, who invented the Icosian game, now also known as Hamilton's puzzle, which involves finding Hamiltonian cycles in the edge graph of a dodecahedron. Hamilton solved this problem by using the Icosian calculus, an algebraic structure based on the roots of unity, which has many equivalences (also invented by Hamilton). This solution does not generalize to arbitrary graphs. The cycle was named after Sir William Rowan Hamilton, who in 1857 invented a puzzle-game that involved hunting for the Hamiltonian cycle. The game, called the Icosian game, was distributed as a dodecahedron graph with a hole at each vertex. To solve the puzzle or win the game one had to use pegs and string to find the Hamiltonian cycle, a closed loop that passed through each hole once.

This paper proposes a new approach to develop route for a given bus stop, consisting of a component based description of the bus stop layout, a modified Hamiltonian path based on graph theory and a route verification method designed for bus stopes. Whether visiting a water park, theme park, or zoo, we plan an efficient route to visit a selected attraction or all of the attractions. This approach has the advantages of efficiency and universality. When a Hamiltonian path is modified, the designers can simply update the graphical data for the bus stop, and the new route information can be computed using the automatic method. Furthermore, the route location efficiency is enhanced by the modified Hamiltonian path.

2. Preliminaries

Definition 2.1. Let V = {v_1, v_2, ……} be the set of vertices and let E = {e_1, e_2, …….} be the set of edges. Then a Graph(or linear graph) G = (V, E) consists of a finite set of objects V and another set E such that each edges e_k is identified with an unordered pair (v_i, v_j) of vertices. And the study of graphs is known as Graph theory.

Definition 2.2. Suppose that G is a graph. If a simple path passes through each vertex of G exactly once, then it is known as a Hamiltonian path. In this path, all the edges may or may not be covered but edges must not repeat.

Definition 2.3. Let G be a graph. If a simple circuit passes through each vertex of G exactly once, then it is known as a Hamiltonian circuit or Hamiltonian cycle. In other words, a Hamilton Circuit is a Hamilton Path that starts and ends at the same vertex.

Definition 2.4. A simple graph G is said to be maximallly non-Hamiltonian if it is not Hamiltonian, but in addition, any edge connecting two non-adjacent vertices forms a Hamiltonian graph.

Definition 2.5. A graph G is said to be a Hamiltonian graph if it contains one Hamiltonian cycle.

Definition 2.6. Development is a process that involves growth, progress, positive change or a combination of physical, economic, environmental, social and demographic components.

Definition 2.7. A route is defined as the pairing between the characteristics of a destination and the path to that destination.

3. Main Results

Theorem 3.1. *If G is a simple graph with n bus stopes where $n \geq 3$ and $d(s) \geq n/2$ for every stop 's' of G, then G is Hamiltonian.*

Proof. Let if possible the result is not true. So, the graph G is Non-Hamiltonian. Then for somen ≥ 3; there is a non-Hamitonian graph in which every bus stop has degree at least n/2. Therefore, any proper spanning supergraph has every bus stop with degree at least n/2 because any proper spanning supergraph can be obtained by introducing more routes in G.

Therefore, there is a Maximal Non-Hamiltonian graph of G with n bus stopes and $d(s) \geq n/2$ for each bus stop 's' in G.

But the graph G cannot be complete, since if G is complete graph K_n then for $n \geq 3$ it would be a Hamiltonian graph. Therefore, there are two nonadjacent bus stopes 's' and 't' in G.

Now, let G + st be the supergraph of G obtained by introducing a route 'st'. Then, G + st must be a Hamiltonian graph because G is Maximal Non-Hamiltonian graph.

Also, if C is a Hamiltonian cycle of G + st then C must contain the route 'st'. Otherwise it will be a Hamiltonian cycle in G.

Therefore, choosing such a cycle $C \equiv t_1\ t_2. . .t_n\ t_1$, where $t_1 = s$ and $t_n = t$ (the bus route $t_n\ t_1$ is just ts i.e. st). So, the cycle C contains the route 'st'.

Now let,

$P = \{t_i \in C$: there is a bus route from s to t_{i+1} in G$\}$

and

$Q = \{t_j \in C$: there is a bus route from t to t_j in G$\}$

Then, $t_n \notin Q$, since otherwise there would be a route from t to $t_n = t$, i.e., a loop, which is impossible because G is simple graph. Also, $t_n \notin P$ (interpreting t_{n+1} as t_1), since otherwise we get again a loop, this time from s to $t_1 = s$. Therefore, $t_n \notin P \cup Q$.

Let $|P|$, $|Q|$ and $|P \cup Q|$ denote the number of elements in P, Q, and $P \cup Q$, respectively.

Therefore, we have

$|P \cup Q| < n$ (1)

Also, for each route incident with u, there corresponds precisely one bus stop v_i in S.Hence,

$|P| = d(s)$ (2)

Similarly, we can write

$|Q| = d(t)$ (3)

Now, if t_k is a bus stop belonging to both P and Q, there is a bus route r_1 joining s to t_{k+1} and a route r_2 joining t to t_k. Therefore, we have

$$C' \equiv t_1 t_{k+1} t_{k+2}. . .t_n t_k\ t_{k-1}. . .t_2 t_1$$

as a Hamiltonian cycle in G, which is a contradiction, since G is non-Hamiltonian (Figure 1). This shows that there is no bus stop t_k in $P \cap Q$, i.e., $P \cap Q = \phi$.

Therefore, $|P \cup Q| = |P| + |Q|$.

Hence, from Equations (1), (2), and (3), we have

$d(s) + d(t) = |P| + |Q| = |P \cup Q| < n$

which is impossible.

Since, in G, d(s) ≥ n/2 and d(t) ≥ n/2, hence, d(s) + d(t) ≥ n.

Therefore, we find a contradiction. So our assumption was wrong.

Thus, we conclude that G is a Hamiltonian cycle.

Theorem 3.2. *Let G be a simple graph with n bus stopes and let 's' and 't' be non-adjacent bus stopes in G such that d(s) + d(t) ≥ n. Let G + st denote the supergraph of G obtained by joining 's' and 't' by a bus route. Then, G is Hamiltonian iff G + st is Hamiltonian.*

Proof. Necessary Part– Suppose that G is Hamiltonian. Then we have to show that G + st is Hamiltonian.

We have the following figure:

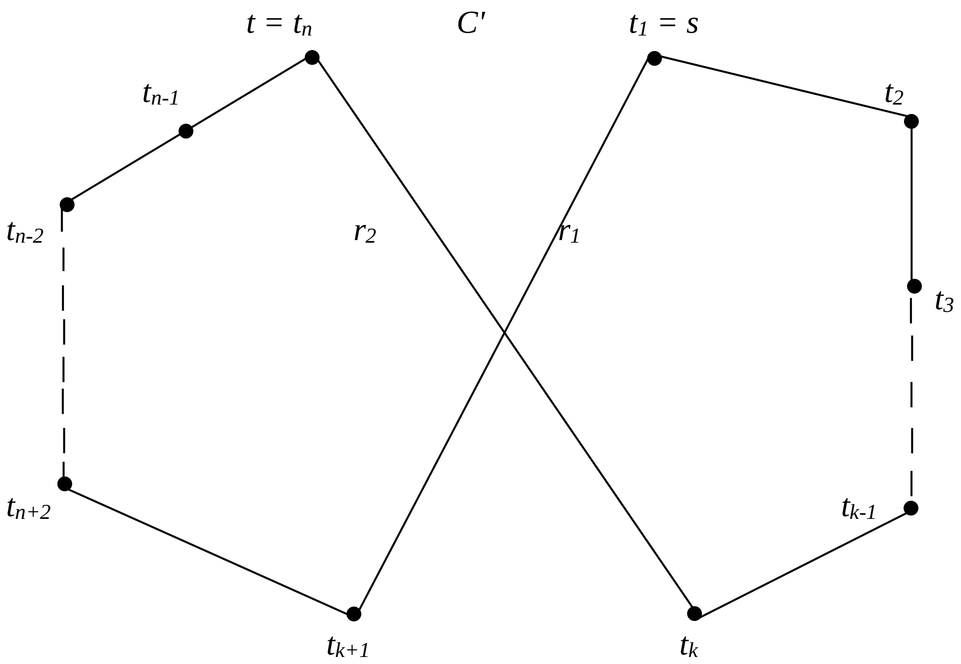

Figure 1. A Hamiltonian cycle C'

Since, G is Hamiltonian. Therefore, from Theorem 3.1, we can say that the supergraph G + st must be Hamiltonian.

Sufficient Part–Suppose that G + st is Hamiltonian. Then we have to show that G is Hamiltonian.

Since, G + st is a Hamiltonian.

Therefore, by Theorem 3.1, it is clear that G is not Hamiltonian.

Hence, we can obtain the inequality

d(s) + d(t) < n.

But by hypothesis,

$d(s) + d(t) \geq n$.

Therefore, we can say that G must be Hamiltonian.

Theorem 3.3. *A simple graph with n bus stopes, where n>2, is Hamiltonian if the sum of the degrees of every pair of nonadjacent bus stopes is at least n.*

Proof. Suppose a graph G with n bus stopes satisfying the given inequality conditionis not Hamiltonian. So it is a subgraph of the complete graph K_n with fewer bus routes.

Now, we add routes to the graph by joining nonadjacent bus stopes until we obtain a graph H such that the addition of one more route joining two nonadjacent bus stopes in H, which produce a Hamiltonian graph with n bus stopes.

Let 'x' and 'y' be two nonadjacent bus stopes in H. Therefore, they are nonadjacent in G.

Since, $d(x) + d(y) \geq$ in G.

$\Rightarrow \quad d(x) + d(y) \geq$ in H.

Now, if we join the nonadjacent bus stopes x and y, then the resulting graph is Hamiltonian. So, there is a Hamiltonian path between the bus stopes x and y in graph H.

Also, if we write $x = t_1$ and $y = t_n$, then we have the following Hamiltonian path:

$$t_1 \text{ ——— } t_2 \text{ ——— } \ldots t_{i-1} \text{ ——— } t_i \text{ ——— } t_{i+1} \text{ ——— } \ldots . t_{n-1} \text{ ——— } t_n$$

Figure2. A Hamiltonian path from t_1 to t_n

Now, suppose the degree of t_1 is γ in graph H. Again suppose that there is a route between t_1 and t_i in this graph.

Then H is Hamiltonian which is the existence of a route between t_{i-1} and t_n.

Therefore, whenever bus stopes t_1 and t_i are adjacent in H, stopes t_n and t_{i-1} are notadjacent (Figure 1). Hence, this is true for $1< i <n$.

So, $d(t_n) \leq (n-1) - \gamma$, since thedegree of t_1 is γ. This implies that the sum of the degrees of the two nonadjacent bus stopes in G is less than n; which is contradiction to our hypothesis.

Therefore, any connected graph satisfying the given condition is Hamiltonian.

So, we have the following figure:

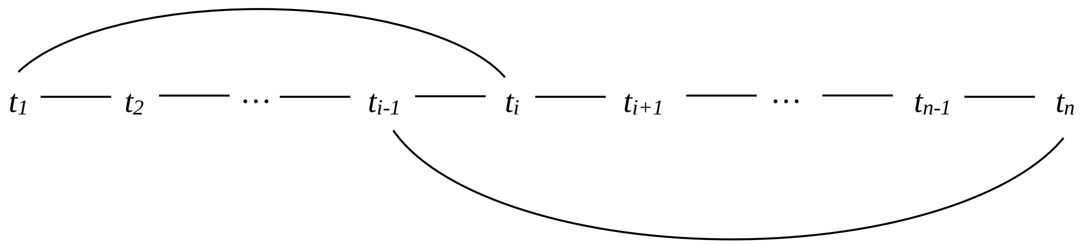

Figure 3. A Hamiltonian cycle $t_1t_it_{i+1}\ldots t_nt_{i-1}\ldots t_1$

Also, we have the following figure:

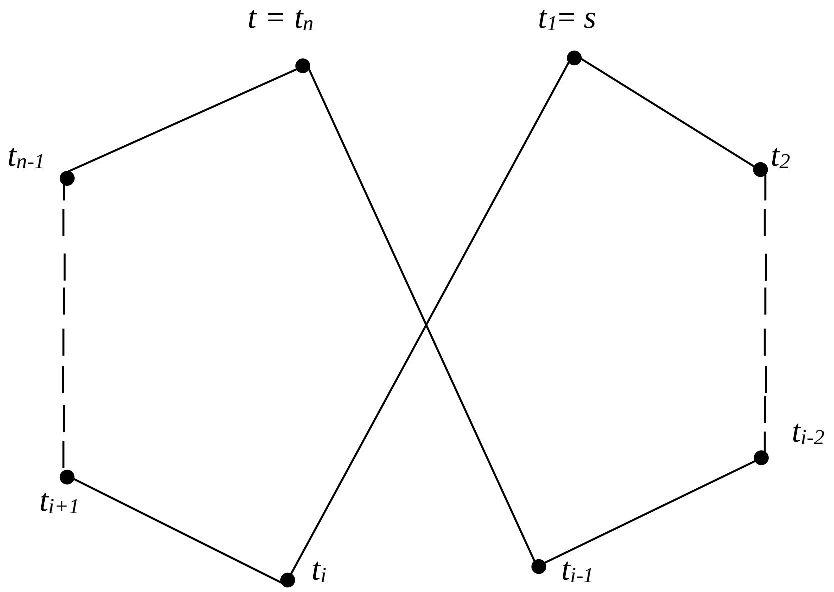

Figure 4. Representation of Hamiltonian cycle $t_1t_it_{i+1}\ldots.t_nt_{i-1}\ldots t_1$

4. Conclusion

In this paper, in a simple graph with n bus stopes, we obtained that G is Hamiltonian cycle iff G + st is Hamiltonian, where ‘s’ and ‘t’ are non-adjacent bus stopes in G such that $d(s) + d(t) \geq n$ and supergraph obtained by joining ‘s’ and ‘t’ by a route. We have used the Hamiltonian path to represent the efficiency of including each bus stop within the route. Thus, the obtained result is efficient to elucidate the methodology.

5. References

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