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Sum of Three Squares Is Equal to a Square

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Abstract

I have developed a new mathematical relation of conjugate numbers so that sum of three square numbers is equal to a single square number. This is true for all natural numbers belong to N>1 (N is Set of Natural Numbers).If a and b are two successive numbers,that is, b = (a +1) then

a^2 +b^2 +(ab)^2 = (ab + 1)^2 for all positive integral values of a and b >1.

But, I have also proved that above theorem is true for fractional and imaginary numbers when difference between two such numbers is 1.

Introduction-Conjugate natural numbers are infinite. But I have developed a mathematical formula with any two conjugate natural numbers where sum of three squares is equal to a single square number. This is found to be a property of conjugate natural numbers.

In mathematics, Legendre's three-square theorem states that a natural number can be represented as the sum of three squares of integers

n= x^2 + y^2 +z^2

if and only if n is not of the form n= 4^a(8b + 7) for nonnegative integers a and b.

The first numbers that can not be expressed as the sum of three squares are 7,15,23,28,31,39,47,55,60,63,71,...................

Pierre de Fermat gave a criterion for numbers of the form 8a + 1 and 8a+ 3 to be sums of a square plus twice another square, but did not provide a proof. N.Beguelin noticed in 1774 that every positive integer which is neither of the form 8n + 7 , nor of the form 4n, is the sum of three squares , but he did not provide a satisfactory proof. In 1796 Gauss proved his Eureka theorem that every positive integer n is the sum of 3 triangular numbers; this is equivalent to the fact that 8n + 3 is a sum of three squares.

In particular , Gauss counted the number of solutions of the expression of an integer as a sum of three squares , and this is a generalisation of yet another result of Legendre, whose proof is incomplete.

With Lagrange's four-square theorem and the two – square theorem of Girard ,Fermat and Euler , the Wiring's problem for k+ 2 is entirely solved.

One of the most investigated topics in additive theory is the representation of integers by sums of squares and more generally; by quadratic forms. For instance, the classical problem of finding formulas for the number of ways of expressing aninteger as the sum of squares.

In 1801 ,Gauss going way beyond Legendre ,actually obtained a formula for the number of primitive representations of an integer as a sum of three squares.

As the brief history says that an integer is represented as sum of three squares,I have tried to show that sum of three squares is equal to a single square and I am successful to develop a generalised formula to show that sum of three squares is equal to a single square always.

Sum of Three Squares is equal to a Square

Generally, we know that sum of two squares is equal to a square .According to Pythagorus,-a^2 + b^2=c^2 where a is perp./base is base/perp. & c is hypotenuse ofa rt.angled triangle.

But I have developed a formula where, a^2 + b^2 + c^2 = d^2 such that a=n, b= (n+1), c={ n(n+1)} and d={ n(n + 1) + 1}.Thus with two conjugate natural numbers n and (n + 1) ,sum of three squares is equal to a square is possible(n>1).

Therefore, the general formula is,-

n^2 + (n + 1)^2 +{n(n + 1)}^2 = {n(n+ 1) + 1 }^2 where n belongs to N and n>1(N is set of natural numbers).

For examples-

2^2+3^2+6^2=7^2,

3^2+4^2+(12)^2=(13)^2,

4^2 +5^2 +(20)^2 = (21)^2,

5^2 +6^2 +(30)^2 = (31)^2,

6^2 + 7^2 +(42)^2 = (43)^2,

7^2 +8^2 +(56)^2 = (57)^2,

.................................,

(100)^2+(101)^2+(10100)^2=(10101)^2 ; so on.

Thus , it has been proved that for any two consecutive positives integers n and (n+1) ,where n>1, sum of three squares is equal to a single square.It is an exception toPythagoras theorem.

Now,I am going to prove that above theorem is also true for consicutive fractional or decimal numbers-

Following results are obtained—

(2.2)^2 + (3.2)^2 + (2.2 x 3.2)^2 = {(2.2 x 3.2) +1}^2,

(3.2)^2 + (4.2)^2 +(3.2 x 4.2)^2 = {(3.2 x 4.2) +1}^2,

(4.2)^2 + (5.2)^2 + (4.2 x 5.2)^2 = {(4.2 x 5.2) +1}^2,

.... and so on.

Therefore, if a and b are two consicutive or successive fractional or decimal positive numbers of difference 1,a>2 and b=a+1 then a^2 +b^2 +(ab)^2 = (ab +1)^2 holds true.

This theorem is equally true for imaginary numbers as below- (2 + i)^2 + (3

+ i)^2 +{(2 + i)(3 +i)}^2 ={(2+i)(3+i) +1}^2,

(3 +i)^2 + (4+i)^2 + {(3+i)(4+i)}^2 = {(3+i)(4+i) +1}^2,

.... and so on.

Again,if the imaginary is,a=(2 +2i) and b=(3 +2i) then also,

(2 +2i)^2 +(3 +3i)^2 +{(2 +2i)(3 +2i)}^2 = {(2 +2i)(3 +2i) +1}^2 holds true.

So, it is clear that if a and b are two consitive or successive imaginary numbers,s.t.,

b-a=1,then a^2 +b^2 +(ab)^2 =(ab +1)^2 is true.Therefore, this theorem is true for integral,fractional and imaginary numbers as well.

Application

Sum of squares theorems have found various applications in applied number theory, such as cryptography and integer factoring algorithms. They are often used as intermediate steps in the proofs of other theorems in elementary number theory.
Sum of squares is used in calculating statistics, such as variance, standard error, and standard deviation. It is also used in performing ANOVA (or analysis of variance), which is used to tell if there are differences between multiple groups of data.

The sum of squares is a measure of deviation from the mean. In statistics, the mean is the average of a set of numbers and is the most commonly used measure of central tendency. The arithmetic mean is simply calculated by summing up the values in the data set and dividing by the number of values. The sum of squares is a measure of deviation from the mean. In statistics, the mean is the average of a set of numbers and is the most commonly used measure of central tendency. The arithmetic mean is simply calculated by summing up the values in the data set and dividing by the number of values.

In statistics, the explained sum of squares (ESS), alternatively known as the model sum of squares or sum of squares due to regression (SSR – not to be confused with the residual sum of squares (RSS) or sum of squares of errors), is a quantity used in describing how well a model, often a regression model, represents the data being modelled. In particular, the explained sum of squares measures how much variation there is in the modelled values and this is compared to the total sum of squares (TSS), which measures how much variation there is in the observed data, and to the residual sum of squares, which measures the variation in the error between the observed data and modelled values.

Sum of squares is a statistical technique used in regression analysis to determine the dispersion of data points. In a regression analysis, the goal is to determine how well a data series can be fitted to a function that might help to explain how the data series was generated. Sum of squares is used as a mathematical way to find the function that best fits (varies least)from the data.

Conclusion- This formula is useful in combining three squares to get a square of equal area. This formula is useful in mathematical modelling and theory of numbers.Generally,sum of two squares is equal to a single square is known all as Pythagoras theorem. But, there is an exception to this theorem which is proved by me in this paper. Besides this theorem is applicable to integral, fractional and imaginary numbers where difference between two consecutive numbers is 1.

Sum of squares refers to the sum of the squares of numbers. It is basically the addition of squared numbers. The squared terms could be 2 terms, 3 terms, or ‘n’ number of terms, first n even terms or odd terms, set of natural numbers or consecutive numbers, etc. This is basic math, used to perform the arithmetic operation of addition of squared numbers. In this article, we will come across the formula for addition of squared terms with respect to statistics, algebra, and for n number of terms.

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