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Infinite Series for Square- Roots

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Introduction

The **square root of 2** (approximately 1.4142) is a positive real number that, when multiplied by itself, equals the number 2. It may be written in mathematics as 2^1/2 and is an algebraic number. Technically, it should be called the principal square root of 2, to distinguish it from the negative number with the same property.

Geometrically, the square root of 2 is the length of a diagonal across a square with sides of one unit of length; this follows from the Pythagorean theorem. It was probably the first number known to be irrational. The fraction 99/70 (≈ **1.4142**857) is sometimes used as a good rational approximation with a reasonably small denominator.

The Babylonian clay tablet YBC 7289 (c. 1800–1600 BC) gives an approximation of 2^1/2 in four sexagesimal figures, 1.24 51 10, which is accurate to about six decimal digits, and is the closest possible three-place sexagesimal representation of 2^1/2 as 1+24/60 + 51/60^2 +10/60^3 = 305470/216000 = 1.41215686274509803 9…..

In ancient Roman architecture, Vitruvius describes the use of the square root of 2 progression or *ad quadratum* technique. It consists basically in a geometric, rather than arithmetic, method to double a square, in which the diagonal of the original square is equal to the side of the resulting square. Vitruvius attributes the idea to Plato. The system was employed to build pavements by creating a square tangent to the corners of the original square at 45 degrees of it. The proportion was also used to design atria by giving them a length equal to a diagonal taken from a square, whose sides are equivalent to the intended atrium's width.

There are some interesting properties involving the square root of 2 in the physical sciences:

- The square root of two is the frequency ratio of a tritone interval in twelve-tone equal temperament music.
- The square root of two forms the relationship of f-stops in photographic lenses, which in turn means that the ratio of *areas* between two successive apertures is 2.
- The celestial latitude (declination) of the Sun during a planet's astronomical cross-quarter day points equals the tilt of the planet's axis divided by 2^1/2.

  Greek mathematician Archimedes produced the first known summation of an infinite series with a method that is still used in the area of calculus today.

  Srinivasa Ramanujan (1887-1920), the man who reshaped twentieth-century mathematics with his various contributions in several mathematical domains, including mathematical analysis, infinite series, continued fractions, number theory, and game theory is recognized as one of history's greatest mathematicians.

In 1914, Ramanujan found a formula for *infinite series for pi,* which forms the basis of many algorithms used today. Finding an *accurate approximation of π (pi)* has been one of the most important challenges in the history of mathematics.

Every real number except zero has two square roots. The principal square root of most numbers is an irrational number with an infinite decimal expansion. As a result, the decimal expansion of any such square root can only be computed to some finite-precision approximation. However, even if we are taking the square root of a perfect square integer, so that the result does have an exact finite representation, the procedure used to compute it may only return a series of increasingly accurate approximations.

The continued fraction representation of a real number can be used instead of its decimal or binary expansion and this representation has the property that the square root of any rational number (which is not already a perfect square) has a periodic, repeating expansion, similar to how rational numbers have repeating expansions in the decimal notation system.

Procedures for finding square roots (particularly the square root of 2) have been known since at least the period of ancient Babylon in the 17th century BCE. Heron's method from first century Egypt was the first ascertainable algorithm for computing square root. Modern analytic methods began to be developed after introduction of the Arabic numeral system to western Europe in the early Renaissance. Today, nearly all computing devices have a fast and accurate square root function, either as a programming language construct, a compiler intrinsic or library function, or as a hardware operator, based on one of the described procedures.

Abstract

I have formulated a general rule for finding infinite series for square-root of positive real integers.
More clearly, I have invented a method to find infinite series for square-root of all square natural numbers plus one .Mathematically,
(n^2 + 1)^1/2 = n + 1/2n+ 1/2n+ 1/2n+ 1/2n+ ……. to infinity where n belongs to N (set of all natural numbers). The derivation of the general infinite series lies the solution of the general equation
X^2 +nx – 1 = 0.

Original paper-
Considering, 1/2+ 1/2+ 1/2+ 1/2+……. = x, I get, x^2 +2x – 1 = 0 and therefore, x = - 1 + 2^1/2 or,
2^1/2 = 1 + 1/2+ 1/2+ 1/2+ 1/2+ ……..to infinity.
Again considering, 1/4+ 1/4+ 1/4+ 1/4+ …….= x, I get, x^2 + 4x – 1 = 0 and therefore,
X = - 2 + 5^1/2 or, 5^1/2 = 2 + 1/4+ 1/4+ 1/4+ 1/4+ ………to infinity.
Similarly, I get,
10^1/2 = 3 + 1/6+ 1/6+ 1/6+ 1/6+ ………to infinity.
17^1/2 = 4 + 1/8+ 1/8+ 1/8+ 1/8+ ………to infinity.
26^1/2= 5 + 1/10+ 1/10+ 1/10+ 1/10+ …….to infinity and so on. Therefore, mathematically,
or generally, I can say,

$(n^2 + 1)^{1/2} = n + 1/2n+ \ 1/2n+ \ 1/2n+ \ 1/2n+ \ldots$ to infinity for all n belongs to N (Set of Natural numbers).These are all beautiful mathematical or numerical systematic infinite series for square roots of all square natural numbers plus one.

The solution lies in the solution of the general equation of $x^2 + nx - 1 = 0$ where I get,

$X = \{ - n +/- (n^2 + 4)^{1/2}] /2 = -(n/2) +/- [(n/2)^2 + 1] ^{1/2} = - m +/- (m^2 + 1)^{1/2}$

Where , n = 2m and m belongs to N (set of natural numbers).

Here, the negative (-'ve) sign of the roots of x is ignored because ,the value of square-root of any positive real number can not be negative.

Thus,I have invented a general rule for expressing the value of a square-root of a positive real number in an infinite series as-

Square-root of (square of n + 1) = n + 1/2n+ 1/2n+ 1/2n+ ……….. to infinity where n is any natural number belonging to N.

Some examples of infinite sets-

1)Set of all points in a plane is an infinite set.

2)Set of all points in a line segment is an infinite set.

3)Set of all positive integers which is multiple of 3 is an infinite set.

4)W = {0, 1, 2, 3, ……..} i.e. set of all whole numbers is an infinite set.

5)N = {1, 2, 3, ……….} ...

6)Z = {……

Applications of infinite series-

In mathematics, a series can be defined as the process of adding an endless number of numbers or quantities to a specific starting number or amount over and over again. Series are used in many fields of mathematics, including the study of finite structures, such as combinatorics, which is used in the formation of functions. The study of series is essential for understanding calculus and its generalisation, as well as for understanding mathematical analysis. Beyond their use in mathematics, infinite series are also widely employed in a variety of quantitative sciences such as statistics, physics, computer science, finance and other related fields. Infinite series, the sum of infinitely many numbers related in a given way and listed in a given order. Infinite series are useful in mathematics and in such disciplines as physics, chemistry, biology, and engineering

They are used in engineering to analyse current flow and sound waves, among other things. In physics, infinite series can be used to determine how long it takes a bouncing ball to come to rest or how long it takes a swinging pendulum to come to a complete halt.

Conclusion-

I have given a formula for generating various infinite serieses of square-roots of real numbers as to the need of any problem.Like infinite series there are multiple examples of infinite sets and items around us: the stars in the midnight sky, water droplets, and the millions of cells in the human body. But in mathematics, the ideal example of an infinite set is a set of natural numbers. The set of natural numbers is unlimited and has no end.

Every nonnegative real number *x* has a unique nonnegative square root, called the *principal square root*.Every positive number *x* has two square roots,one is +x^½ and other is -x^½. The two roots can be written more concisely using the ± sign as +/-x^½. Although the principal square root of a positive number is only one of its two square roots, the designation "*the* square root" is often used to refer to the principal square rootThe square root of a nonnegative number is used in the definition of Euclidean norm (and distance), as well as in generalizations such as Hilbert spaces. It defines an important concept of standard deviation used in probability theory and statistics. It has a major use in the formula for roots of a quadratic equation; quadratic fields and rings of quadratic integers, which are based on square roots, are important in algebra and have uses in geometry. Square roots frequently appear in mathematical formulas elsewhere, as well as in many physical laws.The Taylor series of (1 +x)^½ about $x = 0$ converges for $|x| \leq 1$, My formula ,(1 + n^2)^½ = n + 1/2n+ 1/2n+ 1/2n+ ….is an expression of the square-root of a real positive number in an infinite series in a completely new way..

Reference-

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