International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Material Derivative in the Presence of Dyons and Macroscopic Maxwell Equations

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Abstract

We defined dyon charges in the coupled form of Dirac distributions. The general currents formed by these dyon charges are also defined in coupled form. Microscopic Maxwell equations with these sources are obtained by using material derivative and with the help of Gauss law. It should be noted that in this formulation, the speed of the charges is assumed to be constant on average. The transition from microscopic Maxwell equations to macroscopic Maxwell equations is applied using averaging process. Finally, the macroscopic Maxwell equations describing electromagnetic theory are obtained in symmetrical form in the presence of dyons.

Keywords: Dyons, Material derivative, Macroscopic Maxwell equations

1. Introduction

The symmetry constituted by the electric charge and the magnetic charge has always stimulated interest. In this context, many theories have been developed. One of them is dyon particles which carry both an electric charge and a magnetic charge at the same time. These particles have brought a different perspective to symmetry.

Poincaré [1] studied the dynamics of electric charge in the field of a fixed magnetic charge. Thomson [2] studied the same problem and gave more detailed explanations. Dirac [3,4] obtained the quantization condition of the electric charge in relation to magnetic charge. Schwinger [5-8] generalized quantization condition to dyons but couldn't construct consistent field theory of dyons. Later Zwanziger [9] also gave quantization condition for the dyons.

Zor [10] obtained microscopic Maxwell equations in the presence of dyons using material derivative. We expand this theory to macroscopic domain, which explains practical works. The microscopic theory of point charges conducts on the dynamics of a small number of charges. In this work, the effect of electric and magnetic charges on each other has been neglected in the presence of dyons.

2. Formulation

New theories can be developed by using dyon particles that have electric and magnetic charges on them at the same time. Theories developed at the microscopic domain can be extended to the macroscopic domain.

We have dyon charges are moving and positioned at $\vec{r}_a(t)$, $a = 1, 2, \ldots, n$. We can define charge densities for these dyon particles using the (δ) Dirac distributions as

$$\begin{pmatrix}\rho_e(\vec{r},t)\\ \rho_g(\vec{r},t)\end{pmatrix}=\sum_{a=1}^{n}\begin{pmatrix}e_a\\ g_a\end{pmatrix}\delta(\vec{r}-\vec{r}_a(t)). \tag{1}$$

The current densities of moving charges with the velocity $\vec{v}_a(t)$ can be written as

$$\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}=\sum_{a=1}^{n}\begin{pmatrix}e_a\\ g_a\end{pmatrix}\vec{v}_a(t)\delta(\vec{r}-\vec{r}_a(t)). \tag{2}$$

We can write the current density definition for dyons,

$$\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}=\vec{v}\begin{pmatrix}\rho_e(\vec{r},t)\\ \rho_g(\vec{r},t)\end{pmatrix}. \tag{3}$$

Gauss equations of the dyons which has charge densities ($\rho_e(\vec{r},t)$, $\rho_g(\vec{r},t)$) in microscopic medium written as

$$\vec{\nabla}\cdot\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}=\begin{pmatrix}\rho_e(\vec{r},t)\\ \rho_g(\vec{r},t)\end{pmatrix}. \tag{4}$$

Here ($\vec{e}(\vec{r},t),\vec{b}(\vec{r},t)$) are dyons electric and magnetic fields, respectively. These Gauss equations help us to derive Maxwell's equations.

We can define the fields of the charges on moving body using material derivatives. Thus we apply material derivative to the fields of dyon

$$0=\frac{d}{dt}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}=\frac{\partial}{\partial t}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}+(\vec{v}\cdot\vec{\nabla})\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}. \tag{5}$$

If we use the vector equality

$$\vec{v}\left[\vec{\nabla}\cdot\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]=\left(\vec{v}\cdot\vec{\nabla}\right)\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}+\vec{\nabla}\times\left[\vec{v}\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right] \tag{6}$$

on (5) and substitute Gauss equations in (6), we have

$$\vec{v}\begin{pmatrix}\rho_e(\vec{r},t)\\ \rho_g(\vec{r},t)\end{pmatrix}=-\frac{\partial}{\partial t}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}+\vec{\nabla}\times\left[\vec{v}\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]. \tag{7}$$

If we supposed the charges flow with the constant velocity $\vec{v}$, these charges can create currents as (4). We substitute (4) in (7) and get

$$\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}=-\frac{\partial}{\partial t}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}+\vec{\nabla}\times\left[\vec{v}\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]. \tag{8}$$

We can use the equalities

$$\vec{v}\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}=c\begin{pmatrix}\vec{b}(\vec{r},t)\\ -\vec{e}(\vec{r},t)\end{pmatrix} \tag{9}$$

for convenience. Thus we yield microscopic Maxwell Ampere and Faraday equations using (9)

$$\frac{\partial}{\partial t}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}=-\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}+c\vec{\nabla}\times\begin{pmatrix}\vec{b}(\vec{r},t)\\ -\vec{e}(\vec{r},t)\end{pmatrix}. \tag{10}$$

These are symmetric equations in microscopic domain with dyons.

We can transform the charge densities, current densities and field quantities in microscopic domain to macroscopic domain using averaging process. We can write these averaged values directly into the linear differential equations.

$$\bar{\rho}_e(\vec{r},t)=\frac{1}{\sqrt{\varepsilon_0}}\rho_{fe}(\vec{r},t), \tag{11}$$

$$\bar{\vec{j}}_e(\vec{r},t)=\frac{1}{\sqrt{\varepsilon_0}}\vec{j}_{fe}(\vec{r},t), \tag{12}$$

$$\bar{\rho}_g(\vec{r},t)=\sqrt{\mu_0}\rho_{fg}(\vec{r},t), \tag{13}$$

$$\bar{\vec{j}}_g(\vec{r},t)=\sqrt{\mu_0}\vec{j}_{fg}(\vec{r},t), \tag{14}$$

$$\bar{\vec{e}}(\vec{r},t)=\sqrt{\varepsilon_0}\vec{E}(\vec{r},t), \tag{15}$$

$$\bar{\vec{b}}(\vec{r},t)=\frac{\vec{B}(\vec{r},t)}{\sqrt{\mu_0}}, \tag{16}$$

where (ε_0, μ_0) are the permittivity and permeability of vacuum. If we use these new quantities in (10) and (4), the macroscopic symmetric Maxwell equations can be obtained

$$\frac{\partial}{\partial t}\begin{pmatrix}\mu_0\varepsilon_0\vec{E}(\vec{r},t)\\ \vec{B}(\vec{r},t)\end{pmatrix}=-\mu_0\begin{pmatrix}\vec{j}_{fe}(\vec{r},t)\\ \vec{j}_{fg}(\vec{r},t)\end{pmatrix}+\vec{\nabla}\times\begin{pmatrix}\vec{B}(\vec{r},t)\\ -\vec{E}(\vec{r},t)\end{pmatrix}, \tag{17}$$

$$\vec{\nabla}\cdot\begin{pmatrix}\vec{E}(\vec{r},t)\\ \vec{B}(\vec{r},t)\end{pmatrix}=\begin{pmatrix}\frac{1}{\varepsilon_0}\rho_{fe}(\vec{r},t)\\ \mu_0\rho_{fg}(\vec{r},t)\end{pmatrix}. \tag{18}$$

The equations are in the coupled form. Consequently, these equations express the electromagnetic theory in macroscopic medium with dyons.

3. Conclusion

The assumption that the electric and magnetic charges are in the same particle (dyon) allows many theoretical studies. In this work, we accepted the existence of dyons and obtained microscopic Maxwell equations using Gauss law and applying material derivative operation. We obtained the symmetric macroscopic Maxwell equations by applying the averaging process to the fields and sources of microscopic Maxwell equations. Thus we conducted the theory that obtained in microscopic domain to macroscopic theory.

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