

An Efficient Method to Obtain Initial Basic Feasible Solution of a Transportation Problem

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Abstract

Transportation Problem (TP) is the study of strategy for shipping the product from the origin (supplier) to the destination (consumers) and the optimal shipment cost of the TP obtained from the initial basic feasible solution (IBFS). IBFS is the prime element of TP which helps to optimize the TP objective function. Existing methods for finding IBFS could not give the nearest value to minimal cost everywhere. Hence an innovative technique Known as TOCM-MT provides IBFS, which is the closest to an optimal solution. Many researchers have used this TOCM-MT technique through thousands of real-life TP problems, in every time got a better result than VAM, JHM, and TDM1 techniques. Hence, this technique is best for obtaining IBFS. This research is illustrated the TOCM-MT approach through time minimizing, transshipment, and profit maximization problems with some numerical examples.

Keywords: Transportation Problem; Initial basic feasible solution; Optimal Solution; Total opportunity cost matrix.

1. INTRODUCTION

THE transportation problem is one of the most important problems in various sciences. This problem Transport some products from source to destination with minimal transportation costs, Meet supply and demand constraints. TP's goal is to achieve the lowest total cost as the best solution. First developed by Hitchcock. It can be applied to real-world problems such as: Personal Assignments, Task Assignments, Flow Shop Planning Issues and Vehicles routing.

There are two steps to get the optimal solution from TP. The first step is to find the initial basic feasible solution (IBFS) and the second step is to find the optimal solution of this IBFS. To solve transport problems, you need to start with IBFS. Finding an IBFS is important because it is a fundamental solution to achieve the best possible solution. The IBFS results may be similar or closer to the optimal solution values. Better IBFS results allow you to reduce the number of iterations to get the optimal solution. In this research, IBFS and transport issues are found.

VAM, JHM, and TDM1 have been explored to find IBFS, but do not always provide the best. Then TDM1 was chosen for the following reasons: The highest penalty (HP) is chosen arbitrarily, with the largest units directly assigned to the cheapest cell. Therefore, the optimal solution is not always obtained. Some studies used the TOCM matrix as the starting matrix and found IBFS to be superior. Therefore, in this study,

we integrate TOCM with a modified TDM1 called Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) to determine the IBFS of TP.

2. LITERATURE REVIEW

Many studies on IBFS have been performed by multiple researchers, the three most popular being the Northwest Corner Method (NCM). Least Cost Method (LCM) and Vogel's Approximation Method (VAM). Several methods to find IBFS based on LCM have been developed by researchers such as Juman and Hoque, Babu, Dhurai, Kousalya and Malarvizhi, Ahmed et al., and Deshmukh. Juman and Hoque proposed the Juman-Hoque Method (JHM) to obtain IBFS. It was different in that it started with an infeasible solution and ended up with an efficient IBFS. Experiments have shown that JHM leads to the lowest total cost in 16 out of 18 transportation problems.

VAM is generally the best known early and basic viable solution. It has long been studied and modified by researchers such as Hosseini, Soomro et al, Rashid, Hakim, Goyal, Das et al, Azad and Hossain, Alkubaisi, Seethalakshmy, and Srinivasan. Hosseini proposed Total Difference Method 1 (TDM1) to obtain his IBFS of TP. VAM computes all row and column penalties, while TDM1 computes only row penalties. Another difference is how penalties are calculated. Penalty of TDM1 is more complete than VAM. This is so because TDM1 penalty is the sum of differences between the least and other costs, but VAM penalty is the difference between two least costs. Azad and Hossain took average row and column penalties into consideration. Alkubaisi calculated the penalty value using the median cost. The penalty value could be determined using an alternative method proposed by Seethalakshmy and Srinivasan. The column penalty is the difference between the maximum cost and the minimum cost, whereas the row penalty is the difference between the two greatest costs.

The Total Opportunity Cost Matrix (TOCM), developed by Kirca and Satir, was first used. By combining the row and column opportunity cost matrices, it converts the original TP matrix into an initial matrix. Every element is subtracted from the row/column opportunity cost matrix by the least expensive cost. In order to provide a workable solution to the transportation issue, Khan et al. suggested TOCM SUM. TOCM was utilized by Dubey and Shrivastava to enhance VAM. To determine the most fundamentally workable solution (BFS) to the transportation issue, Islam et al. presented the Total Opportunity Cost Table (TOCT). The distribution indicator for each cell or the TOCM was specified by Khan et al. Meenakshi and Mathirajan integrated TOCM and VAM to determine the transportation issue's lowest total cost.

1. Introduction

The Total Opportunity Cost Matrix (TOCM) was introduced by Kirca and Satir. Transform the matrix transportation problem from the original matrix to the initial matrix by adding row and column possibilities. The Opportunity row subtracts the lowest cost from each element in the row. The Opportunity column subtracts each element in the column with the lowest cost in it. TOCM is the sum of opportunity in rows and columns.

Total Opportunity Cost Matrix – Minimal Total

Transportation Problem (TP) deals with delivering the product from the sources to the destinations and Initial Basic Feasible Solution (IBFS) is necessary to obtain an optimal solution. The previous methods related to it did not always provide the satisfied result all the time. Therefore a new method called

Total Opportunity Cost Matrix – Minimal Total (TOCMMT) was proposed to determine IBFS as a basic solution to solve TP. The objective is to achieve a total cost with similar or closer values to the optimal solution.

Notations

m	Total number of supply	n	Total number of demand
T_i	Supply i		
R_j	Demand j		
C_{ij}	Transportation cost from the supply i to		demand j
Y_{ij}	Allocation made from the supply i and		demand j

Abbreviations

LP	Largest Penalty
MC	Minimum Cost
ROM	Row Opportunity Matrix
COM	Column Opportunity Matrix
TOCM	Total Opportunity Cost Matrix
TC	Total Cost
OV	Objective Value

Mathematical Model of Transportation problem

The objective of the TP can be formulated as follow:

$$\begin{aligned} \text{Min } Z &= \sum_{i=1}^m \sum_{j=1}^n C_{ij} Y_{ij} \\ \text{Subject to } \sum_{j=1}^n Y_{ij} &= T_i \text{ for } i = 1, 2, \dots, m & \sum_{i=1}^m Y_{ij} &= R_j \text{ for } j = 1, 2, \dots, n \\ \text{where } Y_{ij} &\geq 0 \end{aligned}$$

Algorithm for TOCM-MT Approach

The steps of TOCM-MT method can be described as follows:

Step 1: Consider an original TP matrix $m \times n$ with cost C_{ij} , demands $R_j, j = 1, 2, \dots, n$ and supplies $T_i, i = 1, 2, \dots, m$. When total supply and total demand are not equal, then we have to add a dummy row or column.

Step 2: Obtain ROM from the original TP by finding the MC of each row and then subtracting each cost in the row with the MC.

Step 3: Obtain COM from the original TP by finding the MC of each column and then subtracting each cost in the column with the MC.

Step 4: Obtain the TOCM where the entries are the sum of the entries of ROM and COM.

Step 5: Find the penalty P_i for each row where the penalty is the total difference between the minimum cost MC_i and other costs in the row, as shown in Equations (1) and (2).

$$MC_i = \min (C_{ij}), j = 1, 2, \dots, n \tag{1}$$

$$P_i = \sum_{j=1}^n (C_{ij} - MC_i) \tag{2}$$

Step 6: Choose the Largest Penalty (LP) as shown in Equation (3). Use the following tie-breakers in case of tie in the given order:

- (i) Select LP with the smallest C_{ij} .
- (ii) In case of a tie in (i), select penalty with the greatest total cost TC_i as shown in Equation (4).

(iii) In case of a tie in (ii), select penalty with the maximum allocation of Y_{ij} .

$$LP = \max (P_i), i = 1, 2, \dots, m \quad (3)$$

$$TC_i = \sum_{j=1}^n C_{ij} \quad (4)$$

Step 7: Select the minimum cost (MC)

from the largest penalty. In case of a tie (i.e. equal MC), select MC with the maximum allocation of Y_{ij} .

Step 8: Check the value of MC. If MC is not equal to zero, then go to step 9, else if MC equal to zero then select the LP from the first LP (LP_1) or second LP (LP_2). Select the LP by comparing each cost cell in LP_1 and each cost cell in LP_2 as shown in

Equation (5), where C_{1j} is the cost at LP_1 and C_{2j} is the cost at LP_2 . Total OV_{1j} is the sum of OV_{1j} and Total OV_{2j} is the sum of OV_{2j} . LP is selected on the basis of Equation (6).

$$\text{Total } OV_i = \sum_{j=1}^n OV_{ij}$$

$$\text{where, } OV_{2j} = \begin{cases} 1 & \text{if } C_{1j} < C_{2j}, j = 1, 2, \dots, n \\ 0 & \text{if } C_{1j} \geq C_{2j}, j = 1, 2, \dots, n \end{cases} \quad (5)$$

(5)

$$\text{Total } GV_{2j} = \sum_{j=1}^n OV_{2j}$$

$$\text{where, LP} = \begin{cases} LP_1 & \text{if Total } OV_{1j} \geq \text{Total } OV_{2j} \\ LP_2 & \text{if Total } OV_{1j} < \text{Total } OV_{2j} \end{cases} \quad (6)$$

Step 9: Allocate the maximum possible units Y_{ij} to the cell having minimum cost of LP.

Step 10: Cross out the satisfied column or row by adjusting the supply and demand.

Step 11: Without considering the cross out rows and columns recalculate the penalty.

Step 12: Repeat steps 6–11 until all the rows and columns are satisfied.

Step 13: Finally, calculate the Total Cost Transportation Problem (TCTP) by combining TOCM-MT with the penalty and the original TP.

$$TCTP = \sum_{i=1}^m \sum_{j=1}^n C_{ij} Y_{ij}$$

2. MINIMIZATION TRANSPORTATION PROBLEM Consider four bases of operations B_i and three targets T_j . The tops of bombs per aircraft from any base that can be delivered to any target are given in the table below.

	T ₁	T ₂	T ₃	Supply
B ₁	8	6	5	150
B ₂	6	6	6	150
B ₃	10	8	4	150
B ₄	8	6	4	150
Demand	200	200	200	

The daily sortie capacity of each of the four bases is 150 sorties per day. The daily requirement in sorties over each individual target is 200.

Solution:

Step 1: Construct an original Transportation Problem. Step 2: Construct ROM from the original TP by subtracting each cost of the row from the least cost in that row. Here 5, 6, 4 & 4 are the minimum cost of row 1, 2, 3 & 4 respectively.

Table 1

Row opportunity matrix.

	T ₁	T ₂	T ₃	Supply
B ₁	3	1	0	150
B ₂	0	0	0	150
B ₃	6	4	0	150
B ₄	4	2	0	150
Demand	200	200	200	

Step 3: Construct COM from the original TP by subtracting each cost of the row from the least cost in that row. Here 6, 6 & 4 are the least cost of column 1, 2 & 3 respectively.

Table 2

Column opportunity matrix.

	T ₁	T ₂	T ₃	Supply
B ₁	2	0	1	150
B ₂	0	0	2	150
B ₃	4	2	0	150
B ₄	2	0	0	150
Demand	200	200	200	

Step 4: Construct the TOCM in which the entries are the sum of the row and column opportunity matrix.

Table 3 TOCM matrix.

	T ₁	T ₂	T ₃	Supply
B ₁	5	1	1	150
B ₂	0	0	2	150
B ₃	10	6	0	150
B ₄	6	2	0	150
Demand	200	200	200	

Step 5: Find the penalty for each row. Penalty 1 of row 1 is 4, row 2 is 2, row 3 is 16 and row 4 is 8.

Table 4

	T ₁	T ₂	T ₃	Supply	Penalty 1
B ₁	5	1	1	150	4
B ₂	0	0	2	150	2
B ₃	10	6	0	150	16
B ₄	6	2	0	150	8
Demand	200	200	200		

Step 6: Select the LP from LP_1 and LP_2 by comparing each cost cell in LP_1 with costs of LP_2 . The LP is LP_2 is 16 at row 3. Select the MC from the largest penalty. The MC is 0.

Table 5

	T ₁	T ₂	T ₃	Supply	Penalty 1
B ₁	5	1	1	150	4
B ₂	0	0	2	150	2
B ₃	10	6	0	150	0
B ₄	6	2	0	150	8
Demand	200	200	50		

Step 7: Adjust the supply and demand then cross out the satisfied supply B₃. Recalculate the penalty 2.

Step 8: Select the LP. The LP₁ is 8 and LP₂ is 4. By comparing each cost cell in LP₁ with costs of LP₂ we can get the LP as LP₂ is 8 at row 4. **Table 6**

	T ₁	T ₂	T ₃	Supply	Penalty 2
B ₁	5	1	1	150	4
B ₂	0	0	2	150	2
B ₄	6	2	0	50	100
Demand	200	200	0		

Step 9: By adjusting supply and demand, cross out the satisfied demand T₃. Recalculate the penalty 3 without considering the cross out rows and columns. **Table 7**

	T ₁	T ₂	Supply	Penalty 3
B ₁	5	1	15	0
B ₂	0	0	150	0
B ₄	6	2	100	4
Demand	200	50		

Step 10: Recalculate the Penalty 4 and Penalty 5 respectively. **Table 8**

	T ₁	T ₂	Supply	Penalty 4
B ₂	0	0	150	0
B ₄	0	50	50	4
Demand	200	0		

Table 9

	T ₁	Supply	Penalty
			5
B ₂	0	150 / 0	0
B ₄	6	50 / 0	6
Demand	200 / 150 / 0		

Step 11. Finally, calculate the Total Cost of Transportation Problem (TCTP) by combining TOCM-MT with the penalty and original transportation problem. **Table 10**

	T ₁	T ₂	T ₃	Supply
B ₁	8	6	5	150
B ₂	6	6	6	150
B ₃	10	8	4	150
B ₄	8	6	4	150
Demand	200	20 0	200	

Hence the allocation units given from Final Table are as follows:

$$X_{12} = 150, X_{21} = 150, X_{33} = 150, X_{41} = 50, X_{42} = 50, X_{43} = 50$$

$$\begin{aligned} \text{TCTP} &= 6(150) + 6(150) + 4(150) + 8(50) + 6(50) + 4(50) \\ &= \text{Rs.}3, 300. \end{aligned}$$

3. MAXIMIZATION TRANSPORTATION PROBLEM

Generally, in transportation problem the objective is to minimize. However, the transportation problem can also be useful for maximization. The procedure is as follows. Step-1. Convert the maximization problem into minimization problem where the entries are obtained by subtracting each element from the highest element of the table. Step-2. Find an initial feasible solution by using TOCM-MT method. Step-3. Find an optimal solution by using MODI method.

Experimental Result

A company has 3 factories. It provides readymade garments 4 dealers. The demand, supply and the transportation costs are given below in the following matrix. Obtain the distribution to maximize the profit.

	D ₁	D ₂	D ₃	D ₄	Supply
o ₁	15	51	42	33	23
o ₂	80	42	26	81	44
o ₃	90	40	66	60	33
Demand	23	31	16	30	

Step 1: Convert the maximization problem into minimization by subtracting all the elements from the largest element in the given transportation table. Here, the maximum element is 90. So subtract each value from 90. **Table 1**

	D ₁	D ₂	D ₃	D ₄	Supply
o ₁	75	39	48	57	23
o ₂	10	48	64	9	44
o ₃	0	50	24	30	33
Demand	23	31	16	30	

Step 2: Construct the TOCM in which the entries are the sum of the row and column opportunity matrix. **Table 2**

	D ₁	D ₂	D ₃	D ₄	Supply
o ₁	111	0	33	66	23
o ₂	11	48	95	0	44
o ₃	0	61	24	51	33
Demand	23	31	16	30	

Step 3: Select the Largest Penalty from LP_1 and LP_2 by comparing each cost cell in LP_1 with costs of LP_2 . The LP is LP_1 is 210 at row 1. **Table 3**

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty 1
o ₁	111	23 0	33	66	23 / 0	210
o ₂	11	48	95	0	44	154
o ₃	0	61	24	51	33	136
Demand	23	31	16	30		

Step 4: Find the largest penalty and allocate the cell with minimum cost (MC). **Table 4**

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty 2
o ₂	11	48	95	30 0	14	154
o ₃	0	61	24	51	33	136
Demand	23	8	16	0		

Step 5: Adjust supply and demand then cross out the satisfied demand D_4 . Recalculate the penalty 3.
 Step 6: Select the LP and allocate the cell having minimum cost (MC) from the largest penalty. The MC is 0. **Table 5**

	D_1	D_2	D_3	Supply	Penalty
					3
o_2	14	11	48	95	0
o_3	0	61	24	33	85
Demand	9	8	16		

Step 7: Adjust supply and demand then cross out the satisfied supply o_2 . Recalculate the penalty 3 without considering the cross out rows and columns. **Table 6**

	D_1	D_2	D_3	Supply	Penalty
					4
o_3		9	61	24	24
	0				85
Demand	0	8	16		

Step 8: Recalculate the Penalty 5 and Penalty 6 respectively. **Table 7**

	D_2	D_3	Supply	Penalty	Penalty
				5	6
o_1	8	16	24	24 / 8 / 37	61
	61	24	0		
Demand	8 / 0	16 / 0			

Table 8

	D_1	D_2	D_3	D_4	Supply
o_1	75	39	23	48	57
					23
o_2	10	14	48	64	9
					30
o_3	0	9	50	8	24
					16
					30
Demand	23	31	16	30	

Use MODI method for finding an optimal solution. The initial basic feasible solution has $m+n-1$ allocations, that is $3+4-1=6$ allocations in independent positions.

Now, take $u_3 = 0$, we get $v_1 = 0, v_2 = 50, v_3 = 24$.

Similarly, $u_1 + v_2 = 39$ or $u_1 + 50 = 39$ or $u_1 = -11$.

Again, $u_2 + v_1 = 10$ or $u_2 + 0 = 10$ or $u_2 = 10$.

In the same way, $u_2 + v_4 = 9$ or $10 + v_4 = 9$ or $v_4 = -1$.

Compute empty cell evaluations $\Delta_{ij} = C_{ij} - (u_i + v_j)$.

Table 9 ($[C_{ij}]$ matrix)

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	75		48	57	23
S ₂		48	57		44
S ₃				30	33
Demand	23	31	16	30	

Table 10 ($[u_i + v_j]$ matrix)

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	-		13	-	23
S ₂		60	34		44
S ₃				-1	33
Demand	23	31	16	30	

Table 11 ($[C_{ij} - (u_i + v_j)]$ matrix)

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	86		35	69	23
S ₂		-12	30		44
S ₃				31	33
Demand	23	31	16	30	

Select the empty cell evaluation $\Delta_{22} = -12$ as it is negative. So allocate θ to the cell (2, 2) and form a closed loop followed by alternatively subtracting and adding the amount of this allocation to other corners of the loop.

Table 12

	D ₁	D ₂	D ₃	D ₄	Supply
O ₁			23		
O ₁	75	39	48	57	23
O ₂	θ 14	$+\theta$		30	
O ₂	- 10	48	64	9	44
O ₃	+ 9	- 8	16		
O ₃	θ 0	θ 50	24	30	33
Demand	23	31	16	30	

So, $\theta = \min(14, 8) = 8$ units from the cell (3, 2) to (2, 2). **Table 13**

	D ₁	D ₂	D ₃	D ₄	Supply
O ₁	75	23 39	48	57	23
O ₂	6 10	8 48	64	30 9	44
O ₃	17 0	50	16 24	30	33
Demand	23	31	16	30	

Now, take $u_2 = 0$, we get $v_1 = 10, v_2 = 48, v_4 = 9$.

Similarly, $u_1 + v_2 = 39$ or $u_1 + 48 = 39$ or $u_1 = -9$.

Again, $u_3 + v_1 = 0$ or $u_3 + 10 = 0$ or $u_3 = -10$.

In the same way, $u_3 + v_3 = 24$ or $-10 + v_3 = 24$ or $v_3 = 34$.

Compute empty cell evaluations $\Delta_{ij} = C_{ij} - (u_i + v_j)$. **Table 14** ($[C_{ij}]$ matrix)

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	75		48	57	23
S ₂			64		44
S ₃		50		30	33
Demand	23	31	16	30	

Table 15 ($[u_i + v_j]$ matrix)

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	1		25	0	23
S ₂			34		44
S ₃		38		-1	33
Demand	23	31	16	30	

Table 16 ($[C_{ij} - (u_i + v_j)]$ matrix)

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	74		23	57	23
S ₂			30		44
S ₃		12		31	33
Demand	23	31	16	30	

All the empty cell evaluations are positive. Hence we reached at the optimal stage. The final solution table is given in table 17. **Table 17**

	D ₁	D ₂	D ₃	D ₄	Supply
O ₁	15	5 1	42	33	23
O ₂	6 80	8 42	26	30 81	44
O ₃	17 90	40	16 6	6 60	33

Demand	23	31	16	30	
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The allocation units for this solution becomes:

$$X_{12}= 23, X_{21} = 6, X_{22}=8, X_{24}=30, X_{31}=17, X_{33}=16$$

$$\begin{aligned} \text{Cost} &= 51 \times 23 + 80 \times 6 + 42 \times 8 + 81 \times 30 + 90 \times 17 + 66 \times 16 \\ &= \text{Rs.}7,005. \end{aligned}$$

4. TIME-MINIMIZING TRANSPORTATION PROBLEM

Instead of focusing on reducing the cost of transportation, time minimizing transportation problems aim to do the opposite. For instance, in the military, the time of supply is more valuable than the cost of transportation, hence it is always preferable to reduce the overall time of supply rather than the cost. These issues are frequently seen in emergency services, including the military, hospital administration, fire services, etc.

Time minimization transportation problems are actually quite similar to the cost-minimization transportation problems, with the exception that the unit time t_{ij} is used in place of unit cost C_{ij} , which is the cost per unit of time to transport the items from i th origin to j th destination.

First by applying TOCM-MT method, we have to find the initial basic feasible solution. Now if \tilde{T}_j is the largest transportation time associated with the j th feasible plan, then we have to find such plan that gives minimum \tilde{T}_j .

Mathematical model of TP

The objective of the TP can be formulated as follow:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n t_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} = S_i \text{ for } I = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = D_j \text{ for } j = 1, 2, \dots, n$$

where $x_{ij} \geq 0$

Notations m Total number of supply n Total number of demand

S_i Supply I

D_j Demand j

t_{ij} Transportation time from the supply I to demand j x_{ij} Allocation made from the supply I and demand j

Algorithm

Step-1. First apply TOCM-MT method and find the initial basic feasible solution.

Step-2. Compute \tilde{T}_j for the basic feasible solution and cross out all the non-basic cells for which $t_{ij} \geq \tilde{T}_j$.

Step-3. Create a loop for the \tilde{T}_j -corresponding basic cells such that as the values in the cell's corners are moved around, this cell's values tend (not necessarily) to zero and none of the variables become zero. Then the solution is optimal, if such a closed path cannot exist, otherwise go to step-2. **Step-4.** Continue the process unless an optimum basic feasible solution is determined.

Experimental Result

Consider the following transportation problem. Find the minimum shipping time.

	W ₁	W ₂	W ₃	Supply
F ₁	16	20	12	200
F ₂	14	8	18	160
F ₃	26	24	16	90
Demand	180	120	150	

Step-1. Find an initial basic feasible solution using TOCM-MT method. **Table 1** (TOCM matrix)

	W ₁	W ₂	W ₃	Supply
F ₁	6	20	0	200
F ₂	6	0	16	160
F ₃	22	24	4	90
Demand	180	120	150	

Table 2

	W ₁	W ₂	W ₃	Supply	Penalty 1
F ₁	6	20	0	200	26
F ₂	6	0	16	160	22
F ₃	22	24	4	90	38
Demand	180	120	150		

Table 3

	W ₁	W ₂	W ₃	Supply	Penalty 1
F ₁	6	20	0	200	26
F ₂	6	0	16	160	22
F ₃	22	24	4	90	38
Demand	180	120	150	0	

Table 4

	W ₁	W ₂	W ₃	Supply	Penalty 2
F ₁	6	20	0	60	26
F ₂	6	0		160	22
Demand	180	120	0		

Table 5

	W ₁	W ₂	Supply	Penalty
				3
F ₁	6	60 20	0	14
F ₂	6	0	160	6
Demand	40	120		

Table 6

	W ₁	W ₂	Supply	Penalty
				4
F ₁	6	40 0	120 0	6
Demand	0	0		

Table 7

	W ₁	W ₂	W ₃	Supply
F ₁	16	140 20	12 60	200
F ₂	14	40 8	120 18	160
F ₃	26	24	16 90	90
Demand	180	120	150	

Here, $t_{11} = 16, t_{13} = 12, t_{21} = 14, t_{22} = 8, t_{33} = 16$.

$\tilde{T}_1 = \max\{16, 12, 14, 8\} = 16$.

Cross out the non-basic cells, where $t_{ij} > \tilde{T}_1$.

Table 8

16	140	20	12	60
14	40	8	120	18
26	24	16	90	

Now we cannot form any closed loop. So, the solution is optimal.

Hence, the shipping time is 16 time unit.

5. TRANSHIPMENT PROBLEM

The term “transshipment problem” refers to a situation where a commodity is readily available and regularly shifts between sources or destinations before arriving at its final destination.

Here are some key features of the transshipment problems:

- a. In transportation problem, there are m sources and n destinations whereas, in transshipment problem, there are m+n sources and destinations.
- b. If S_i is the i th source and D_j is the j th destination then the transportation costs from S_i to S_j is zero and the transportation costs from S_i to S_j or S_i to D_i are not equal.
- c. In order to solve the transshipment problem, we must find the optimum solution to the transportation problem. We then solve the transshipment problem in the same way.
- d. $2m+2n-1$ number of basic variables are there in basic feasible solution. We are left with $m+n-1$ basic variables after eliminating the variables that appear in the $m+n$ diagonal cells.

Experimental Result

Consider the following transshipment problem with two sources and two destinations, the costs for shipment in rupees are given below. Determine an optimum shipping schedule.

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁	0	1	6	2	4
S ₂	1	0	8	2	5
D ₁	0	7	0	3	
D ₂	1	0	1	0	
Demand			3	6	

Step-1. Each given source and destination in the transshipment problem may be referred as a source or destination. If we now assume that there is no demand at any of the destinations S₁, S₂ or any of the sources D₁, D₂ and that there is no amount available at any of them, then a hypothetical quantity of supply and demand known as “buffer stock” is added to all the points in order to have a supply and demand. The buffer stock is 9 units.

Table 1 An original transportation problem.

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁	0	1	6	2	¹³
S ₂	1	0	8	2	¹⁴
D ₁	0	7	0	3	⁹
D ₂	1	0	1	0	⁹
Demand	9	9	12	15	

Table 2 TOCM matrix.

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁	0	2	12	4	¹³
S ₂	2	0	16	4	¹⁴
D ₁	0	14	0	6	⁹
D ₂	2	0	2	0	⁹
Demand	9	9	12	15	

Table 3 (Penalty 1 for each row)

	S ₁	S ₂	D ₁	D ₂	Supply	Penalty 1	
S ₁	0	2	12	4	13	18	
S ₂	2	0	9	16	4	5	22
							20
							4
D ₁	0	14	0	6	9		
D ₂	2	0	2	0	9		
Demand	9	0	12	15			

Table 4 (Penalty 2 for each row)

	S ₁	D ₁	D ₂	Supply	Penalty 2	
S ₁	0	12	4	13		
S ₂	2	16	4	0	5	16
						16
D ₁	0	0	6	9	6	
D ₂	2	2	0	9	4	
Demand	4	12	15			

Table 5 (Penalty 3 for each row)

	S ₁	D ₁	D ₂	Supply	Penalty 3	
S ₁	0	12	4	9	4	6
D ₁	0	0	6	9	6	
D ₂	2	2	0	9	4	
Demand	4	12	15			

Table 6 (Penalty 4 for each row)

	D ₁	D ₂	Supply	Penalty 4	
S ₁	12	4	0	9	8
D ₁	0	6	9	6	
D ₂	2	0	9	2	
Demand	12	6			

Table 7 (Penalty 5 for each row)

	D ₁	D ₂	Supply	Penalty 5
D ₁	0	9	6	0
D ₂	2	3	0	6
Demand	0	0		

Table 8

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁	0	4		9	13
S ₂	2	5	9		14
D ₁			0	9	9
D ₂			2	3	6
Demand	9	9	12	15	

Use MODI method for finding an optimal solution.

Now, take $u_1 = 0$, we get $v_1 = 0$.

Similarly, $u_2 + v_1 = 2$ or $u_2 = 2$.

Again, $u_2 + v_2 = 0$ or $v_2 = -2$.

In the same way, $u_1 + v_4 = 4$ or $v_4 = 4$ and $u_4 + v_3 = 2$ or $v_3 = 6$ and $u_3 = -6$.

Table 9 ($[C_{ij}]$ matrix)

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁		2	12		13
S ₂			16	4	14
D ₁	0	14		6	9
D ₂	2	0			9
Demand	9	9	12	15	

Table 10 ($[u_i + v_j]$ matrix)

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁		-2	6		13
S ₂			8	6	14
D ₁	-6	-8		-2	9
D ₂	-4	-6			9

Demand	9	9	12	15	
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Table 11 ($[C_{ij} - (u_i + v_j)]$ matrix)

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁		4	6		13
S ₂			8	-2	14
D ₁	6	22		8	9
D ₂	6	6			9
Demand	9	9	12	15	

Select the empty cell evaluation $\Delta_{24} = -2$ as it is negative. So allocate θ to the cell (2, 4) and form a closed loop followed by alternatively subtracting and adding the amount of this allocation to other corners of the loop. **Table 12**

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁	θ 0 4			θ 4 9	
S ₂	θ 2 5	θ 9		$+\theta$	
D ₁	0	14	0 9	6	9
D ₂	2	0	2 3	0 6	9
Demand	9	9	12	15	

So, $\theta = \min(5, 9) = 5$ units from the cell (1, 4) to (2, 4).

Table 13

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁	0 9 2			4 12	13
S ₂	2	θ 9	16	4 5	14
D ₁	0	14	0 9		9
D ₂	2	0	2 3	0 6	9
Demand	9	9	12	15	

Now, take $u_1 = 0$, we get $v_1 = 0, v_4 = 4$.

Similarly, $u_2 + v_4 = 4$ or $u_2 = 0$.

Again, $u_2 + v_2 = 0$ or $v_2 = 0$.

In the same way, $u_4 + v_4 = 0$ or $u_4 + 4 = 0$ or $u_4 = -4$ and $u_4 + v_3 = 2$ or $-4 + v_3 = 2$ or $v_3 = 6$ and $u_3 = -6$. Compute empty cell evaluations $\Delta_{ij} = C_{ij} - (u_i + v_j)$. **Table 14** ($[C_{ij}]$ matrix)

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁		2	12		13
S ₂	2		16		14
D ₁	0	14		6	9
D ₂	2	0			9
Demand	9	9	12	15	

Table 15 ($[u_i + v_j]$ matrix)

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁		0	6		13
S ₂	0		6		14
D ₁	-6	-6		-2	9
D ₂	-4	-4			9
Demand	9	9	12	15	

Table 16 ($[C_{ij} - (u_i + v_j)]$ matrix)

	S ₁	S ₂	D ₁	D ₂	Supply
S ₁		2	6		13
S ₂	2		10		14
D ₁	6	20		8	9
D ₂	6	4			9
Demand	9	9	12	15	

All the empty cell evaluations are positive and hence, we reached at the optimal stage. The final solution table is given in table 13.

Total cost = Rs. $(4 \times 9 + 2 \times 5 + 2 \times 3)$ = Rs.52. The shipping cost is Rs.52.

CONCLUSION

An optimal solution of the objective function for TP depends on non-degeneracy IBFS. This study provides a TOCM-MT technique for obtaining IBFS, which leads to a direct optimal solution for TP. After analyzing VAM, JHM, and TDM1 methods, including TOCM-MT, through various applications, it concludes that TOCM-MT gives the best result and can apply to solving real-life TP problems.

REFERENCES

1. Juman ZAMS, Hoque MA. An efficient heuristic to obtain a better initial basic feasible solution to the transportation problem. Appl Soft Comput J 2015; 34:813-26.
2. Deshmukh NM. An Innovative Method for Solving Transportation. Int J Phys Math Sci 2012; 2(3):86-91.

3. Bilqis Amaliah, Chastine Fatichah, Erma Suryani. Total opportunity cost matrix-minimal total: A new approach to determine initial basic feasible solution of a transportation problems. *Egyptian Informatics Journal* 20(2019) 131-141.
4. Hosseini E. Three new methods to find initial basic feasible solution of transportation problems. *Appl Math Sci* 2017; 11(37):1803-14.
5. Kousalya P, Malarvizhi P. A new technique for finding initial basic feasible solution to transportation problem. *Int J Eng Manage Res* 2016; 6(3):26-32.
6. Soomro AS et. al. Modified Vogel's Approximation method for solving transportation problem. *Math Theory model* 2015; 5(4):32-43.
7. Kirca O, Satir A. A Heuristic for obtaining an initial solution for the transportation problem. *J Oper Res Soc* 1990; 41(9):865-71.
8. Juman ZAMS, Hoque M.A. A heuristic solution technique to attain the minimal total cost bounds of transporting a homogeneous product with varying demands and supplies. *Eur J Oper Res* 2014; 239(1):146-56.
9. Hakim MA. An alternative method to find initial basic feasible solution of a transportation problem. *Ann Pure Appl Math* 2012; 1(2):203-9.
10. Seethalakshmy A, Srinivasan N. An improved algorithm to obtain initial basic feasible solution for the transportation problem. *Int J Sci Res* 2015; 6(4):1225-9.