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Estimated Solutions of Intigral Equations Arising In Some Applications of Science and Engineering

P Madhura Subhashini

Associate Professor, B V Raju College

Abstract:
We present the numerical solution of the Fredholm Integral Equations by using the analytic method (Adomian Decomposition Method). To exhibit the correctness and efficacy of the projected method (ADM), some numerical examples have been performed. A Fredholm integral equations is solved by ADM which gives us the fairly accurate solution of the problem that tends to the exact solution of the problem.

Keywords:Adomian Decomposition Method, Integral Equations,Fredholm Integral Equations, Numerical Example. Adomian Decomposition Method

Adomian Decomposition Method

The Adomian Decomposition method (ADM) is very commanding technique which considers the inexact solution of a nonlinear equation as an infinite series which essentially converges to the exact solution in this paper, ADM is proposed to solve some first order, second order and third order differential equations and integral equations. The Adomian Decomposition method (ADM) was firstly introduced by George Adomain in 1981. This method has been applied to solve differential equations and integral equations of linear and nonlinear problem in Mathematics, Physics, Biology and Chemistry up to know a large number of research paper have been published to show the possibility of the decomposition method.

Proposed method for solving the Fredholm integral equation.

The type of integral equation in which the restrictions of the integration are constant, in which a and b are constant are called the Fredholm Integral equations, and is given as

$$\emptyset(x) = f(x) + \rho \int_a^b K(x,t)\emptyset(t)(t)dt \tag{1}$$

Where the function and the kernel are given in the advance, and ρ is a parameter. In this division, the procedure of the Adomian decomposition method is used. The Adomian decomposition method connecting the decomposing of the unknown function (x) of any equation into a addition of an infinite number of constituents defined by the
decomposition serie

$$\emptyset(x) = \sum_{n=0}^{\infty} \emptyset_n(x) \tag{2}$$

Or In the same way

$$\emptyset(x) = \emptyset_1(x) + \emptyset_2(x) + \emptyset_3(x) \pm \cdots$$

When the constituents $\emptyset_n(x), n \geq 0$ will be resolved.
The Adomain decomposition method analyze itself which discover the components $\emptyset_0$(x), $\emptyset_1$(x),, $\emptyset_2$(x)…

To classify the recurrence relation , we substitute (2) into the Fredholm integral equation (1) to get

$$\sum_{n=0}^{\infty} \emptyset_n(x) = f(x) + \int_a^b K(x,t) \sum_{n=0}^{\infty} \emptyset_n(t)dt \qquad (3)$$

The zeroth component $\emptyset_0$(x) is spotted by all terms that are not comprises under the integral sign. This signifies that the components $\emptyset_n(x), n \geq 0$ of the unknown function $\emptyset(x)$ are totally resolved by the recurrence relation.

$\emptyset_0$(x) = f(x), $\emptyset_{n+1}(x) = \int_a^b K(x,t) \sum_{n=0}^{\infty} \emptyset_n(t)dt \ , n \geq 0$

Or correspondingly

Thus the solution of the Fredholm Integral equation (1) is easily acquired in a series form by make use of he series as assumption in (2)

Applications of Fredholm Integral Equations:
Consider the linear Fredholm integral equation

$$\mathbf{1.}\ \emptyset(\mathbf{x}) = \mathbf{sinx} + \mathbf{1} + \int_{\mathbf{0}}^{\boldsymbol{\pi}} \sum_{\mathbf{n=0}}^{\infty} \emptyset(\mathbf{t})\mathbf{dt}$$

Let $\emptyset(x) = \sum_{n=0}^{\infty} \emptyset_n(x)$

$$\emptyset_0(x) = \sin x + 1$$

$$\sum_{n=0}^{\infty} \emptyset_n(x) = sinx + 1 + \int_0^{\Pi} \sum_{\pi=0}^{\infty} \emptyset_{n-1}(t)\, dt$$

Now $\phi_{n+1}(x) = \int_0^{\Pi} \sum_{\pi=0}^{\infty} \emptyset_{n-1}(t)\, dt$

$$\emptyset_1(x) = \int_0^{\pi} \emptyset_0(t)dt$$

$= \int_0^{\pi}(sint + 1)dt$
$= \int_0^{\pi} sint + \int_0^{\pi} dt$
$= (-cost)_0^{\pi} + (t)_0^{\pi}$
$= (-cos\pi - cos0) + (\pi - 0)$
$= (-1 - 1) + (\pi - 0)$

$$\emptyset_1(x) = -2 - \pi$$

$$\emptyset_2(x) = \int_0^{\pi} \emptyset_1(t)dt$$

$= \int_0^{\pi}(-2-\pi)dt$
$= (-2-\pi)(t)_0^{\pi}$
$=(-2-\pi)(\pi-0)$

$$\emptyset_2(x) = -2\pi - \pi^2$$

$$\emptyset_3(x) = \int_0^{\pi} \emptyset_2(t)dt$$

$= \int_0^{\pi}[-2\pi-\pi^2]dt$
$= [-2\pi-\pi^2](t)_0^{\pi}$
$= [-2\pi-\pi^2](\pi-0)$

$$\emptyset_3(x) = -2\pi^2 - \pi^3$$

$$\emptyset \quad (x) = sinx + 1 + (-2 - \pi - 2\pi - \pi^2 - 2\pi^2 - \pi^3 + \cdots)$$

$$\emptyset \quad (x) = sinx - 1 + (\ -3\pi - 3\pi^2 - \pi^3 + \cdots)$$

2).$\boldsymbol{\emptyset(x) = x + e^{-x} + x^2 \int_0^1 \sum_{n=0}^{\infty} t\emptyset(t)dt}$

$$Let\ \emptyset(x) = \sum_{n=0}^{\infty} \emptyset_n\,(\mathrm{x})$$

$= \sum_{n=0}^{\infty} \emptyset_n(x) = x + e^{-x} + x^2 \int_0^1 \sum_{n=0}^{\infty} t\emptyset(t)dt$
Where $\emptyset_0(x) = x + e^{-x}$
$= \emptyset_{n+1}(x) = x^2 \int_0^1 \sum_{n=0}^{\infty} t\emptyset(t)dt$

$$\emptyset_1(x) = x^2 \int_0^1 t\emptyset_0(t)dt$$

$= x^2 \int_0^1 t(t+e^{-t})dt$
$= x^2[\int_0^1 t^2\,dt + \int_0^1 te^{-t}dt]$
$= x^2[\left(\frac{t^3}{3}\right)_0^1 + (-te^{-t})_0^1 - (-e^{-t})_0^1]$
$= x^2[\left(\frac{1}{3} - \frac{1}{0}\right) + (-e^{-1} - 0e^0) - (-e^{-1} - e^0)]$
$= x^2(\frac{1}{3} - e^{-1} + e^{-1})$

$$\emptyset_1(x) = \frac{x^2}{3}$$

$$\emptyset_2(x) = x^2 \int_0^1 t\emptyset_1(t)dt$$

$= x^2 \int_0^1 t(\frac{t^2}{3})dt$
$= \frac{x^2}{3} \int_0^1 t^3dt$
$= \frac{x^2}{3}\left(\frac{t^4}{4}\right)_0^1$

$=\frac{x^2}{3}(\frac{1}{4}-\frac{0}{4})$

$$\emptyset_2(x)=\frac{x^2}{12}$$

$$\emptyset_3(x)=x^2\int_0^1 t\emptyset_2(t)dt$$

$=x^2\int_0^1 t(\frac{t^2}{12})dt$

$=\frac{x^2}{12}\left(\frac{t^4}{4}\right)_O^1$

$=\frac{x^2}{12}\left(\frac{1}{4}-\frac{1}{0}\right)$

$$\emptyset_3(x)=\frac{x^2}{48}$$

$:\emptyset(x)=x+e^{-x}+(\frac{x^2}{3}+\frac{x^2}{12}+\frac{x^2}{48}+\cdots)$

$$\emptyset(x)=x+e^{-x}+\frac{x^2}{3}\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\right)$$

3) $\boldsymbol{\emptyset(x)=\frac{2}{3}+5x+x\int_0^1 t\emptyset(t)dt}$

$$Let\ \emptyset(x)=\sum_{n=0}^{\infty}\emptyset_n(t)$$

$$\emptyset_0(x)=\frac{2}{3}+5x$$

$$\emptyset_{n+1}(x)=x\int_0^1\sum_{n=0}^{\infty}t\emptyset(t)dt$$

$$\emptyset_1(x)=x\int_0^1 t\emptyset_0(t)dt$$

$=x\int_0^1 t(\frac{2}{3}+5t)$

$=\frac{2}{3}x\int_0^1 tdt+5x\int_0^1 t^2dt$

$=\frac{2}{3}x\left(\frac{t^2}{2}\right)_1^0-5x\int_0^1 t^2\,dt$

$=\frac{2}{3}x\left(\frac{t^2}{2}\right)_0^1+5x\left(\frac{t^3}{3}\right)_0^1$

$=\frac{2}{3}x\left(\frac{1}{2}-\frac{0}{2}\right)+5x\left(\frac{1}{3}-\frac{0}{3}\right)$

$=\frac{2}{6}x+\frac{5}{3}x$

$=\frac{2x+10x}{6}$

$=\frac{12x}{6}$

$=\emptyset_1(x)=2x$

$$\emptyset_2(x)=x\int_0^1 t\emptyset_1(t)dt$$

$=x\int_0^1 t(2t)dt$

$= x2\int_0^1 (t^2)\,dt$

$= 2x\left(\frac{t^3}{3}\right)_0^1$

$= 2x\left(\frac{1}{3} - 0\right)$

$= \emptyset_2(x) = \frac{2x}{3}$

and so on

$$\emptyset(\boldsymbol{x}) = \frac{2}{3} + 5x + 2x + \frac{2x}{3} + \cdots$$

4) $\emptyset(\boldsymbol{x}) = \boldsymbol{e}^{x} + \mathbf{1} + \int_0^1 \emptyset \boldsymbol{t}\, \boldsymbol{dt}$

$$Let\ \emptyset(x) = \sum_{n=0}^{\infty} \emptyset_n(x)$$

$$\emptyset(x) = \emptyset_1(x) + \emptyset_2(x) + \emptyset_3(x) + \cdots$$

$$\sum_{n=0}^{\infty} \emptyset_n\,(x) = f\,(x) + J_a^b k(x,t) \sum_{n=0}^{\infty} \emptyset_n(t)dt$$

$$Let \emptyset_0(x) = e^x + 1$$

$$\emptyset_{n+1}(x) = \int_0^1 \sum_{n=0}^{\infty} \emptyset_n(t)dt$$

$$This\ implies\ \emptyset_0(x) = e^x + 1$$

$$\emptyset_1(x) = \int_0^1 \emptyset_0(t)dt$$

$= \int_0^1 (e^t + 1)tdt$

$= \int_0^1 (te^t + t)dt$

$= \int_0^1 te^t dt + \int_0^1 tdt$

$= t\int_0^1 e^t dt - \int_0^1 e^t + \int_0^1 tdt$

$= [\mathrm{te}^t - e^t + \frac{t^2}{2}]_0^1$

$= (1.e^1 - e^1 + \frac{1}{2} - 0 + 1 - 0)$

$= 1 + \frac{1}{2}$

$= \frac{2+1}{2}$

$$\emptyset_1(x) = \frac{3}{2}$$

$$\emptyset_2(x) = \int_0^1 \emptyset_1(t)dt$$

$= \int_0^1 t \cdot \frac{3}{2} dt$

$= \frac{3}{2}\int_0^1 t\,dt$

$= \frac{3}{2}\left(\frac{t^2}{2}\right)_0^1$

$=\frac{3}{4}(1)$

$$\emptyset_2(x)=\frac{3}{2^2}$$
$$\emptyset_3(x)=\int_0^1 \emptyset_2(t)dt$$

$=\int_0^1 \frac{3}{4}tdt$
$=\frac{3}{4}\int_0^1 tdt$
$=\frac{3}{4}(\frac{t^2}{2})_0^1$
$=\frac{3}{8}$

$$\emptyset_3(x)=\frac{3}{2^3}$$
$$\emptyset(x)=\emptyset_1(x)+\emptyset_{2(x)}+\emptyset_3(x)+\cdots$$
$$\emptyset(x)=e^x+1+\frac{3}{2}+\frac{3}{2^2}+\frac{3}{2^3}+\cdots$$

Conclusion

The aspire of this paper is to employ the Adomain Decomposition Method for solving the Fredholm Integral Equation . It can be visibly seen that decomposition method for the Fredholm Integral Equation is equivalent to consecutive approximation method.Even though the Adomain decomposition method is very burly and useful tool for solving the integral equations.

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