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A Fuzzy Inventory Replenishment Model for Deteriorating and Ameliorating Items under Linear Trend in Demand and Partial Backlogging.

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Abstract:
This research paper deals with an inventory replenishment model for both deteriorating and ameliorating items under linear trend in demand over a fixed time horizon. This paper aimed to develop the model with shortages which are partially backlogged. The model is developed under the fuzzy environment. Optimal technique is used to solve the model considering Signed Distance Method (SD). The ordering cost, purchase cost, holding cost, deteriorating cost, ameliorating cost, shortage cost and opportunity cost are all considered as fuzzy cost parameters. Finally the fuzzy model is illustrated with the help of a numerical example and a sensitivity of the optimal solution towards the changes in the values of all fuzzy cost parameters is also furnished, and a concluding remark is given expressing the model development and also future research works in consequence of the present.

Keywords: Inventory, replenishment, fuzzy, deteriorating, ameliorating, linear trend in demand, shortages and partial backlogging.

Subject classification: AMS Classification No. 90B05

1. Introduction:

In the present business sectors, the highly business asset plays an important role in production and warehouses management. Large number of raw materials is required for starting production which is based on highly financial structure. The basic feature of the inventory system stands on supply chain management policy all over. In inventory models, there are uncertainties in the cost coefficients. The present paper deals with the importance of fuzzy in uncertain coefficients to find optimal solution using method of defuzzification, namely signed distance method (SD).

Due to competitive market, the replacement policy or exchange policy is more attractive to the customers nowadays. Many shopping sites like Flipkart, Amazon, Zomato, etc provide the opportunity to replace/exchange the deteriorating production like food items, dry foods, and clothes etc. which are commonly used in every family. Again the non-instantaneous policy applied in cosmetics, eggs, fish etc. are attracted by the customer and getting more benefit after using fresh products.

Inventory management system has an important role in any business enterprises. Most of the inventory models are based on demand and supply. Haris [1] developed first an Economic Order Quantity (EOQ) model. Later Hadly and Whitin [2] considered an inventory model assuming both types of parameters constant and variable form. Consequently research development on inventory model has been focused to crisp concept (deterministic model) where several demand pattern like constant, linear, quadratic, cubic, exponential, price depended etc. are assumed considering deteriorating and ameliorating items. In this context, we remember many researchers like Donaldson [3], Hwang [4], [5]. Nodoust et al [6], Mallick et al [7], Laily et al [8] etc who have several research papers published in different national and international journals.

Fuzzy is one of the most prominent techniques to find optimal solution under uncertainty nature. The inventory cost coefficients such as ordering cost, purchase cost, holding cost, deteriorating cost, ameliorating cost, shortage cost and opportunity cost are very difficult to assume during determination of the optimal solution of the fuzzy model. Zadeh [9] is the first effort to discuss the fuzzy set theory which is based on the uncertainty of the cost parameters. Later Bellman and Zadeh [10] explained in their research works on how to integrate the fuzzy constraints. Kao and Hsu [11] developed the inventory model under trapezoidal fuzzy demand rate. Gradually many developments are followed by Sujit [12], Hossen [13], Yadav [14] etc.

In practice, some customers would like to wait for backlogging during that shortage period, but other would not. Consequently, the opportunity cost due to lost sales should be considered in the modelling. Zhao[16], Giri et al. [17]. Skouri et al [18], Biswaranjan Mandal [19].[20] etc assumed that the backlogging rate was a fixed fraction of demand rate during this period. However, in some inventory system, for some fashionable commodities , the length of waiting time for next replenishment become main factor for determining whether the backlogging will be accepted or not. The longer the waiting time is, the smaller the backlogging rate would be. Therefore backlogging rate is variable and is dependent on the waiting time for the next replenishment.

In this paper, we developed a fuzzy inventory model for deteriorating and ameliorating items with linear trend in demand and partial backlogging over a fixed time horizon. The purpose of the study is to defuzzify the total inventory cost function using signed distance method (SD). The model is illustrated by a suitable numerical example and a sensitivity analysis for the optimal solution towards changes in the fuzzy cost parameters is discussed. Lastly a concluding remark is given expressing the nature of the present model and also future research work in consequence of the present model.

2. Definitions and Preliminaries :

We have stated the following definitions for development of the fuzzy inventory model.

a) A fuzzy set X on the given universal set is a set of order pairs and defined by

$$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)) : x \in X\}, \text{ where } \lambda_{\tilde{A}} : X \to [0,1] \text{ is called membership function.}$$

b) A fuzzy number $\tilde{A}$ is a fuzzy set on the real number R, if its membership function $\lambda_{\tilde{A}}$ has the following properties

(i). $\lambda_{\tilde{A}}(x)$ is upper semi continuous.

(ii). $\lambda_{\tilde{A}}(x)$ =0, outside some interval $[a_1, a_4]$

Then $\exists$ real numbers a_2 and a_3, $a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$ and decreasing on $[a_3, a_4]$ and $\lambda_{\tilde{A}}(x)$ =1 for each $x \in [a_1, a_2]$.

c) A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is represented with membership function $\lambda_{\tilde{A}}$ as

$$\lambda_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ 1, a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, a_3 \le x \le a_4 \\ 0, otherwise \end{cases}$$

d) Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, the arithmetical operations are defined as:

(i) $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

(iii) $\tilde{A} \Theta \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iv) $\tilde{A} \phi \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$

(v) $\alpha \otimes \tilde{A} = \{ \begin{matrix} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \ge 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), \alpha < 0 \end{matrix}$

e) Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a fuzzy set defined on R, then the Signed Distance Method(SD) of $\tilde{A}$ is defined as

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_G(\alpha)] d\alpha = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

3. Notations and Assumptions:

The present inventory model is developed under the following notations and assumptions:

Notations:

(i) I(t) : On hand inventory at time t.

(ii) R(t) : Demand rate.

(iii) Q : On-hand inventory.

(iv) θ : The constant deterioration rate where $0 \le \theta < 1$

(v) A(t) : Two parameter Weibull distributed ameliorating rate.

(vi) T : The fixed length of each production cycle.

(vii) $\tilde{A}_0$: The fuzzy ordering cost per order during the cycle period.

(viii) $\tilde{p}_c$: The fuzzy purchasing cost per unit item.

(ix) $\tilde{h}_c$: The fuzzy holding cost per unit item.

(x) $\tilde{d}_c$: The fuzzy deterioration cost per unit item.

(xi) $\tilde{a}_c$: The fuzzy cost of amelioration per unit item.

(xii) $\tilde{s}_c$: The fuzzy cost due to shortage per unit item.

(xiii) $\tilde{o}_c$: The fuzzy opportunity cost per unit item.

(xiv) $\tilde{TC}$: The fuzzy total cost of the system per unit time.

Assumptions:

(i). Lead time is zero.
(ii). Replenishment rate is infinite but size is finite.
(iii). The time horizon is finite.
(iv). There is no repair of deteriorated items occurring during the cycle.
(v). Amelioration and deterioration occur when the item is effectively in stock.
(vi). The demand rate is linear tended function of time

R(t) = $a+bt$, $a \geq 0, b \geq 0$ where a is the first demand level and b is the second demand level.

(vii). The amelioration rate A(t) with the Weibull distribution

A(t) = $\alpha\beta t^{\beta-1}$, $0 \leq \alpha < 1$, $\beta \geq 1$ where α is scale and β is shape parameter. When β = 1, A(t) becomes a constant which is the case of an exponential nature of amelioration; when β <1, the amelioration rate is decreasing with time, and when β >1, it is increasing with time.

(viii). Shortages are allowed and they adopt the notation used in Abad[15], where the unsatisfied demand is backlogged and the fraction of shortages backordered is $e^{-\delta t}$, where δ is a positive constant and t is the waiting time for the next replenishment. We also assume that $te^{-\delta t}$ is a nested function.

4. Fuzzy Model development and Solution:

The inventory system consists Q units of the product at the beginning of each cycle. Due to the effect of demand, deterioration and amelioration, the inventory level decreases in [0, t_1] and becomes zero at t = t_1. The shortages occur during time period [t_1, T] which are partially backlogged. The behaviour of the model at any time t can be described by the following differential equations:

$$\frac{dI(t)}{dt}+(\theta-\alpha\beta t^{\beta-1})I(t)=-(a+bt), 0\le t\le t_1 \qquad (4.1)$$

And $$\frac{dI(t)}{dt}=-(a+bt)e^{-\delta(T-t)}, t_1\le t\le T \qquad (4.2)$$

The boundary conditions are $I(0)=Q$ and $I(t_1)=0$ (4.3)

The solutions of the equations (4.1) and (4.2) using (4.3) are given by the following

$$\text{I(t)} = Q(1-\theta+\alpha t^{\beta})-\{at+(\frac{b}{2}-\frac{a\theta}{2})t^2+\frac{b\theta}{3}t^3+\frac{a\alpha\beta}{\beta+1}t^{\beta+1}+\frac{b\alpha\beta}{2(\beta+2)}t^{\beta+2}\},$$

$$0\le t\le t_1 \qquad (4.4)$$

And $$\text{I(t)} = (\frac{a+bt}{\delta}-\frac{b}{\delta^2})e^{-\delta(T-t)},\ t_1\le t\le T \qquad (4.5)$$

Since $I(t_1)=0$, we get the following expression of on-hand inventory from the equation (4.4)

$$\text{Q} = at_1+(\frac{b}{2}+\frac{a\theta}{2})t_1^{\ 2}+\frac{b\theta}{3}t_1^{\ 3}-\frac{a\alpha}{\beta+1}t_1^{\ \beta+1}-\frac{b\alpha}{\beta+2}t_1^{\ \beta+2} \qquad (4.6)$$

The total inventory holding during the time interval [0, t_1] is given by

$$I_T = \int_0^{t_1} I(t)dt = \int_0^{t_1}[Q(1-\theta+\alpha t^{\beta})-\{at+(\frac{b}{2}-\frac{a\theta}{2})t^2+\frac{b\theta}{3}t^3+\frac{a\alpha\beta}{\beta+1}t^{\beta+1}+\frac{b\alpha\beta}{2(\beta+2)}t^{\beta+2}\}]dt$$

$$= \frac{a}{2}t_1^{\ 2}+(\frac{b}{3}+\frac{a\theta}{6})t_1^{\ 3}+\frac{b\theta}{8}t_1^{\ 4}-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\ \beta+2}-\frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\ \beta+3} \qquad (4.7)$$

The total number of deteriorated units during the inventory cycle is given by

$$D_T = \theta\int_0^{t_1} I(t)dt$$

$$= \theta[\frac{a}{2}t_1^{\ 2}+(\frac{b}{3}+\frac{a\theta}{6})t_1^{\ 3}+\frac{b\theta}{8}t_1^{\ 4}-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\ \beta+2}-\frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\ \beta+3}] \qquad (4.8)$$

The total number of ameliorating units during the inventory cycle is given by

$$A_T = \int_0^{t_1}\alpha\beta t^{\beta-1}I(t)dt = \alpha\{\frac{a}{\beta+1}t_1^{\ \beta+1}+\frac{b}{\beta+2}t_1^{\ \beta+2}\} \qquad (4.9)$$

The total number of shortages during the period [t_1, T] is given by

$$S_T = \int_{t_1}^{T} -I(t)dt = -e^{-\delta(T-t_1)}[\frac{a+b(T-t_1)}{\delta^2} - \frac{2b}{\delta^3}] \quad (4.10)$$

The amount of lost sales during the period [t_1,T] is given by

$$L_T = \int_{t_1}^{T} R(t)\{1-e^{-\delta(T-t)}\}dt = \int_{t_1}^{T} (a+bt)\{1-e^{-\delta(T-t)}\}dt$$

$$= a\{T-t_1-\frac{1}{\delta}(1-e^{-\delta(T-t_1)})\} - \frac{b}{2}(T^2-t_1^2) - b\{\frac{T}{\delta}-\frac{1}{\delta^2}-(\frac{t_1}{\delta}-\frac{1}{\delta^2})e^{-\delta(T-t_1)}\} \quad (4.11)$$

Fuzzy Cost Components:

The fuzzy total cost over the period [0, T] consists of the following cost components:

(1).Fuzzy ordering cost **(FOC)** over the period [0,T] = $\tilde{A}_0$

(2). Fuzzy purchasing cost **(FPC)** over the period [0,T] = $\tilde{p}_c$ I(0) = $\tilde{p}_c$ Q

$$= \tilde{p}_c \; [at_1 + (\frac{b}{2}+\frac{a\theta}{2})t_1^{\,2} + \frac{b\theta}{3}t_1^{\,3} - \frac{a\alpha}{\beta+1}t_1^{\,\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\,\beta+2}]$$

(3). Fuzzy holding cost for carrying inventory **(FHC)** over the period [0,T] = $\tilde{h}_c\, I_T$

$$= \tilde{h}_c[\frac{a}{2}t_1^{\,2} + (\frac{b}{3}+\frac{a\theta}{6})t_1^{\,3} + \frac{b\theta}{8}t_1^{\,4} - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\,\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\,\beta+3}]$$

(4). Fuzzy cost due to deterioration **(FCD)** over the period [0,T] = $\tilde{d}_c\, D_T$

$$= \tilde{d}_c\, \theta[\frac{a}{2}t_1^{\,2} + (\frac{b}{3}+\frac{a\theta}{6})t_1^{\,3} + \frac{b\theta}{8}t_1^{\,4} - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\,\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\,\beta+3}]$$

(5).Fuzzy amelioration cost **(FAMC)** over the period [0,T] = $\tilde{a}_c\, A_T$

$$= \tilde{a}_c\, \alpha\{\frac{a}{\beta+1}t_1^{\,\beta+1} + \frac{b}{\beta+2}t_1^{\,\beta+2}\}$$

(6).Fuzzy cost due to shortage **(FCS)** over the period [0,T] = $\tilde{s}_c\, S_T$

$$= -\tilde{s}_c\, e^{-\delta(T-t_1)}[\frac{a+b(T-t_1)}{\delta^2} - \frac{2b}{\delta^3}]$$

(7). Fuzzy opportunity Cost due to lost sales **(FOPC)** over the period [0,T] $= \tilde{o}_c\, L_T$

$$= \tilde{o}_c[a\{T-t_1-\frac{1}{\delta}(1-e^{-\delta(T-t_1)})\} - \frac{b}{2}(T^2-t_1^2) - b\{\frac{T}{\delta}-\frac{1}{\delta^2}-(\frac{t_1}{\delta}-\frac{1}{\delta^2})e^{-\delta(T-t_1)}\}]$$

The average fuzzy total cost per unit time of the system during the cycle [0,T] will be

$$\tilde{TC}(t_1) = \frac{1}{T}\ [\ \langle FOC\rangle + \langle FPC\rangle + \langle FHC\rangle + \langle FCD\rangle + \langle FAMC\rangle + \langle FCS\rangle + \langle FOPC\rangle]$$

$$= \frac{1}{T}[\ \tilde{A}_0 + \tilde{p}_c\ [at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^{\,2} + \frac{b\theta}{3}t_1^{\,3} - \frac{a\alpha}{\beta+1}t_1^{\,\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\,\beta+2}]$$

$$+\ (\tilde{h}_c + \tilde{d}_c\,\theta)[\frac{a}{2}t_1^{\,2} + (\frac{b}{3} + \frac{a\theta}{6})t_1^{\,3} + \frac{b\theta}{8}t_1^{\,4} - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\,\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\,\beta+3}]$$

$$+\ \tilde{a}_c\,\alpha\{\frac{a}{\beta+1}t_1^{\,\beta+1} + \frac{b}{\beta+2}t_1^{\,\beta+2}\}\ - \tilde{c}_s\,e^{(\lambda+\delta)t_1}[\frac{a+b(T-t_1)}{\delta^2} - \frac{2b}{\delta^3}]$$

$$+\tilde{o}_c[a\{T - t_1 - \frac{1}{\delta}(1 - e^{-\delta(T-t_1)})\} - \frac{b}{2}(T^2 - t_1^{\,2}) - b\{\frac{T}{\delta} - \frac{1}{\delta^2} - (\frac{t_1}{\delta} - \frac{1}{\delta^2})e^{-\delta(T-t_1)}\}]] \qquad (4.12)$$

For minimum, the necessary condition is $\frac{d\,\tilde{TC}(t_1)}{dt_1} = 0$

This gives $\tilde{p}_c\ \{\ a + (b + a\theta)t_1 + b\theta t_1^{\,2} - a\alpha t_1^{\,\beta} - b\alpha t^{\beta+1}\ \}$

$$+(\tilde{h}_c + \tilde{d}_c\,\theta)\{at_1 + (b + \frac{a\theta}{2})t_1^{\,2} + \frac{b\theta}{2}t_1^{\,3} - \frac{a\alpha\beta}{\beta+1}t_1^{\,\beta+1} - \frac{b\alpha\beta}{\beta+1}t_1^{\,\beta+2}\}\ +\ \tilde{a}_c\,\alpha\{at_1^{\,\beta} + bt_1^{\,\beta+1}\}$$

$$-\tilde{s}_c\,e^{-\delta(T-t_1)}[\frac{a+b(T-t_1)}{\delta} - \frac{3b}{\delta^2}]\ +\ \tilde{o}_c[a(e^{-\delta(T-t_1)} - 1) + bt_1(e^{-\delta(T-t_1)} + 1)] = 0 \qquad (4.13)$$

For minimum the sufficient condition $\frac{d^2\,\tilde{TC}(t_1)}{dt_1^{\,2}}$ >0 would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1 which is evaluated using (4.13)

The optimal values Q^* of Q and $\tilde{TC}^*$ of $\tilde{TC}(t_1)$ are obtained by putting the value $t_1 = t_1^*$ from the expressions (4.6) and (4.12).

5. Numerical Example:

Let us consider a fuzzy inventory system with the following cost parameters in appropriate units as

$\tilde{A}_0$ = (400, 500, 600, 700) ; $\tilde{p}_c$ =(2, 4, 6, 8) ; $\tilde{h}_c$ = (2, 3, 4, 5) ; $\tilde{d}_c$ = (8, 9, 10, 12) ; $\tilde{a}_c$ = (4, 6, 8, 10) ; $\tilde{s}_c$ = (8, 10, 12, 14) ; $\tilde{o}_c$ = (10, 12, 14, 16) ; a = 300 ; b = 10 ; α = 0.001 ; β = 2 ; θ = 0.8 ; δ = 0.5 ; T = 1 year

Solving the equation (4.13) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.63 year; Q^* = 237.41 units and $\tilde{TC}^*$ = Rs. 12200.94

It is checked that this solution satisfies the sufficient condition for optimality.

6. Sensitivity Analysis and Discussion.

We now study the effects of changes in the fuzzy cost parameters $\tilde{A}_0$, $\tilde{p}_c$, $\tilde{h}_c$, $\tilde{d}_c$, $\tilde{a}_c$, $\tilde{s}_c$ and $\tilde{o}_c$ on the optimal on-hand inventory quantity (Q^*) and the fuzzy optimal total cost ($\tilde{TC}^*$) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one fuzzy cost parameter at a time and keeping remaining cost parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the fuzzy cost parameters on the model

Changing parameter	% change in the system parameter	% change in	
		Q^*	TC^*
$\tilde{A}_0$	-50	No effect	- 2.25
	-20		- 0.90
	+20		0.90
	+50		2.25
$\tilde{p}_c$	-50	38.13	8.54
	-20	14.35	3.33
	+20	-13.24	-3.21
	+50	-31.14	-7.83
$\tilde{h}_c$	-50	14.41	4.31
	-20	5.30	1.58
	+20	-4.79	-1.43
	+50	-11.16	-3.34
$\tilde{d}_c$	-50	39.11	11.69
	-20	12.65	3.78
	+20	-10.07	-3.02
	+50	-21.86	-6.55
$\tilde{a}_c$	-50	0.01	0.004
	-20	0.005	0.002
	+20	-0.005	-0.002
	+50	-0.01	-0.004
$\tilde{s}_c$	-50	-57.65	-53.53
	-20	-25.30	-23.99
	+20	28.70	28.18
	+50	58.96	60.05
$\tilde{o}_c$	-50	-9.07	-3.71
	-20	-3.30	-1.34
	+20	2.93	1.19
	+50	7.75	2.73

Analyzing the results of table A, the following observations may be made:

(i) The fuzzy optimal total cost ($\tilde{TC}^*$) increases or decreases with the increase or decrease in the values of the fuzzy cost parameters $\tilde{A}_0$, $\tilde{s}_c$ and $\tilde{o}_c$. On the other hand it increases or decreases with the decrease or increase in the values of the system parameters $\tilde{p}_c$, $\tilde{h}_c$, $\tilde{d}_c$ and $\tilde{a}_c$. The results obtained show that $\tilde{TC}^*$ is highly sensitive towards changes of the cost parameter $\tilde{s}_c$, moderate sensitive towards the changes of $\tilde{A}_0$, $\tilde{p}_c$, $\tilde{h}_c$, $\tilde{d}_c$, $\tilde{a}_c$ and $\tilde{o}_c$, and less sensitive towards the changes of $\tilde{a}_c$.

(ii) The optimal on-hand inventory quantity (Q^*) increases or decreases with the increase or decrease in the values of the fuzzy cost parameters $\tilde{s}_c$ and $\tilde{o}_c$. On the other hand Q^* increases or decreases with the decrease or increase in the values of the cost parameters $\tilde{p}_c$, $\tilde{h}_c$, $\tilde{d}_c$ and $\tilde{a}_c$. The results obtained show that Q^* is highly sensitive towards changes of $\tilde{p}_c$, $\tilde{d}_c$ and $\tilde{s}_c$, moderate sensitive towards the changes of $\tilde{h}_c$ and $\tilde{o}_c$, and less sensitive towards the changes of . It is insensitive towards changes of $\tilde{A}_0$.

Hence, proper adequate attention must be taken to estimate the fuzzy shortage cost parameter $\tilde{s}_c$ during evaluation of the total optimal inventory cost.

7. Concluding Remarks :

The present paper deals with a fuzzy-based inventory management system where the optimal inventory cost function is obtained using fuzzy approach under an uncertain environment. The demand function is linear trend in demand and ameliorating rate is Weibull distributed along with constant rate of deteriorating items. Shortages are considered in the present model which are partially backlogged. The inventory cost under fuzzy sense is optimized using a method of defuzzification, namely signed distance method(SD). Numerical illustration and sensitivity analysis are performed and it is observe that few cost parameters are highly sensitive whereas few are moderate and less sensitive. A future work will be further incorporated in the present model by introducing inflation and time discounting, trade credit policy or profit based inventory model under this imprecise environment.

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