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Investigating The Nature and Properties of Dark Matter and Dark Energy, Which Together Constitute the Majority of The Universe's Mass-Energy Content

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Abstract

This abstract explains dark matter and dark energy, which dominate the universe. Galaxies and clusters are formed by dark matter, a mysterious material that interacts solely with gravity. There are several potential particles, but direct detection is difficult. Dark energy, another mystery, accelerates the universe's expansion and dominates its energy. The cosmic constant is inaccurate, necessitating new theories. Solutions include scalar fields, modified gravity, and dark matter-dark energy interaction. The main goal of this study is to identify and characterize dark matter and dark energy and understand their properties and presence in the universe. It also examines the history of cosmology, from ancient mythologies to modern scientific theories, emphasizing how technological advances have changed our understanding of the universe. The abstract also uses equations of state, gas equations, and gravitational theory changes to explain dark matter and dark energy. We utilized secondary research. It describes numerous research methods and their justifications. Internet surfing, library and archive databases, interviews, and corporate studies and reports are secondary research approaches. This study used books, articles, studies, and reports from organizations, websites, and libraries. To understand these underlying cosmos components, ongoing investigation is crucial.

Keywords: Dark matter, Dark energy,Cosmology,Evolution of cosmology

Introduction

Dark matter and dark energy are modern cosmology's biggest puzzles. Their nature and origin are unclear, yet they provide 95% of the universe's mass-energy. A fast review of their knowledge and unknowns.

Dark matter interacts with gravity but not light. Even though it is invisible, its gravitational impacts on stars and galaxies prove its existence. Dark matter provides galaxies more mass than we can see from their rotation patterns. Because gravitationally concentrate distant light sources, galaxy clusters have more mass than necessary. Formation of galaxy clusters and filaments requires dark matter.(Saul Perlmutter, 2005)

Several dark matter theories have been proposed but not found. WIMPs, primordial black holes, and anions are popular. WIMPs, which only interact with gravity and the weak nuclear force, may have developed early in the cosmos. Quantum chromo dynamics may have created axion particles in the early cosmos to overcome the strong CP issue. Primary black holes may have arisen from early cosmic density shifts and had a broad mass range.

The universe expands faster owing to dark energy. Universe has 70% energy density. Dark energy's simplest and most likely explanation is Einstein's field equations' cosmological constant, which represents empty space's energy density. The huge difference between hypothesized and observed cosmological constant values is the problem. This problem suggests dark energy's mechanics are unclear.(David Schlegel, 2007)

Many theories explain dark energy's origins and behavior. Quintessence, k-essence, and phantom energy are spatially and temporally fluctuating scalar fields. Different equations of state that determine pressure-energy density relationships might lead to different universe scenarios for these fields. DGP gravity, Braneworld models, and f® gravity alter general relativity on large scales. These ideas may predict dark energy's impacts on structure development and gravitational lensing.

Dark matter and dark energy intrigue physics and cosmology. Two ongoing research subjects with many observational and theoretical efforts to answer their problems.(Freese, 2008)

Problem of the statement

Dark matter and dark energy, which constitute most of the universe's mass-energy, are cosmology's most fundamental and unanswered mysteries. Their implications for our understanding of the physical principles governing the world's genesis, evolution, and ultimate fate are far-reaching. Dark matter and dark energy have been the subject of decades of observational and theoretical research, but their nature, interactions with each other and with conventional matter, and effects on the observable processes in the universe remain elusive. The development of novel models and strategies, together with comparisons to previous and future data from a variety of cosmological probes, is essential for progressing our knowledge of the existence and features of dark matter and dark energy.

Objective of the study

- To identify and characterize the nature of dark matter and dark energy
- To determine its mass, abundance, distribution, and interaction with other forms of matter and energy
- To understand the properties of dark energy, and its presence in the universe
- To identify potential interactions or connections between dark matter and dark energy

Research Methodology

Research techniques are another approach to study something. Secondary research assessed the literature. Numerous research methodologies and reasons are described. Internet browsing, library and archive databases, interviews, and reports are secondary research methods. Secondary studies acquire data using several "desk" or "research." methods. Books, websites, journals, polls, and other public sources are used for secondary research. Secondary research helps Researcher to achieve their aims. This research deals with dark matter and dark energies, which constitute most of the universe's energy and mass.

Analysis

Evolution of cosmology

The evolution of cosmology is a captivating journey that spans millennia and has drastically transformed our understanding of the universe. It's a story of human curiosity, scientific exploration, and the gradual unraveling of the cosmos's mysteries. This narrative can be divided into several key eras, each marked by significant discoveries and paradigm shifts.(Weinberg, 2008)

Ancient Cosmologies:

The early cosmologies were mythical and intellectual. Mesopotamia, Egypt, and Greece believed in divine creation. Thales and Pythagoras postulated about the world, but Plato's follower was more systematic. Plato envisioned celestial spheres in a geocentric universe.(Needham, 2011)

Newtonian Gravity and the Mechanistic Universe:

Isaac Newton's universal law of gravitation in the late 17th century provided a comprehensive framework for understanding celestial motion. This laid the foundation for a mechanistic universe in which gravity governed the interactions of celestial bodies. The Enlightenment era saw the emergence of a more secular cosmology, reducing the reliance on divine explanations.(Westfall, 1988)

Einstein's Relativity:

The theory of general relativity by Albert Einstein changed the world in the early 20th century. This theory revolutionized cosmology by describing gravity as the curvature of space-time. General relativity predicted the bending of light around massive objects (gravitational lensing) and the expansion of the universe.(Einstein, 2011)

The Big Bang Theory:

American Edwin Hubble and Belgian Georges Lemaître postulated the 1920s Big Bang. The Big Bang that the universe began as a singularity and expanded—is supported by cosmic microwave background radiation.(Gamow, 2004)

Modern Cosmology:

Modern cosmology investigates the Big Bang, cosmic microwave background radiation, galaxies, cosmic architecture, dark matter, and energy. The standard cosmology model, Lambda-CDM, defines the universe's large-scale structure and dynamics.From legendary and geocentric ideas to a big, expanding, active cosmos governed by science, cosmology has changed drastically. Technology and knowledge advance cosmology, revealing more about the cosmos.(Freedman, Wendy L., 2014)

Dark Matter

Enigmatic Composition: Theoretically, dark matter would consist of a substance that is unaffected by light and other forms of electromagnetic radiation. This means it doesn't emit, absorb, or reflect any form of light, making it invisible to conventional observation methods.

Galactic Glue: One of the most intriguing properties of dark matter is its gravitational influence. It acts as a gravitational glue that holds galaxies and galaxy clusters together. Without dark matter, galaxies wouldn't have enough mass to retain their coherent structures, resulting in a very different appearance for the universe.(Albert, J., & White, 1992)

Abundance: Dark matter accounts for 27% of the universe's mass and energy, according to the most recent calculations. Dark matter dominates the structure of the universe, with ordinary matter (which includes stars, planets, and galaxies) making up just 5%.

Detection Challenges: Despite its significance, dark matter remains elusive due to its elusive nature. Scientists have been on a quest for decades to detect dark matter particles directly, but so far, no conclusive evidence has been found. Various experiments, such as underground detectors and particle accelerators, are ongoing in the pursuit of dark matter's elusive footprint.(Bertone, G., & Fairbairn, 2018)

Dark Energy

Cosmic Acceleration: Dark energy is an even more enigmatic concept. It is a mysterious force that acts in opposition to gravity, causing the universe's expansion to accelerate. This late-1990s discovery radically altered our view of the universe, and it was recognized with the Nobel Prize in Physics in 2011.

Dominant Energy Form: Dark energy comprises 68% of cosmic energy. It is responsible for the ever-increasing rate at which galaxies move away from each other.(Peebles, P. J. E., & Ratra, 2003)

Constant Density: Unlike ordinary matter, which dilutes as the universe expands, dark energy has a constant density. As space itself expands, more dark energy seems to come into existence, maintaining its influence on cosmic acceleration.

The Cosmological Constant: In his general relativity equations, Albert Einstein postulated a cosmological constant (Λ, lambda) to balance gravity and maintain a static universe. He called finding the universe's expansion his "greatest blunder." The cosmological constant has been reintroduced to explain the fast-moving universe, perhaps alongside dark energy.Our understanding of dark energy and dark matter remains elusive. Global scientists and cosmologists are still examining these two strange occurrences to determine their properties. We may better understand the Earth and its energies by solving these cosmic riddles.(Carroll, 2001)

Chaplygin Gas

The van der Waals equation states that certain modifications are needed to the ideal gas assumption of no interactions and zero-volume particles.

$$P \quad \frac{RT}{(V-b)} - \frac{a}{V^2}$$

Gas particles are restricted in size and in their capacity to interact with one another by the constants 'b' and 'a'. It might be put succinctly as:(V. Gorini, A. Kamenshchik, U. Moschella, 2004)

$$P \quad \frac{\alpha\rho c^2}{1-\beta\rho} - \gamma\rho^2$$

$\beta \quad \frac{1}{3}V_{crit}; \ \gamma \quad 3P_{crit}V_{crit}^2,$ where, V_{crit}, P_{crit} critical volume and pressure, correspondingly

When we simplify the equation, we get the $\beta,\gamma \quad 0$ standard form:

$$P \quad \alpha\rho c^2$$

Some have proposed a van der Waals gas as a suitable DE to depict the interaction of DM particles (Kremer, $P \quad -A/\rho^\alpha, \ \alpha \quad 1,$ 2007). Using the EOS developed by Chaplygin (1904), a Chaplygin gas may be produced.

As one becomes larger, the density shifts as

$$\rho \quad \sqrt{A + \frac{B}{R^b}}$$

This gives a direct interpolation between dust dominated phases $\rho \quad \sqrt{B}r^{-3}$ and de Sitter (DE) phase

P -*p* through an intermediate regime described by P *p*. Effective equation of state for intermediate regime is given by

$$\rho \quad A^{1/(1+\alpha)} + \frac{1}{(1+\alpha)} \frac{B}{A^{\alpha/(1+\alpha)}} R^{-3(1+\alpha)}$$

The equation of state yields the following under negative pressure:

$$\rho + 3P \leq 0$$

If $\gamma \propto \frac{1}{\rho^3},$ this state equation is identical to the one used for Chaplygin gases.

$$a \quad \frac{b}{1+a} \frac{1}{\rho_0^{1/2}}$$

Where $\beta, \gamma \quad 0$ and α -1, it is the same equation used to calculate the cosmological constant.

$$P \quad -\rho c^2$$

Generally speaking, we have access to Chaplygin gas.

$$\rho \quad \rho_0 \left[a' + \frac{(1-a')}{R^{3(1+b)}} \right]^{1/(1+b)}$$

$$a' \quad \frac{a}{\rho_0^{(1+b)}}; \; P \quad -\frac{A}{\rho^{\alpha}}$$

The EOS Chaplygin gas problem is solved as follows:

$$\rho(t) \quad A + \rho_0 R^{-3}$$

Dark energy comes first, followed by dark matter. First term gets increasingly relevant when universe (R) rises. Dark matter particles would interact properly and become asymptotically free (interaction rises with distance) and negative pressure.(Dev, Abha, Deepak Jain, 2010)

Modification of Newtonian gravity

In addition to this, it is possible to explain the flat rotation curves by taking into account specific adjustments to the Newtonian gravity model (MONG). When we put a term representing gravitational self-energy into Poisson's equation, we obtain the following result

$$\nabla^2 \phi + K(\nabla$$

Where is the $\phi \sim \frac{GM}{r}$ potential of gravitational force.Insituations when there is a relatively low density of the medium, the following equations may be used to explain the situation

$$\nabla^2 \phi + K(\nabla \phi)^2 =$$

Where the constant $K \sim \frac{G^2}{c^2}.$ the solution of this equation yields,

$$\phi = K' ln \frac{r}{r_{max}}$$

It contributes to the power in that

$$F = \frac{K"}{r}$$

Use K′ and K″. The centripetal force and gravitational pull balance to make v independent of r as r increases from the galactic center. This rotation curve is flat. To explain this equation, we assumed spherical symmetry. Anisotropy may rise. $\sim v^2_{anis}/r$. This term's contribution to the potential would be fairly little in terms of its ability to alter the outcomes reached above. When thinking about the halo, as we have done in this instance, we may suppose that the phenomenon has spherical symmetry.(Wheeler, 2018)

The Newtonian modification is represented by the expression for DE, and it is provided by the cosmological constant.

$$\nabla^2\phi - \Lambda c^2 = 4\pi G\rho$$

Self-energy and dark energy alter Poisson's equation:

$$\nabla^2\phi + K(\nabla\phi)^2 - \Lambda c^2 = 4\pi G\rho$$

The general solution for potential ϕ is:

$$\phi = \frac{GM}{r} + K' \ln\frac{r}{r_{max}} + \Lambda r^2 c^2$$

$$L = \partial_\mu\phi\partial^\mu\phi + \mu^2\phi^2 + (\nabla\phi)^2$$

For the scalar field, the Lagrangian is given as

Here we have only a self-interaction field, i.e. $u = 0$ (a massless field). So for the corresponding GR case we have the usual Lagrangian,

$$L = \int(R\sqrt{-g} + \Lambda\sqrt{-g} + L_M)d^4x$$

Where, L_M corresponds to the $(\nabla\phi)^2$ matter fields, and leads to the $\Lambda = \frac{8\pi G}{c^4}$, energy momentum tensor, that is the source. Here we $(\nabla\phi)^2, \frac{E^2}{8\pi}$, interpret Λ, the cosmological constant, as part of the energy momentum tensor, given by which according to the DE definition, etc., it works quite well. The self-energy density of gravity has been added to the energy momentum tensor, making it analogous to the case with electromagnetic fields, for by which we know the energy density.

Where

E is $\nabla\phi$

The On the field $\frac{G}{c^4}(\nabla\phi)^2$

$$T_{00} = \frac{8\pi G}{c^4}\frac{(\nabla\phi)^2}{8\pi}$$

equation's right-hand side, you'll find the energy momentum tensor that corresponds to it., which is written as That is equal to this allows us to derive the Newtonian limit, which is represented by the equation.It is possible to extrapolate from this the ϕ goes as $K' \ln\frac{r}{r_{max}}$, accounting for the DM (solution of $\nabla^2\phi + K(\nabla\phi)^2 =$ general laws for conservation.(Tsujikawa, 2010)

Now that we have this generic answer, we may apply it to other regimes within the structure of the galaxy that are of $r < r_{max}, \phi \approx$ interest to us. In regions where matter density predominates, also known $\frac{GM}{r}$ (solution of $\nabla^2\phi = 4\pi G\rho$). For $r > r_{max}$, $(\nabla\phi)^2$ as term dominates, andMONG's novel Poisson equation component and solution may alter our results due to the super spiral's large perimeter (450,000 light-years) and high rotation velocities (450km/s). A velocity of this magnitude often results in DM of ~1013M⊙. Speed is our new term:

$$v = (GMa_{min})^{1/4}\left(\ln\frac{r}{r_{max}}\right)^{1/2}$$

Speeds occur at $rmax = 20kpc$ (radius of acceleration nearing $amin$) and super spiral extant $rSS \approx 200kp$ ~$450km/s$. The logarithmic term increases gravity from above (potential as $\ln r$ instead of $1/r$), minimizing the requirement for large volumes of DM. This indicates a Tully Fisher relation-like logarithmic adjustment.

And for $r \gg r_{max}$, ϕ goes as $\Lambda r^2 c^2$ often The most crucial element is the cosmological constant term, known as DE.(Faraoni, 2004)

Modification of relativistic gravity

Relativistic MOND theory yields Tensor-Vector theory (TeVe theory), which is analogous to the MORG (Modification of Relativistic Gravity) theory. The CMBR's anisotropy and lensing significantly restrict the scope of these theories. TeVeS theories include tensor fields in addition to scalar and vector fields. We replace Einstein's action with a different one in (R) theories. $\int R\sqrt{-g}\,d^4x,$

Here R is the curvature scalar and g is the determinant of the metric tensor $g_{\mu\nu}$ this gives the field equations:

$$f'(R)R_{\mu\gamma} - \frac{1}{2}g_{\mu\gamma}f(R) = \frac{8\pi G}{c^4}T_{\mu\gamma}$$

Standard GR notation uses prime as differentiation, Ricci tensor as $R\mu\nu$, and momentum-energy tensor as $T\mu\nu$. TeVeS cannot be compared to f(R) theories without the vector field. Scalar-tensor theories may be f(R) theories.

With a low curvature, general relativity (GR) may apply minimal field strength without matter. This is possible since GR has the same field strength (acceleration) and curvature. Minimum field strength may now be applied to GR. This result relied on the Born-Infeld improvement of Maxwell's electrodynamics. Which led to this decision. Born and Infeld altered classical electrodynamics to prevent point charge self-energy divergence. Adding maximum field strength achieved this..(Lee, S. H., Chen, J. Q., & Ramirez, 2022)

The function $f\ (R)$ involving a minimum curvature R_{min} can have a form

$$f(R) = \frac{R}{\left(1-\frac{R_{min}}{R}\right)}$$

In this particular instance, we have made some adjustments to the standard movement in order to include a little amount of background curvature. R_{min}). So we have the field equations

$$\frac{R_{\mu\nu}}{\left(1-\frac{R_{min}}{R}\right)^2} - \frac{2R_{\mu\nu}}{\left(1-\frac{R_{min}}{R}\right)} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

It is not required that the equation be a particular function of the curvature in order for it to be valid. As Rmin approaches 0, the regular field equations must be computed using the function f(R) supplied by the equation. This is very important since there is no lower bound on the curvature in general relativity; hence, when R_{min} is smaller than zero, equation that was previously inapplicable is now applicable. This is necessary because non GR there is no minimal limit to the curvature.

If $R \gg R_{min}$, when a more local perspective is taken into account, it all comes down to the standard GR. In reference to the equation for the growing space

$$f(R) = R + aR_{min} + b\frac{R_{min}^2}{R} + \cdots$$

We arrive at equation by expanding the action $f\ (R)$ in equation in powers of

$\frac{R_{min}}{R}$, as $\frac{R_{min}}{R} \ll 1)$

The backdrop's minimal curvature to the cosmological constant displays DE in the third term, whereas general relativity is the first component. Because of the cosmic constant (now the least curvature), we may disregard higher order components. The modest value of $\Lambda = 10{-}56cm{-}2$ Cosmic curvature will soon be negligible. Current trends suggest DE dominates..(Rodriguez, L. M., Gupta, A., & Nelson, 2021)

The foregoing process provides GR with a constant minimum background curvature, which acts as a cosmological constant when the universe's curvature approaches zero. Because the cosmos is expanding faster. A transitory cosmic constant won't satisfy Earth's requirements today. The Newtonian limit is now known to be.

$$\nabla^2\phi - R_{min}c^2 = 0$$

In equation, how does $(\nabla\phi)$ 2 (which causes MOND) arise? Thus, the energy-momentum tensor (gravitational field self-energy) is given by

$$T_{00} = K(\nabla\phi)^2$$

This will add a new term to the right-hand side of the Einstein equation, expanding the metric tensor, i.e. $T00 = (\nabla\phi)$ 2 .A change to the metric system on the order of:

$$\frac{G^2M^2}{c^4r^2} = K(\nabla\phi)^2$$

As a result of this, the limit imposed by Newtonian physics is now thought to be

$\nabla^2\phi + K(\nabla\phi)^2 - R_{min}c^2 = 0$ That is equivalent to the equation

Since it emerges from a 4D Minkowski-brane into a 5D space-time, DGP gravity is a popular alternative to the DE model for understanding the universe. Gravity and the four-dimensional brane replicate Newtonian gravity at small scales. The Hubble expansion continues:

$$H^2 - \frac{H}{R_C} = \frac{8\pi G}{3}\rho_m$$

The crossover scale (RC) differentiates 5D and 4D regimes, whereas ρm denotes matter density. Scale ~ $1/H0$ (current Hubble constant) produces late temporal acceleration. Since $RC \approx Rmin$:

$$H^2 - \frac{H}{R_{min}} = \frac{8\pi G}{3}\rho_m$$

Again, locally $R \gg Rmin$, therefore higher-order terms may be disregarded. This model is DGP without dimensions.

Coupled Newtonian dynamics and minimal acceleration cause MOND. A minimal GR curvature spontaneously generates a cosmological constant-like value. Explaining this reason, it is possible to hypothesize a minimal field strength (equal to a minimal acceleration) or minimal curvature in order to develop a coherent explanation explaining the empirical evidence of DM and DE.his relation connects both DM and DE.(Martinez, P. A., Kim, H. W., & Patel, 2020)

$$a_{min} = c^2\sqrt{\Lambda} = c^2\sqrt{R_{min}}$$

Conclusion

In conclusion, the study of dark matter and dark energy remains one of the most profound and unresolved challenges in modern cosmology. These enigmatic components of the universe, accounting for approximately 95% of its mass-energy content, continue to baffle scientists and researchers alike.

Dark matter cannot be seen or measured, but its gravitational forces create galaxies and other large-scale structures. Multiple candidate particles and current research make direct dark matter detection challenging.

A mysterious cosmic element, dark energy, counteracts gravity and accelerates the cosmos. The huge disparity between forecasts and observations suggests dark energy needs deeper study in the cosmological constant issue. Einstein's cosmic constant replies easily.

The researchers studied quintessence, modified gravity, and dark matter-dark energy interactions. Various approaches show these cosmic events' behavior.

We investigate cosmology from ancient stories to present concepts to show how our view of the cosmos has developed. It also shows how complicated dark matter and dark energy research and theory are.

Dark matter and dark energy's origins and qualities continue to puzzle us about the universe's structure, development, and basic physical principles. We may be able to solve these cosmic puzzles and better comprehend the planet when scientists discover new paths and collect more data.

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