

Counting Of Identical Sub Triangles Developed with Parallel St. Lines

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Abstract:

I have given the solution to a problem where a triangle is divided into many sub-triangles with parallel st. lines(increasing order) parallel to each side of the triangle (nos. of parallel st. lines are the same) and counting of total no. of triangles including all identical(arranged in similar order) sub-triangles and the main triangle is done. If the no. of parallel st. lines parallel to each of the three sides of the triangles is (n-1), then the total no. of parallel st. lines including each side of the triangle will be n. Now, I have found a correlation between this total no. of parallel st. lines or total no. of base-lines ‘n’ with the total no. of triangles including all identical sub-triangles thus generated.

According to my formula,

Total no. of triangles, $\Delta_n = \sum(2n - 1) + 3(n - 2) + 1$, for $n \geq 2$, because for $n = 1$, there is only one triangle. $n \in \mathbb{N}$ (set of natural no. s).

Thus, for $n=m$, $\Delta_m = \sum(2m - 1) + 3(m - 2) + 1$

& for $n=(m+1)$, $\Delta_{m+1} = \sum(2m + 1) + 3(m - 1) + 1$

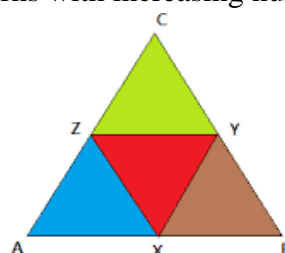
$\therefore \Delta_{m+1} - \Delta_m = 2m + 4 = 2(m + 2)$

Thus, differences between two consecutive triangles with subdivisions of similar patterns and m and (m + 1)base-lines are obtained Here, (2n-1) is an odd no.

and $\sum(2n - 1)$ is summation of odd nos.Original paper-

There are so many puzzles or problems with the counting of triangles that some of them drew the attention of mathematicians.Once Ramanujan tried to prove the existence of God with his famous “God game number”.

Here, I am going to solve an age-old puzzle or problem of counting the subdivisions of a triangle.When a triangle is continuously subdivided into many sub triangles with parallel st.lines parallel to each sides of the triangle in an increasing order then it is not manually possible to count the sub triangles . So it needs a general formula to count them.When I have tried for it I can generate a general formula to count the sub triangles arranged in similarpatterns with increasing number of subdivisions only.



In the adjacent figure a triangle ABC is divided into sub-triangles with st.lines parallel to AB,BC and CA are YZ,ZX and XY respectively.Now, the sub-triangles are AXZ,BXY,CYZ and XYZ .Therefore, the total number of sub-triangles are 4 .But, if the number of parallel lines to each side of the triangle is increased ,the number of sub-triangles will also be increased and at some point the number of sub-triangles can not be counted manually.Here,we need a general formula to count sub-triangles.I have developed a formula not to count all sub-triangles in any number of sub-divisions but to count sub-triangles arranged in similar pattern in any number of sub-divisions..Actually, it is a counting method of sub-triangles in identical pattern in any number of subdivisions with parallel st.lines ‘n’(n is any real positive number) parallel to each side of a triangle.

In the adjacent figure ,there is only 1 parallel st.line to each side of the triangle ABC.The parallel st.lines are YZ,ZX and XY.Now to formulate my formula I consider these parallel st.lines with the original base-lines AB.BC and CA of the original triangle ABC as total number of base-lines and therefore ,here, the total number of base-lines is $n = 2$..Thus the total no. of sub triangles along with the main triangle is 5.

With so many examples I have developed my formula as a general formula to count all sub-triangles in a similar pattern of subdivisions or in identical order with $(n - 1)$ parallel st.lines along with the main triangle.

If the no. of parallel st. lines parallel to each of the three sides of the triangles is $(n-1)$, then the total no. of parallel st. lines including each side of the triangle will be n . Now,I have found a correlation between this total no. of parallelst. lines or total no. of base-lines ‘n’ with the total no. of triangles including all sub-triangles thus generated.

According to my formula,

Total no. of triangles, $\Delta n = \sum(2n - 1) + 3(n - 2) + 1$,for $n \geq 2$, because for $n = 1$,there is only one triangle. $n \in \mathbb{N}$ (set of natural no. s).

Thus, for $n=m$, $\Delta m = \sum(2m - 1) + 3(m - 2) + 1$

& for $n=(m+1)$, $\Delta m + 1 = \sum(2m + 1) + 3(m - 1) + 1$

$\therefore \Delta m + 1 - \Delta m = 2m + 4 = 2(m + 2)$

Thus, differences between two consecutive trianglenumbers are obtained. Here, $(2n-1)$ is an odd no. & $\sum(2n - 1)$ is summation of odd nos.

Thus the number of triangular divisions in a similar pattern along with the main triangle is to be counted for any numerical value of n which is greater than or equal to 2.

Because for $n=1$,there are no divisions and there is only themain triangle.

Use - If a line intersecting two sides of a triangle is parallel to the remaining side, then the smaller triangle created by the parallel line is similar to the larger, original triangle.Thus a lot of small similar triangles can be generated for geometric use.

Triangles can be used to make trusses. Trusses are used in many structures, such as roofs, bridges, and buildings. Trusses combine horizontal beams and diagonal beams to form triangles.

Triangles have many fundamental advantages that make them great for architects as well as interested students: they are quite common, structurally sound, and simple to apply and utilise in everyday life. A triangle's strength comes from its shape, which distributes pressures evenly along its three sides.

In graphic arts, this versatile shape symbolizes balance and stability. Apart from the rule of thirds, many photographers (as well as artists) like to use what's called the golden triangle rule in their composition. It involves arranging elements in a triangle to create a harmonious and symmetrical image.

Conclusion-

In consecutive subdivisions of a triangle with parallel st.lines all sub-triangles in an identical pattern and the increase in the number of sub-triangles in an identical pattern with the increase in the no. of parallel st.lines can be well understood with my formula.

As triangles are useful architectural tools that give strength and stability in the design of buildings and other, therefore sub-triangles are also important in architecture.

For, base-lines, $n=2, 3, 4, \dots$ etc. the no.s of identical sub-triangles along with the main triangle are 5, 13, 23, ...

.etc. Here, the common-differences are 8, 10, 12, ... etc. So, the proportionate increase of identical sub-triangles can be well understood.

References-

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2. The primary triangle-E Fivaz-Depeursinge, A Corboz-Warnery - New York, 1999