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Rough, Soft Approximation Spaces Based on Soft Binary Relations Over ’n’ Nonempty Finite Sets

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Abstract

Soft, rough set theory has evolved as an essential tool for decision making. Many researchers have done a fusion of soft set and rough set over a single universe. In recent years, rough set theory, soft set theory has been extended over two different non-empty sets and fused with fuzzy sets, intuitionistic fuzzy sets, etc. This article aims to generalize soft sets over ’n’ different non-empty sets, introduce new approximation operators and topological approximation operators in approximation space and topological space, respectively and study their properties.

Keywords: Soft set, Rough set, Fuzzy set, Soft rough set, Fuzzy soft rough set, approximation spaces.

1. Introduction

Zadeh (1965) defined the concept of fuzzy set to handle imprecise data. This theory helped in solving problems with uncertainty but had its own difficulties in solving. Thus, in order to model problem with uncertainty, the concepts of rough set and soft set were developed. Pawlak (1982) first defined rough set. These sets were related with upper and lower approximations and generally are crisp sets. Pawlak’s rough sets are contigent on equivalence relation but finding an equivalence relation among the elements of a set was troublesome. Though many relations were used to define rough set theory, it had complications in modelling problems with uncertainty. Hence, Molodtsov (1999) initiated the theory of soft set was. Maji et al., (2003) further investigated the theory of soft sets. This theory has its application in various fields like decision making, game theory, operations etc. Alcantud (2016) studied the relationship between Soft set and fuzzy set. Further, Feng et al., (2010, 2011) studied the relationship between soft, rough and fuzzy sets. Sani Danjuma et al., (2004) reviewed soft set based parameter reduction and decision making were investigated by many authors.

Many researches in topology are being done over single universe. Now, there is a large scope for research in extending the sets over a universe to two or more different universes. Wei-Zhi Wu et al., (2003) studied generalized fuzzy rough sets on two universes. Ruixia Yan et al., (2010) and Daowu Pei et al., (2004) studied rough set over dual universe. Weihua Xu et al., (2013) also studied fuzzy rough set model over two universes. Few other researches over two universes are done using hesitant fuzzy rough sets (Haidong Zhang et al., 2017), dual hesitant fuzzy multigranulation rough set theory (Chao Zhang et al., 2004), multi granulation rough set theory (Bingzhen Sun et al., 2015), an interval valued hesitant fuzzy multi granulation rough sets (Chao Zhang et al., 2016), probabilistic rough sets (Weimin Ma et al., 2012), etc.

Only a few attempts have been done in generalizing the soft sets over single universe to two or more universes. It is well known that soft set plays a vital role in decision making. Extension of soft set over n number of different non-empty finite sets helps the decision maker to make many different decisions at a time and will lead to a wide area of research.

This paper explains the theoretical approach of generalizing the Soft set theory over n different universes. Soft set is approximated corresponding to rough approximation space, soft approximation space using soft binary relations to obtain rough soft set, soft rough set respectively. Soft binary relation also generates a new topology, their basic properties are investigated.

2. Preliminaries

Definition 2.1: A topology τ of a set U is defined by the collection of all subsets of U satisfying the following properties

1. $\emptyset, U \in \tau$
2. The finite intersection of elements of τ is in τ
3. The arbitrary union of elements in τ is in τ

A pair (U, τ) is called a topological space where the elements of τ are said to be open and its complements are closed. An interior is said to be the largest open subset and closure is said to be the smallest closed superset. A Subbasis for a topology on U is a collection of subsets of U whose union equals U. The collection of all union of finite intersection of elements of subbasis generates the topology τ.

The need to represent the partitions of a universe gave rise to a new concept called rough set theory. This theory uses the equivalence classes of a partition of the considered universe.

Definition 2.2: The pair (U, R) denotes Pawlak's approximation space where R is an equivalence relation and $X \subseteq U$. Using R, the following operators were defined:

$\underline{R}(X) = \{x \in U : [x]R \subseteq X\}$,

$\overline{R}(X) = \{x \in U : [x]R \cap X \neq \emptyset\}$.

If $\underline{R}(X) = \overline{R}(X)$, then X is said to be definable set. Otherwise, X is a rough set.

Here, positive region $(X) = \underline{R}(X)$,

Negative region $(X) = U - \overline{R}(X)$,

Boundary region $(X) = \overline{R}(X) - \underline{R}(X)$.

Concerning the quality of an approximation, an accuracy measure has been done. Accuracy is equal to the cardinality of lower approximation divided by the cardinality of upper approximation. If the accuracy equals 1 then the set is exact or definable.

A soft set is different from general set and is a generalization of fuzzy set, deviation from rough set theory. This set was developed to solve problems with uncertainty using a parameter set.

Definition 2.3: A soft set over universe set U is a mapping from the subset of a parameter set to the power set of a universe set.

Definition 2.4: A soft topology S_τ of a set U is defined by the collection of all soft subsets of U satisfying the following properties:
i. $\emptyset, F_A \in S_\tau$.
ii. The finite intersection of soft open sets of S_τ is in S_τ.
iii. Arbitrary union of soft open sets in S_τ is in S_τ.

A pair (U, S_τ) is called a topological space where the elements of S_τ are said to be open and its complements are closed.

3. Rough soft set, Soft rough set

Definition 3.1: A binary relation $R_{m(s,t)}$ on S, T induced by m_k is defined by $(g,j)R_{m(s,t)}(g_1,j_1) \Leftrightarrow \{(g,j)m_k(g_1,j_1)\}$ for each $(g,j),(g_1,j_1) \in S \times T$.

Definition 3.2: Successor neighbourhood of each (g, j) in $S \times T$ is given by
$R_{m(s,t)}(g,j) = \{(g_1,j_1) \in S \times T; (g,j)R_{m(s,t)}(g_1,j_1)\}$.

Definition 3.3: Predecessor neighbourhood of each (g, j) in $S \times T$ is given by
$R_{m(s,t)}(g_1,j_1) = \{(g,j) \in S \times T; (g,j)R_{m(s,t)}(g_1,j_1)\}$.

Definition 3.4: Let $S_1, S_2, \dots S_n$ be nonempty finite sets. K be the subset of a parameter set E. A pair (m,K) or m_k is called soft binary relation over $S_1, S_2, \dots S_n$, if (m,K) is a soft set over $S_1 \times S_2 \times \dots \times S_n$.
Throughout this paper, we consider as $n = 2$, i.e., two non-empty finite sets say S, T.

Definition 3.5: Let S and T be two different nonempty finite sets. K be the subset of a parameter set E. A pair (m,K) or m_k is called soft binary relation over S and T, if (m,K) is a soft set over $S \times T$.

Example 3.6: Let S denote the set of three patients $\{N,Z,C\}$, T denote the set of three diseases $\{Typhoid(Ty), Dengue\ (D), Pneumonia\ (P)\}$. Let E be the set of parameter which define the symptoms of diseases.
$E = \{e_1(fever), e_2\ (breathing\ problem), e_3(joint\ pain), e_4\ (head\ ache)\}$,
$K = \{e_1, e_2\} \subseteq E$.
Let $S \times T = \{(N,Ty),(N,D),(N,P),(Z,Ty),(Z,D),(Z,P),(C,T),(C,D),(C,P)\}$ be the universal set. Then, soft set
$m_k = \{(e_1,\{(N,Ty),(N,P),(Z,Ty),(Z,P),(C,Ty),(C,P)\}),(e_2,\{(N,D),(N,P),(Z,D),(Z,P),(C,D),$
$(C,P)\})\}$
denotes patients and their symptoms along with the possibilities of diseases.

Example 3.7: Let S ={set of all prime numbers less than or equal to 5}$= \{2,3,5\}$, T ={set of all composite numbers less than or equal to 5}$= \{4\}$.
Let $S \times T = \{(2,4),(3,4),(5,4)\}$. Let $E = \{e_1, e_2\}$ where $e_1 = \gcd(a,b) = 1$, $e_2 = \gcd(a,b) = 2$. Let $K = \{e_1\} \subseteq E$. Then, the possibilities of the soft set m_k are $\{(e_1,\{(3,4)\})\}$, $\{(e_1,\{(5,4)\})\}$, and $\{(e_1,\{(3,4),(5,4)\})\}$.

Definition 3.8: Let m_k be a soft binary relation over S, T. $G \times J \subseteq S \times T$ and $(S, T, R_{m(s,t)})$ be a rough approximation space with respect to the parameter set. The approximation operators are defined as follows:

$\underline{S_{apr}}(G \times J) = \{(g,j) \in S \times T; R_{m(s,t)}(g,j) \subseteq (G \times J)\}$

$\overline{S_{apr}}(G \times J) = \{(g,j) \in S \times T; R_{m(s,t)}(g,j) \cap G \times J \neq \emptyset\}$

where $\underline{S_{apr}}(G \times J)$ is the lower rough soft approximation and $\overline{S_{apr}}(G \times J)$ is the upper rough soft approximation over two different universal sets.

If $\underline{S_{apr}}(G \times J) = \overline{S_{apr}}(G \times J)$, then G × J is definable soft set. If $\underline{S_{apr}}(G \times J) \neq \overline{S_{apr}}(G \times J)$, then $G \times J$ is rough soft set.

Example 3.9: Let $S = \{N, Z, C\}$, $T = \{Ty, D, P\}$, $E = \{e_1, e_2, e_3, e_4\}$, and $K = \{e_1, e_2\} \subset E$. Let $G = \{N, Z\} \subseteq S, J = \{D, P\} \subseteq T$. Then,

$S \times T = \{(N,Ty), (N,D), (N,P), (Z,Ty), (Z,D), (Z,P), (C,Ty), (C,D), (C,P)\}$,

$G \times J = \{(N,D), (N,P), (Z,D), (Z,P), (C,Ty)\} \subseteq S \times T$.

Then, the soft set is given by, $m(e_1) = \{(N,Ty), (N,P), (Z,Ty), (Z,P), (C,Ty), (C,P)\}$,

$m(e_2) = \{(N,D), (N,P), (Z,D), (Z,P), (C,D), (C,P)\}$.

The successor and predecessor neighbourhoods are given as follows:

$R_{m(s,t)}(N,Ty) = \{(N,P), (Z,Ty), (Z,P), (C,Ty), (C,P)\}$,

$R_{m(s,t)}(N,D) = \{(N,P), (Z,D), (Z,P), (C,D), (C,P)\}$,

$R_{m(s,t)}(N,P) = \{(N,Ty), (N,D), (Z,Ty), (Z,D), (Z,P), (C,Ty), (C,D), (C,P)\}$,

$R_{m(s,t)}(Z,Ty) = \{(N,Ty), (N,P), (Z,P), (C,Ty), (C,P)\}$,

$R_{m(s,t)}(Z,D) = \{(N,D), (N,P), (Z,P), (C,D), (C,P)\}$,

$R_{m(s,t)}(Z,P) = \{(N,Ty), (N,D), (N,P), (Z,Ty), (Z,D), (C,Ty), (C,D), (C,P)\}$,

$R_{m(s,t)}(C,Ty) = \{(N,Ty), (N,P), (Z,Ty), (Z,P), (C,P)\}$,

$R_{m(s,t)}(C,D) = \{(N,Ty), (N,P), (Z,D), (Z,P), (C,P)\}$,

$R_{m(s,t)}(C,P) = \{(N,Ty), (N,D), (N,P), (Z,Ty), (Z,D), (Z,P), (C,Ty), (C,D)\}$.

$\underline{S_{apr}}(G \times J) = \emptyset$,

$\overline{S_{apr}}(G \times J) = \{(N,Ty), (N,D), (N,P), (Z,Ty), (Z,D), (Z,P), (C,Ty), (C,D), (C,P)\}$.

Thus, $\underline{S_{apr}}(G \times J) \neq \overline{S_{apr}}(G \times J)$.

Therefore, $G \times J$ is a rough soft set.

Proposition 3.10: Let m_k be a soft binary relation over S, T and $(S, T, R_{m(s,t)})$ a rough approximation space. Then for any $G \times J$, $L \times V \subseteq S \times T$, properties satisfied by the approximation operators are as follows:

1. $\underline{S_{apr}}((G \times J) \cap (L \times V)) = \underline{S_{apr}}(G \times J) \cap \underline{S_{apr}}(L \times V)$.
2. $\overline{S_{apr}}((G \times J) \cup (L \times V)) = \overline{S_{apr}}(G \times J) \cup \overline{S_{apr}}(L \times V)$.
3. If $G \times J \subseteq L \times V$, then $\underline{S_{apr}}(G \times J) \subseteq \underline{S_{apr}}(L \times V)$. Similarly, $\overline{S_{apr}}(G \times J) \subseteq \overline{S_{apr}}(L \times V)$.

4. $\overline{S_{apr}}(\emptyset) = \underline{S_{apr}}(\emptyset) = \emptyset$.

5. $\underline{S_{apr}}(S \times T) = \overline{S_{apr}}(S \times T) = S \times T$.

6. $\underline{S_{apr}}(G \times J) = (\overline{S_{apr}}(G \times J)^C)^C$.

Definition 3.11: Let m_k be a soft set over $S \times T$ and $G \times J \subseteq S \times T$. Then, the soft approximation operators defined based on soft approximation space (S, T, m_k) are as follows:
$\underline{S_{Apr}}(G \times J) = \{(g, j) \in S \times T; \forall k \in K; (g, j) \in m(k) \subseteq (G \times J)\}$,
$\overline{S_{Apr}}(G \times J) = \{(g, j) \in S \times T; \forall k \in K; (g, j) \in m(k), m(k) \cap (G \times J) \neq \emptyset\}$.
If $\underline{S_{Apr}}(G \times J) = \overline{S_{Apr}}(G \times J)$, then $G \times J$ is soft definable set. Otherwise, $G \times J$ is soft rough set.

Example 3.12: A Soft set considered in example 3.9 has been taken, by using definition 3.11 the operators obtained is as follows:
$\underline{S_{apr}}(G \times J) = \emptyset$,
$\overline{S_{apr}}(G \times J) = \{(N, Ty), (N, D), (N, P), (Z, Ty), (Z, D), (Z, P), (C, Ty), (C, D), (C, P)\}$.
$\underline{S_{apr}}(G \times J) \neq \overline{S_{apr}}(G \times J)$. Therefore, $G \times J$ is a soft rough set.

Proposition 3.13: Soft set over $S \times T$ be m_k and (S, T, m_k) be the soft approximation space. Then,
$\underline{S_{Apr}}(G \times J) = \cup_{k \in K} \{m(k); m(k) \subseteq (G \times J)\}$,
$\overline{S_{Apr}}(G \times J) = \cup_{k \in K} \{m(k); m(k) \cap (G \times J) \neq \emptyset\}$ for all $G \times J \subseteq S \times T$.

Theorem 3.14: Let m_k be a soft set over $S \times T$ and (S, T, m_k) be a soft approximation space. Then for any $G \times J$, $L \times V \subseteq S \times T$, properties satisfied by the approximation operators are as follows:
1. $\overline{S_{Apr}}(\emptyset) = \underline{S_{Apr}}(\emptyset) = \emptyset$.
2. $\underline{S_{Apr}}(S \times T) = \overline{S_{Apr}}(S \times T) = S \times T$.
3. If $G \times J \subseteq L \times V$, then $\underline{S_{Apr}}(G \times J) \subseteq \underline{S_{Apr}}(L \times V)$.
4. If $G \times J \subseteq L \times V$, then $\overline{S_{Apr}}(G \times J) \subseteq \overline{S_{Apr}}(L \times V)$.

Proof:
1. True from definition.
2. This can be proved from proposition 3.13, substituting $S \times T$ for $G \times J$.
3. Assume $G \times J \subseteq L \times V$. Let $(g, j) \in \underline{S_{Apr}}(G \times J)$. Then by definition 3.5, $\forall k \in m_k$ such that $(g, j) \in m(k) \subseteq G \times J$. Since $G \times J \subseteq L \times V$, $(g, j) \in m(k) \subseteq L \times V$. Therefore, $\underline{S_{Apr}}(G \times J) \subseteq \underline{S_{Apr}}(L \times V)$.
4. Similar to (iii).

Theorem 3.15: Let a soft set over $S \times T$ be m_k and (S, T, m_k) be a soft approximation space. Then for any $G \times J, L \times V \subseteq S \times T$, properties satisfied by the approximation operators are as follows:
1. $\underline{S_{Apr}}((G \times J) \cap (L \times V)) \subseteq \underline{S_{Apr}}(G \times J) \cap \underline{S_{Apr}}(L \times V)$.

2. $\underline{S_{Apr}}((G \times J) \cup (L \times V)) \supseteq \underline{S_{Apr}}(G \times J) \cup \underline{S_{Apr}}(L \times V)$.
3. $\overline{S_{Apr}}((G \times J) \cup (L \times V)) = \overline{S_{Apr}}(G \times J) \cup \overline{S_{Apr}}(L \times V)$.
4. $\overline{S_{Apr}}((G \times J) \cap (L \times V)) \subseteq \overline{S_{Apr}}(G \times J) \cap \overline{S_{Apr}}(L \times V)$.

Proof:
1. $(G \times J) \cap (L \times V) \subseteq (G \times J)$, since $(G \times J) \subseteq (L \times V)$.
From, theorem 3.14 (iii), $\underline{S_{Apr}}((G \times J) \cap (L \times V)) \subseteq \underline{S_{Apr}}(G \times J)$.
Similarly, $\underline{S_{Apr}}((G \times J) \cap (L \times V)) \subseteq \underline{S_{Apr}}(L \times V)$ and so
$\underline{S_{Apr}}((G \times J) \cap (L \times V)) \subseteq \underline{S_{Apr}}(G \times J) \cap \underline{S_{Apr}}(L \times V)$
2. Since $(G \times J) \subseteq (G \times J) \cup (L \times V)$, From, theorem 3.14 (iii), $\underline{S_{Apr}}(G \times J) \subseteq \underline{S_{Apr}}((G \times J) \cup (L \times V))$. Similarly, $\underline{S_{Apr}}(L \times V) \subseteq \underline{S_{Apr}}((G \times J) \cup (L \times V))$. Thus, we have, $\underline{S_{Apr}}((G \times J) \cup (L \times V)) \supseteq \underline{S_{Apr}}(G \times J) \cup \underline{S_{Apr}}(L \times V)$.
3. Let $(g, j) \in \overline{S_{Apr}}((G \times J) \cup (L \times V))$. Then by definition, there exists $k \in K$, such that $(g, j) \in m(k)$ and $m(k) \cap ((G \times J) \cup (L \times V)) \neq \emptyset$. Either $m(k) \cap (G \times J) \neq \emptyset$ or $m(k) \cap (L \times V) \neq \emptyset$, indicating that $(g, j) \in \overline{S_{Apr}}(G \times J)$ or $(g, j) \in \overline{S_{Apr}}(L \times V)$. This shows that $\overline{S_{Apr}}((G \times J) \cup (L \times V)) \subseteq \overline{S_{Apr}}(G \times J) \cup \overline{S_{Apr}}(L \times V)$. To prove that, $(G \times J) \subseteq ((G \times J) \cup (L \times V))$ and from theorem 3.14 (iv), $\overline{S_{Apr}}((G \times J)) \subseteq \overline{S_{Apr}}((G \times J) \cup (L \times V))$. Similarly, $\overline{S_{Apr}}((L \times V)) \subseteq \overline{S_{Apr}}((G \times J) \cup (L \times V))$.
Therefore, $\overline{S_{Apr}}((G \times J) \cup (L \times V)) = \overline{S_{Apr}}(G \times J) \cup \overline{S_{Apr}}(L \times V)$.

4. Similar to (i)

4. Rough approximation space with respect to parameter set, Soft approximation space to a topological space

Definition 4.1: Let $(S, T, R_{m(s,t)})$ be a rough approximation space and τ_{BR} be a soft topology obatined from soft binary relation over S, T. Thus, $(S, T, R_{m(s,t)}, \tau_{BR})$ is said to be BR-topological rough approximation space where the elements of τ_{BR} are BR-soft open and its complements are closed.

Example 4.2: Let $S =$ {Set of all prime numbers less than or equal to 6}, $T =$ {Set of all composite numbers less than or equal to 6}. $S \times T =$ {(2, 4),(2, 6),(3, 4),(3, 6),(5, 4),(5, 6)}
Let $E = \{e_1, e_2\}$, $A = \{e_1\}$. Let $m_k = \{(e_1, \{(3,4), (5,4), (5,6)\})\}$ be the soft set considered.
$R_m(2,4) = R_m(2,6) = R_m(3,6) = \emptyset$,
$R_m(3,4) = \{(e_1, \{(5,4), (5,6)\})\}$,
$R_m(5,4) = \{(e_1, \{(3,4), (5,6)\})\}$,
$R_m(5,6) = \{(e_1, \{(3,4), (5,4)\})\}$.
$S_{BR} = \{\emptyset, (e_1, \{(5,4), (5,6)\}), (e_1, \{(3,4), (5,6)\}), (e_1, \{(3,4), (5,4)\})\}$.
$B_{BR} = \{\emptyset, \{(e_1, \{(5,4), (5,6)\})\}, \{(e_1, \{(3,4), (5,6)\})\}, \{(e_1, \{(3,4), (5,4)\})\}, \{(e_1, \{(3,4)\})\},$
$\{(e_1, \{(5,4)\})\}, \{(e_1, \{(5,6)\})\}\}$.
$\tau_{BR} = \{\emptyset, \{(e_1, \{(5,4), (5,6)\})\}, \{(e_1, \{(3,4), (5,6)\})\}, \{(e_1, \{(3,4), (5,4)\})\}, \{(e_1, \{(3,4)\})\},$
$\{(e_1, \{(5,4)\})\}, \{(e_1, \{(5,6)\})\}, \{(e_1, \{(3,4), (5,4), (5,6)\})\}\}$.

τ_{BR} is a BR-soft quasi discrete topology.

Proposition 4.3: Every rough approximation space need not result in BR-topological approximation space.

The following example proves proposition 4.3.
Example 4.4: Let $S = \{$Set of all prime numbers less than or equal to 5$\}$, $T = \{$Set of all composite numbers less than or equal to 5$\}$. $S \times T = \{(2,4),(3,4),(5,4)\}$
Let $E = \{e_1, e_2\} = A$. Let $m_k = \{(e_1, \{(3,4),(5,4)\}), (e_2, \{(2,4)\})\}$
$R_m(2,4) = \emptyset$,
$R_m(3,4) = \{(e_1, \{(5,4)\})\}$,
$R_m(5,4) = \{(e_1, \{(3,4)\})\}$.
$S_{BR} = B_{BR} = \{\emptyset, \{(e_1, \{(5,4)\})\}, \{(e_1, \{(3,4)\})\}\}$.
$\tau_{BR} = \{\emptyset, \{(e_1, \{(5,4)\})\}, \{(e_1, \{(3,4)\})\}, \{(e_1, \{(3,4),(5,4)\})\}\}$.
Thus, τ_{BR} is not a BR-soft topology.

Definition 4.5: Let (S, T, m_k) be a soft approximation space and τ_{BR} be a topology obtained from soft binary relation over S, T. Thus, (S, T, m_k, τ_{BR}) is said to be BR-topological soft approximation space where the elements of τ_{BR} are BR-soft open and its complements are closed.

Definition 4.6: Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space. For each $m_{ki} \subseteq m_k$, the BR-topological approximation operators are defined as follows:
$S\underline{\tau}_{BR}(m_{ki}) = \cup \{m_{kj} \in \tau_{BR}; m_{kj} \subseteq m_{ki}\}$,
$S\overline{\tau}_{BR}(m_{ki}) = \cap \{m_{kj} \in {\tau_{BR}}^C; m_{ki} \subseteq m_{kj}\}$.
In other words, $S\underline{\tau}_{BR}, S\overline{\tau}_{BR}$ is considered as interior and closure of BR-topological approximation space respectively.

Proposition 4.7: Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space and m_{k1}, m_{k2} be two soft subsets of m_k, then the BR-topological operators satisfies the following properties:
1. $S\underline{\tau}_{BR}(\emptyset) = S\overline{\tau}_{BR}(\emptyset) = \emptyset$.
2. $S\underline{\tau}_{BR}(m_k) = S\overline{\tau}_{BR}(m_k) = m_k$.
3. If $m_{k1} \subseteq m_{k2}$, then $S\underline{\tau}_{BR}(m_{k1}) \subseteq S\underline{\tau}_{BR}(m_{k2})$.
4. If $m_{k1} \subseteq m_{k2}$, then $S\overline{\tau}_{BR}(m_{k1}) \subseteq S\overline{\tau}_{BR}(m_{k2})$.
5. $S\underline{\tau}_{BR}(m_{k1} \cap m_{k2}) = S\underline{\tau}_{BR}(m_{k1}) \cap S\underline{\tau}_{BR}(m_{k2})$.
6. $S\underline{\tau}_{BR}(m_{k1} \cup m_{k2}) \supseteq S\underline{\tau}_{BR}(m_{k1}) \cup S\underline{\tau}_{BR}(m_{k2})$.
7. $S\overline{\tau}_{BR}(m_{k1} \cup m_{k2}) = S\overline{\tau}_{BR}(m_{k1}) \cup S\overline{\tau}_{BR}(m_{k2})$.
8. $S\overline{\tau}_{BR}(m_{k1} \cap m_{k2}) \subseteq S\overline{\tau}_{BR}(m_{k1}) \cap S\overline{\tau}_{BR}(m_{k2})$.

Proposition 4.8: If τ_{BR} is a quasi-discrete BR-topology, then every m_{ki} is a exact soft set. i.e.,
$S\underline{\tau}_{BR}(m_{ki}) = S\overline{\tau}_{BR}(m_{ki}) = m_{ki}$.

The following remarks and corollary establishes the relationship between soft set in rough approximation space and BR-topological rough approximation space.

Remark 4.9: If a soft set is a rough soft set in rough approximation space it need not be a rough soft set in BR-topological rough approximation space.

Remark 4.10: Though BR-topological rough approximation space is generated from rough approximation space, the approximation operators in both the cases need not be equal.

Corollary 4.11: Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space and m_{k1} be a soft subset of m_k. Then,
1. Boundary region of m_{k1} in BR-topological rough approximation space is a subset of Boundary region of m_{k1} in rough approximation space.
2. accuracy of m_{k1} in BR-topological rough approximation space is less than or equal to accuracy of m_{k1} in rough approximation space.

The following example is illustrated to prove the above remarks 4.10, 4.11 and corollary 4.12 .
Example 4.12: Consider the BR-Topology taken in example 4.2, since the topology is quasi discrete then by proposition 4.8 every soft subsets of m_k is exact.
Let $m_{k1} = \{(e_1, \{(3,4)\})\}$. Then, $S\underline{\tau}_{BR}(m_{k1}) = S\overline{\tau}_{BR}(m_{k1}) = m_{k1}$ which implies that m_{k1} is exact soft set, boundary region is empty and accuracy is 1.
Similarly, $\overline{S_{Apr}}(m_{k1}) = \{(e_1, \{(5,4), (5,6)\})\}$ and $\underline{S_{Apr}}(m_{k1}) = \{\emptyset\}$ which implies that boundary region $= \overline{S_{Apr}}(m_{k1})$ and accuracy is 0.

From the above the following can be interpreted:
1. $\overline{S_{Apr}}(m_{ki})$ need not be equal to $S\overline{\tau}_{BR}(m_{ki})$.
2. $\underline{S_{Apr}}(m_{ki})$ need not be equal to $S\underline{\tau}_{BR}(m_{ki})$.
3. Boundary(m_{ki}) in BR-topological rough approximation space is subset of Boundary(m_{ki}) in rough approximation space.
4. Accuracy(m_{ki}) in rough approximation space is less than Accuracy(m_{ki}) in BR-topological rough approximation space which implies that accuracy in BR-topological rough approximation space has been improved.

5. Conclusion

In this paper, a generalization of soft set from single universe to 'n' number of different universes was defined using soft binary relation and approximated over rough approximation space, soft approximation space. Further, new topological approximation space was developed and properties of approximation paces, topological approximation spaces were discussed. This paper has many wide areas to be covered further by making a fusion of the proposed method with other techniques.

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