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Pricing the Illiquidity Premium for Private Market Assets

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Abstract

The purpose of this paper is to examine the effect of illiquidity on the value of assets using an options approach. The paper uses two experiments (Hydropower & Damodaran data) to determine the illiquidity. Evidence suggests that erroneous assumptions of the illiquidity premium priced into an asset may ultimately misrepresent an investments value and the risk-return profile. Determining a liquidity standard and an appropriate independent measurement of illiquidity premiums are compared using common yet varying illiquidity pricing theories. Current insights and illiquidity impacts through measurement practices being followed by asset managers are assessed using data of highly illiquid private market assets. Experimental analysis of illiquidity pricing theories are measured against common industry practices of private market assets to reveal inaccuracies in illiquidity premiums. In this paper, the researcher has conducted two sets of experiments to determine the illiquidity premium. Results of the study conclude that the current illiquidity premium being paid to the investors are substantially below the premium that we find using an options approach.

Introduction

Large volumes of empirical and theoretical literature suggest that both the illiquidity level and the illiquidity risk exist in asset prices. Yet the conclusions of such empirical studies do diverge, both with respect to the magnitudes of illiquidity levels and risk with respect to their relative importance. The liquidity adjusted capital asset pricing model (LCAPM) by Acharya and Pedersen (2005) provides a unified theoretical framework in which both the illiquidity level and different types of illiquidity risk affect asset prices. The LCAPM is a conditional model, implying that both illiquidity betas and illiquidity risk premia potentially vary over time. Earlier empirical implementations of this model assume constant risk premia, yielding unconditional versions of the LCAPM (Acharya and Pedersen, 2005; Lee, 2011).

The seminal paper by Amihud and Mendelson (1986) proposes that the long-term investor benefits from investment in illiquid securities. Swensen (2000) states that: “Because market players routinely overpay for liquidity, serious investors benefit by avoiding overpriced liquid securities and locating bargains in less followed, less liquid market segments. Investors engaging in illiquid investment however also need to monitor their illiquidity risk”. The Economist (Feb. 11, 2010) writes that in the Financial crisis in 2007-2009, illiquidity risk was neglected: “With markets awash with cash and hedge funds, private-equity firms and sovereign-wealth funds all keen to invest in assets, there seemed little prospect of a liquidity crisis.”. When liquidity evaporated as confidence fell, many illiquid investors found themselves forced to fire sales.

The LCAPM suggests that there are three types of illiquidity risk. Firstly, there is the covariance of individual asset illiquidity with market-wide illiquidity, studied by, e.g., Chordia, Roll, and Subrahmanyam (2000). Such commonality in illiquidity is documented for all major equity markets (Brockman, Chung and Perignon, 2009; Karolyi, Lee, and van Dijk, 2011). For the US stock markets, Acharya and Pedersen (2005) find that this is the least important of the three types of illiquidity risk. Secondly, there is the co-variance between individual asset return and market-wide illiquidity. This type of illiquidity risk is extensively studied. Pástor and Stambaugh (2003), Liu (2006), Watanabe and Watanabe (2008), Korajczyk and Sadka (2008), and Lou and Sadka (2011) all show that this illiquidity risk is priced risk in US stock markets, but Hasbrouck (2009) and Asparouhova, Bessembinder and Kalcheva (2010) reach the opposite conclusion. Finally, there is the covariance between individual asset illiquidity and market return, which Acharya and Pedersen (2005) identify as the most important of the different illiquidity risks. Wagner (2011) argues theoretically that this risk is realized when investors simultaneously need to liquidate their positions. Brunnermeier and Pedersen (2009) show that such scenarios materialize when investors simultaneously hit their funding constraints and are forced to sell assets.

These contending liquidity covariances between assets and markets raises a central interest for investors, whether to pay higher prices for more liquid assets than for otherwise similar less liquid assets. If so, how much should the premium be for more liquidity and concurrently what pricing method can best estimate this discount for illiquid assets? In this paper we introduce an option pricing theory to establish a controlled market environment to simulate investor choice. This option framework is a bounded range of value to analyze private market data from thirty currently marketed private infrastructure managed funds using the expected return to serve as the benchmark for an unlevered market cost of capital. Further, a case study calculation follows using two competing models for deriving an illiquidity premium was conducted with return data from the hydropower sector of the infrastructure asset class. The results of a statistical analysis test show conclusive evidence that Longstaff's illiquidity premium method provide an alternative proposition to industry practice. The Longstaff model is based on a backward iteration algorithm. At each exercise period, the algorithm approximates the continuation value, which is the value of the option if it is not exercised.

According to Longstaff (1995, referred to by Finnerty, 2013, p. 576) the accepted wisdom in the business appraisal field is that an appropriate discount for a two-year restriction ranges from 25% to 35% and for a one-year restriction 15% to 25%. Further Longstaff refers to the marketability discount being consistent with this range by defining the value of marketability as the price of a look back put option (where the payoff depends on the maximum or minimum of the underlying asset's price occurring over the life of the option). The private placement discount is often referred to as a marketability discount.

The further outline of this paper is as follows. Chapter 1 discusses the related literature. Chapter 2 describes the methodology that is being applied here. Chapter 3 discusses the results. Finally chapter 4 concludes.

Chapter One – Literature Review

Defining Liquidity

The concept of liquidity in financial contexts can take various forms depending on the participants and their perspectives; therefore, to measure any kind of liquidity or illiquidity its first important to distinguish a basis for the type of liquidity being discussed throughout this paper. An initial perspective of liquidity for this paper refers to the transactability for items of value. Therefore, a defining perspective from a central bank who fundamentally creates a medium (currency) for exchanging items of value will support as the foundation for defining liquidity.

According to the European Central Bank (ECB), liquidity is defined as, "the ability of an economic agent to exchange his or her existing wealth for goods and services or for other assets" (Nikolaou, 2009, p. 10). Further, the ECB denotes three types of liquidities in the financial system. First is the central bank liquidity, denoted as the ability of the central bank to supply the liquidity needed to the financial system. This type refers to the amount of liquidity provided through the central bank auctions to the money market according to the monetary policy stance.

The second type of liquidity to the ECB is the concept of a funding liquidity, defined as the ability of banks to meet their liabilities, unwind or settle their positions as they come due. This means an entity is liquid as long as inflows are bigger or at least equal to outflows.

The third type of liquidity according to the ECB, also the most appropriate for this experiment, is market liquidity. Market liquidity is defined as the ability to trade an asset at short notice, at low cost and with little impact on its price. In experimental terms, the ECB definition of market liquidity for economic agents exchanging value in a market environment creates the opportunity to introduce several empirical liquidity studies.

Examination of Liquidity Viewpoints

The following section portrays various empirical studies of market-based liquidity. The conceptual research of the following authors offers a contextual meaning to liquidity and will ultimately shape the parameters for conducting an experiment to measure and analyze illiquidity. Liquidity is reported to be an elusive financial concept that requires a complex set of measurements to define (Amihud, 2000); however, it has also been stated that liquidity simply refers to "the ease of trading a security" (Amihud, Mendelson, & Pedersen, 2005, p. 2). In a review of the literature on liquidity, both descriptions appear to be true. Liquidity as a concept is dichotomized by the idea of illiquidity, which refers to the difficulty associated with trading or selling an asset or stock (Amihud, 2000; Amihud, Mendelson, & Pedersen, 2005; Damodaran, 2005). This section will define liquidity as a concept that describes the ability of an investor to manage assets or stocks, as well as contrast it with illiquidity.

Damodaran (2005) claims that liquidity is not a fixed state of being but rather a changing quality of an asset or stock that is measured in gradients. Thus, he emphasizes that "liquidity is a continuum" (p. 2) which can be measured quantitatively and which changes over time and in response to market conditions. Further, not just assets but also the market can be analyzed in terms of its liquidity or illiquidity. According to Pastor and Stambaugh (2002), "it is not a stock's liquidity per se that matters but its relationship to overall market liquidity" (Damodaran, 2005, p. 20). Over a 34-year period, Pastor and Stambaugh (2002) found that "stocks whose returns are more sensitive to market liquidity have annual returns that are 7.5% higher than stocks whose returns have low sensitivity to market liquidity" (Damodaran, 2005, p. 20).

A few factors are associated positively with liquidity, including the market capitalization or size of the stock, higher trading volume of a stock or asset, and stock price at the end of year (Amihud, 2000). Lower transactions costs are also correlated with the liquidity of a stock or asset as is the ability or ease of the investor to sell or trade as is desirable (Damodaran, 2005). Articulating this more clearly, Damodaran (2005) states, "the value of liquidity ultimately has to derive from the investor being able to sell at some pre-determined price during the non-trading period rather than being forced to hold until the end of the period" (p. 42). In essence, liquidity is well correlated with the amount of freedom and agency an investor has to manage assets and stocks according to changing needs.

Amihud, Hameed, Kang & Zhang (2012) found that because liquidity is valued by stock market investors and traders, they naturally demand higher returns for stocks that are less liquid. While liquidity may be more desirable because it makes assets easier to sell, the impact of liquidity upon returns appears to be negative while the impact of illiquidity upon returns is positive. Amihud et al. (2005) state, "Liquidity helps explain why certain hard-to-trade securities are relatively cheap" and why small stocks that are typically illiquid earn high returns (the small firm effect)" (p. 3). In essence, while it is safer to deal with more liquid stocks it can potentially be more lucrative to deal with illiquid stocks due to higher returns. Investors willing to absorb the illiquidity risk can anticipate higher returns.

Concept of Illiquidity

Francis Longstaff (2001) discovered a fairly clear distinction between illiquidity definitions mentioned in academic and practitioner types of literature: Defined by academic literature, illiquidity can be approximated by the bid/ask spread and/or transaction costs associated with trading a security. From this perspective illiquidity is the situation in which investors find that they face higher trading and execution costs than at other times or in other markets. In this view, an investor can usually trade whenever desired, albeit at some (potentially high) cost. In practitioner literature, the term illiquidity takes a different form, traders view illiquidity as the situation where their ability to buy or sell securities (at any price) is limited or restricted. This type of illiquidity has to do with the quantity of trades that can be executed rather than with the costs of trading.

Giving a comprehensive overview of liquidity also entails articulating the characteristics of illiquidity as regards the functional manageability of assets. Put simply, if liquidity boils down to the ease an investor has in selling or trading an asset, illiquidity can best be defined as the opposite: the degree of difficulty an investor has in selling or trading an asset (Amihud, 2000; Amihud, Mendelson, & Pedersen, 2005; Damodaran, 2005). As with liquidity, illiquidity is a quality that exists on a continuum and is measured in gradients.

As a relative measurement, illiquidity of an asset or stock tends to be determined by the same characteristics, though in a reversed manner, that determine its liquidity. Therefore, assets or stocks with lower trading volumes and market capitalization, larger transactions costs, lower trading frequency, and held by larger firms tend to be more illiquid (Amihud, 2000). Basically, the less freedom an investor has to trade or sell a stock the greater its illiquidity. The difficulty of managing an asset is also described as its risk; therefore, illiquid assets are said to be more risky than liquid ones.

Because assets with greater illiquidity come with greater risks, investors usually pay less for illiquid assets than liquid ones (Damodaran, 2005). Because assets with greater illiquidity are more risky to investors than stocks with lesser illiquidity, it follows that "stock excess return (RM-Rf)y is traditionally considered as risk premium" (Amihud, 2000, p. 22). In other words, the risk premium is a compensation for investment risk, in which expected stock illiquidity is associated with higher risk premium than lower expected stock volatility. Citing Acharya and Pedersen (2005), Damodaran (2005) claims that "illiquid stocks have annualized risk premiums about 1.1% higher than liquid stocks, and that 80% of this premium can be explained by the covariance between a stock's illiquidity and overall market illiquidity.".

Measuring Illiquidity: Theory

Amihud

The theoretical measure of illiquidity proposed by Amihud (2000) is based on "the average daily ratio of absolute stock return to dollar volume" (p. 1). Amihud (2000) claims that his measure of illiquidity is more effective and feasible than others because it can "construct long time series of illiquidity that are necessary to test the effects over time of illiquidity – both expected and unexpected – on ex ante and contemporaneous stock excess return" (p. 1). His findings and others show that, in general, the higher the illiquidity of a stock, the larger its return: "over time, expected market illiquidity of stocks has a positive effect on the ex-ante stock excess return" (Amihud, 2000, p. 1). These findings are confirmed by Amihud et al. (2012) who claim, "The average return on the most illiquid stocks is significantly higher than the average return on the most liquid stocks" (p. 3). Contra wise, expected market liquidity has a negative impact on stock return: "across stocks and over time, expected stock returns are an increasing function of expected illiquidity" (Amihud, 2000, p. 1).

Damodaran

Damodaran (2005) provides illuminating commentary on the impact of illiquidity upon its valuation of assets by investors. Based on the established tenet that investors pay more for liquid assets and less for illiquid assets, Damodaran (2005) articulates three theories that all arrive at the same conclusion: "an illiquid asset should trade at a lower price than an otherwise similar liquid asset" (p. 17). In the first approach to the finding an asset's value is decreased by present expectations of future transactions costs, which creates a discount upon its value. In the second approach, which also reflects the findings of Amihud (2000), "the required rate of return on an asset is adjusted to reflect its illiquidity, with higher required rates of return (and lower values) for less liquid assets" (Damodaran, 2005, p. 17). With the third approach, decreased liquidity carries value as an option, because an illiquid asset may not be able to be traded or sold when it has reached a high price (Damodaran, 2005). Building on Longstaff (1995) Damodaran describes this as a written put option on the firm's value, as the owner of the firm cannot sell its shares for a specific amount of time. As per Damodaran, Illiquidity discounts can be determinants based on couple of factors such as liquidity of assets owned by the firm, financial health and cash flows of the firm, possibility of going public in the future & size of the Firm.

Measuring Illiquidity: Approach

Discount Rate

Investor aversion to liquidity risk naturally leads to a discount rate for purchasing assets that have higher illiquidity: "because liquidity varies over time, risk-averse investors may require a compensation for being exposed to liquidity risk" (Amihud et al., 2005, p. 2). One way in which this compensation manifests is when illiquid stocks or assets are offered with a discount rate. The discount rate is applied to illiquid shares or assets that cannot be traded for a period of time (Amihud et al., 2005, p. 6). Due to this illiquidity risk, or decrease in an asset's manageability, a discount is applied to its purchase price. The period of time during which an investor cannot trade or sell an asset imparts a burden upon the investor. Put another way, "the price discount due to illiquidity is the present value of the expected stream of transaction costs through its lifetime" (Amihud et al., 2005, p. 8).

In practical terms, the discount rate on illiquid shares is akin to the difference between purchasing a lunch you can eat right now and one you have to wait an hour to eat. The more valuable lunch is the one you can eat now, when you're hungry! Most folks are going to pay less for a lunch they can't eat right away. It's the same way with assets and stocks.

Silber (1991) and Amihud et al., (2005) have stated that, "shares that are restricted from trade for two years have an average discount of 30% relative to shares of the same company with identical dividends that can be traded freely". Typically, the greater the restrictions imposed on trading, the greater the discount on the valuation and purchase amount of the asset or stock. Potentially, buying at a discount could yield greater returns when the shares are sold; however, it does eclipse the capacity to trade shares if they peak in value during the holding period. So, in a sense, buying illiquid assets is a gamble that is both acknowledged and compensated with the discount rate. Finally, in regard to private businesses, "the illiquidity discount is likely to vary across both firms and buyers, which renders rules of thumb useless" (Damodaran, 2005). Revealing that due to so many variables and metrics present in private businesses and ownership models, a common one size fits all discount could be unrealistic to measure.

Option Approach

Chaffe (1993) valued the liquidity discount by considering the option to sell the security at the free market price (without the restriction). The value of this put option can be considered as the illiquidity discount. Chaffe (1993) estimated the value of a European put option at the free market price and showed that this was between 28 and 41 %, for a stock with a two year required holding period and a volatility between 60 and 90 percent.

Longstaff (1995) valued the illiquidity discount emphasizing the unrealized gains of not being able to sell the security for a certain period. He estimated the value of a lookback option. These models for certain assumptions may however tend to result in very high discounts.

Damodaran (2005) used a more pragmatic approach where he increased the strike price to reflect the unrealized gains and calculated the value of a standard European put option on this higher strike price weighted by the probability that this higher strike price would not be exceeded. In his words:

"*Not being able to trade on this investment for a period (say, 2 years) undercuts this discipline and it can be argued that the value of illiquidity is the product of the value of the put option (estimated using a strike price set 25% above the purchase price and a 2 year life) and the probability that the stock price will rise 25% or more over the next 2 years" Damodaran (2005 p. 42)*

Katsanis advocated the application of a short put option model for estimating the DLOM. According to Katsanis, the shout put option value serves as an estimate of the marketability and liquidity value embedded within the marketable share value so that the following relationship exists:

Marketable Share Value = Short Put Value + Nonmarketable Share Value
The short put option model is essentially a modification of the Chaffe model. If the risk-free interest rate exceeds the dividend yield and the dividend yield is not zero, the put value concluded by the Chaffe model is multiplied by an exponential adjustment factor based on the expected dividend yield of the subject company security.

Bajaj Study
This first order of research was proven by the "*Bajaj Study,"* which showed that Illiquidity effects on prices do exist both theoretically and empirically. This study of illiquidity value entailed comparing prices of restricted stock versus unrestricted stock, IPO's and private placements, and showed each area agreed that liquidity was preferred, as evidenced through market prices. The Bajaj Study concluded that the marketability of an asset refers to the degree to which an asset can be converted to cash quickly, without incurring large transaction costs or price concessions. All else equal, the more marketable an asset, the higher the price an investor will be willing to pay for the asset. The lack of marketability of an asset is costly to investors because it potentially causes the investor to miss opportunities to allocate capital to alternative uses, such as liquidity or portfolio rebalancing (Bajaj, 2001).

Chapter Two: Methodology
To initialize an illiquidity measurement, the methodology of this experiment begins with an examination into the framework and quality of the data utilized.

Valuation data of hydropower plants, an infrastructure sub-sector were captured for the illiquidity experiment within this paper. These data points were retained through privately held companies. Given this, relevant data in line with this empirical study was aggregated using audited valuation reports provided by private equity asset managers. Common sizing the hydropower data was made by analyzing the investment holding structures to which the assets were held within a fund portfolio, commonly the collection of hydropower plants are grouped together by region & type for presentation purposes. Respecting the privacy of the owners and asset managers, the names of the funds, assets and plants are presented anonymously in this experiment. Since the arguments presented in this paper are made with private market valuations, the illiquidity measurement experiment must acknowledge such data biases and shortfalls with room for future improvement.

Infrastructure
An asset class is generally viewed as highly illiquid due to the combination of the real asset characteristics and high transaction costs. Although the choice behind using this specific asset class is somewhat arbitrary, its high degree of illiquidity could serve as a clear and valuable variable. Yet determining the fair illiquidity compensation could be universally applied to any data in any asset class.

Infrastructure as an asset class could also be taken rather broadly; therefore, it is important to distinguish and define what exactly makes an asset an infrastructure asset. The infrastructure asset management industry is currently in a growing state, still it is in infancy compared to other asset classes.

According to Ibbotson (2009), “Infrastructure is the physical assets, facilities, and systems that enable society to function”. These assets include systems and modes of communication, transportation, energy and utilities, and basic social assets, such as schools and hospitals. Ibbotson (2009) further asserts that infrastructure is comprised of, “long-lived, real assets that are costly and time-consuming to replace, often without immediate substitutes, that typically generate relatively stable cash flows that increase with inflation”.

Hydropower
Hydropower happens to be a subsector within class of Energy sector, which in turn happens to be a sub class of Infrastructure sector and is regarded as one of the oldest and most primitive forms of energy. Fundamentally, all hydropower plants operate in a similar fashion, wherein water flowing over a turbine is used to drive energy from a generator. This highly simplified illustration outlines the basic business model underpinning a rigorous construction involved to building the plant and selling the energy thus created. Although hydropower assets can be broken down into various plant types and revenue schemes, the objective of this research is to ultimately price the assets illiquidity, as a result all plant types will be treated as equal in the study.

Hydropower assets are an ideal infrastructure subsector choice for this experiment due to their relatively long and useful lives. Hydropower plants “require substantial capital investment, but they offer extremely low operating costs and long operating lifespan of around 40–50 years, which is most of the cases can be extended to 100 years with some rehabilitation” (Fichtner, 2015). These characteristics are beneficial to this experiment in a variety of ways:

1. Firstly, the relative stability of a plant’s business model and its relative simplicity adds to the valuation work.
2. Secondly, the fact that plants have such a long, useful life provides an opportunity for future refinement of this study and other offshoot experiments, which could potentially record long data streams on a single asset relative to other energy producing infrastructure assets.

Case Data
The largest threat to this experiment and ones like it is not related to the limited access to relevant data for drawing conclusions. However, the challenge lies in the fact that the sector itself adheres to limited reporting standards resulting in a limited number of common variables across all valuation asset reports.

The same is prevalent in the sector since it is considered to be niche asset class within infrastructure, which in most of cases itself is considered to be niche class of assets, along with the combined impact of private market access. As a result, in order to effectively overcome the challenge and conduct the experiment, a filtering process was adopted with the data to determine a minimum number of variables needed to proceed further with the analysis. During this process, the following criteria were used to identify each of the hydropower asset (in order to develop conclusive evidence for pricing the liquidity premium):

- Asset acquisition price and time
- Consistent valuation method used throughout the asset holding period
- Regularly reported valuations (preferably monthly or quarterly)
- The construction of the valuation discount rate not containing an illiquidity premium

Taking into consideration the above, data fitting the minimum criteria above was gathered from eighty-seven (87) hydropower plants, all geographically located in Western Europe. Post data collection and upon further review, it was established that close proximity of the identified plants also offered an added advantage of all the plants operating under similar market conditions with transparent energy prices, especially financial schemes, making it possible to extrapolate data. The selected 87 plants, with corresponding energy generation capacities and values, are shown in table 1.1. It is to be noted that each of the selected plant was acquired at a single point of time, operated under a consistent valuation methodology and published reports on a quarterly basis.

Illiquidity Compensation: Challenging the Rule of Thumb

In contrast to such empirical research and proven price favourability of investors towards liquidity, private market practice seemingly bares a minimum premium, if anything, to investors. Evidence of this minimization is found in the lack of explicit or implicit illiquidity factors presented to investors. Assets offered with an explicit incentive to the investor will commonly take a rule of thumb form, typically 1%. Other illiquid assets without explicit illiquidity premiums can and do offer implicit incentives commonly built into the discount rate of valuations and / or cost of capital projections.

Considering the private markets, establishing a market medium will provide a benchmark to compare the empirical illiquidity premium reconstructions. The medium, although arbitrary, is commonly found in the private markets to provide investors with a 1% discount as rule of thumb for the asset's illiquidity (Dr Brian Cadman 2013). Therefore, the primary constructions throughout this paper will establish actual data from the private market sector and calculate the illiquidity premium under the main assertions provided by Longstaff (1995), who conveys a method to derive an upper bound illiquidity premium using option pricing theory. The results of the experiment are reviewed both from qualitative and quantitative perspectives. These findings are then compared against a common industry practice to place a value of 1% to the risk premium. Throughout this paper, 1% will serve as a baseline rule of thumb for an illiquidity comparative analysis.

The rule of 1% was discovered in this niche of infrastructure assets, a marketplace to which assets can be generally viewed as illiquid; yet, at best, this general rule of thumb can be commonly seen in discount rate constructions.

An investment illiquidity premium is often offered without much explanation or basis as to the calculation in the private markets. However, in public markets numerous competing theories for fair value compensation for an investor's illiquidity do stand ground, typically through transparent pricing. Private markets tend towards non-transparency when it comes to illiquidity premiums; therefore, a public market advantage exists. Accordingly, this paper challenges the fair value when investors inability to sell privately held assets is evident and the relevant data is notoriously limited. Further, challenges of the fair value argument also arise when the asset class in question is further illiquid due to being a niche asset class within the privately held markets.

As regards the primary focus to calculate an appropriate illiquidity premium as measured against the rule of thumb, it's a worthy consideration to know if investors value such marketability (illiquidity) in the first place.

The Experiment
Framework
The composition of this experiment involves comparing an illiquidity premium of 1%, commonly given to illiquid asset investors, by taking into consideration the empirical study of Francis Longstaff, wherein pricing of marketability restriction (illiquidity) premium has been done through a hypothetical upper bounded selling price using option theory. The upper bound argument by Longstaff (1995) is centered around the concept that a hypothetical investor with a disciplined market timing ability will always sell when the price reaches a certain level above the cost. This perfect timing construct to derive an upper bound concludes that if the investor has this marketability restriction relaxed, the cash flow to the investor would be the same as if they swapped the future value for the maximum price attained by selling at the disciplined upside price. It may be noted that the securities considered under Longstaff's research were US Treasuries, which are used mainly for their consistency. Hydropower assets were chosen amongst other infrastructure assets in an attempt to follow Longstaff's narrative for upper bound theory, and due to the approximate characteristics Hydropower plants exhibit to fixed income assets—including a predictably long and useful life, consistency of generation and ordinarily transparent energy prices.

A continuation of Longstaff's illiquidity study was exemplified through the work of Aswath Damodaran, NYU professor and published author, who compared various valuation and illiquidity premium pricing methods. In an exhaustive study *Marketability and Value: Measuring the Illiquidity Discount,* Damodaran (2005) scopes various methods for pricing an asset's illiquidity premium, including Longstaff's results using options. This paper is based on comparing the rule of thumb, by following the pricing Illustration by Damodaran's hypothetical case study, with hydropower plant data. According to Damodaran (2005), an important clarification with regards to option-based illiquidity pricing is that "liquidity does not give you the right to sell an asset at today's market price anytime over the next T years". Damodaran (2005) concludes the studies by claiming, "What it does is give you the right to sell at the prevailing market price anytime over the next T years". Therefore, the framework of the current experiment is to argue that the value of illiquidity is the product of the value of the put option estimated using a strike price set X% above the purchase price and a T-year life and the probability that the asset will rise X% or more over the next T years (Damodaran, 2005). The increase with X% reflects Longstaff's idea of illiquidity as the impossibility to sell and realize to be able to sell at the highest price. Hence, we do not model the lookback option but capture this component by increasing as in Damodaran the exercise price of the standard option with X%. (pls refer table 1.2)

Ultimately, deriving an illiquidity premium using option theory requires three calculation based on the Damodaran's' illustration that determine:

a) the price of a put option of what maturity and what exercise price using a common financial industry formula with an upper bound for a strike price;
b) the probability the asset value will increase beyond that upper bound; and, (to calculate the upper bound, we calculate the value of cost of equity, Standard Deviation & Investor Upside)
c) ultimately the value of liquidity itself. (the value of liquidity has been calculated by multiplying the Put option with Probability and Z-Inverse value)

The value of the illiquidity is the product the value of the option and the probability that the price rises by more than 9%.

To enhance the value of this experiment calculating the illiquidity premium using Damodaran's illustration, a second sub-experiment was conducted and compares the analysis/calculations of both the experiment. Experiment -I uses the standard deviation from Damodaran database and Experiment –II used standard deviation derived from the hydropower data itself. In both, the experiments calculated the Value of liquidity for Year 1 & Year II by keeping all the other variables constant except this standard deviation. The purpose of conducting the two experiments is to check the results by using different standard deviation, so that we can compare both the results.

Chapter Three: Results

In this chapter we show the results of the calculation of the illiquidity premium on the basis of the approach by Damodaran as discussed in the previous chapter. We first calculate the value of the put option.

Step 1 - Option Pricing Model

The put option pricing model used in the Damodaran illustration was a financial industry standard for option pricing known as the Black-Scholes formula. An option is defined as "a security giving the right to buy or sell an asset, subject to certain conditions, within a specific period of time" (Black-Scholes, 1972). The put option gives the owner a right to sell. Under American style the option can be exercised at any time, but in the European style the right can be exercised only on a specified future date (Black-Scholes, 1972). Underlying the concept of this illiquidity derivative is the put option to sell at a predetermined price. The Black-Scholes formula used to derive the put option price is made through the following four calculations.

$$d_1 = \frac{\ln\left(\frac{S_0}{x}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sqrt[\sigma]{T}}$$

$$d_2 = d_1 - \sqrt[\sigma]{T}$$

$$Call\ Option\ Price\ (C) = S_0 N(D_1) - XN(D_2)e^{-rf\,T}$$

$$\boldsymbol{Put\ Option\ Price\ (P) = C + \left(Xe^{-rfT}\right) - S_0}$$

The put option price is ultimately derived through a put-call parity. The variables needed to run these calculations will be assessed and quantified from the hydropower case data. The following variables are based on current market data from the assets.

(S_0) – Price (Unlevered Production Weighted Value)

The beginning price variable has been determined by an acquisition, denoting time-zero, given the actual exchange in value for the assets. This differs from future valuation reports based on the acquirer's assessment of fair value. The ability to track the asset's value over time can only be made possible by introducing a uniquely constructed value based on presented data. This new price construction is called the *unlevered production weighted value* (UPWV).In the said case, the UPWV is factored from the asset level and combined with energy production capacity. The necessity to build a price around the energy production value is due to the nature of the data available. As a result, the fund tracks the value, at the asset level, since company, as in many assets under management, take on varying degrees of leverage and follow on acquisition activity over time. Therefore, to accurately track a consistent set of hydropower plant values based on limited information, the optimal choice is to base plant value from a weighted level of energy production capacity.

The second level construct involved in determining the value is unwinding a common sizing tactic used by asset managers to determine the amount of leverage involved at plant level. The equity values in this case have numerous partnerships and co-financing structures in place that need to be unwound to calculate the true equity of investors' illiquidity premiums. The fund level takes on no debt but does own a small minority stake in the asset that must be accounted for during the process. The price recalculation needs to account for the fund's equity stake, asset level and plant level leverage, and the correlated annual energy production capacity measured in Gigawatts. The resulting unit (UPWV) is the new asset price determined by the following equation:

$$UPWV = Equity\ Value * \frac{GWh\ Production_{T_{Zero}}}{GWh\ Production_{T_{Current}}}$$

1. Equity Value

The equity value is the reported fair equity value from the fund manager. In the case of time zero, the equity value is the acquisition price. The equity value is the market value, calculated with a discount rate not including an illiquidity premium on projected cash flows from the plants.

2. GWh Production at Time-Zero

The sum production capacity measured in gigawatts per hour of all the plants at time zero.

3. GWh Production at Time-Current

The sum production capacity measured in gigawatts per hour of all the plants at the current period under analysis.

Based on the hydropower case data presented in table 1.1, the variable price (UPWV) is $\boldsymbol{S_0 = 271.55m.}$

(X) - Strike Price (Upper Bound)

In this illiquidity pricing formula, the upper bound must be determined, at which a disciplined investor will always sell, and will take the place of the option strike price. Damodaran (2005) also acknowledges that such a disciplined investor is hypothetical, hence translating into the fact that number chosen for the upper bound is subjective.

The hydropower case data in this study was taken from an investment fund and the fund's private placement memorandum. The fund offers investors a projected 7-9% IRR over the life of the fund per annum. Although this range is on an annual basis and the assets are measured on quarterly intervals, the 9% is a natural upper bound for the period of one year. Therefore, it is deduced that the assets upside per year is 9%, and this number will be sensibly used for the upper bound, having the strike price calculated from the price at time zero multiplied by 1 + 9% for a one-year period option, concluding $\boldsymbol{X = 295.99m}$. The reason for considering the one-year period is that to capture the current macroeconomic changes in the Industry. Moreover, other illiquidity studies such as Pre IPO & Restricted Stocks has been considered the period between 6 months to 18 months only.
However, we have calculated the value of X, for 2+9% i.e two years period as well, the two year period option, coming $\boldsymbol{X = 322.63\text{m}}$

(T) - Time

Time will be denoted both as the starting point in the hydropower case data (acquisition price) set to time-zero (T_0), and the subsequent fair valuations marking forward time periods ($T_1, T_2, T_3 \dots T_x$). Each time period will denote one calendar quarter. An asset transaction for time-zero is used to create a new start for the acquirer and a true sense of fair value.

The hydropower case data time-zero starts with a set of plants acquired in 2017, and subsequently reports the fair values of these hydropower assets on a quarterly basis for the five following reporting periods. Post-acquisition of the plants in Year 2017, we have analysed the hydropower assets (Production, Fair value, and capital structure) on a quarterly basis for the five following reporting periods.
The time period of 1 year was chosen in this experiment and will illustrate the length of illiquidity alongside the length of the option. Therefore time $\boldsymbol{T = 1}$will also denote one year and T=2 will denote two years

(r) - Risk-Free Rate

The model assumes that the riskless interest rate will be constant throughout the holding period of the investor in the fund. Since the hydropower plant values have been standardized in Euro denominations, German Government bonds will be used as the risk-free proxy. The 10 Year German bonds were chosen as the closest duration to our time T. At the time this study was conducted, these 10-Year bonds were yielding 0.46% (Bloomberg 2018); therefore, making $\boldsymbol{r_f = 0.46}$% per annum**.** Longstaff (1995) points out that the upper bound formula assumes that the investor who executes trades with perfect market timing ability will be also be reinvesting at this risk-free rate.

(σ) – Historical Volatility

Measuring the asset volatility is a necessary component in the Black-Scholes option pricing formula, as with any mean-variance model. The Research paper has calculated the volatility in two ways.
The asset Volatility has been considered from Damodaran website for Power sector in Germany. The annual volatility figure of $\boldsymbol{\sigma = 18.96}\%$ was derived from Damodaran website for power sector in Germany.
"Equity volatility" measures changes in the total equity capital of the company and "asset volatility," measures changes in value for the total business enterprise, essentially its total assets. The exact relationship between the common equity volatility, and the asset volatility in Merton's model was given by the expression from Asset vs. Equity Volatility

$$\sigma_E = \Delta_E * (A_0/E_0) * \sigma_A$$

In the Calculation, Damodaran website volatility has been used in the option pricing formula for determining the illiquidity premium. The reason for using the Damodaran Volatility is because its reflect the Comparable Companies Assets standard deviation for Power sector in Germany.
The results of step 1, to calculate a European style put option using the Black-Scholes model. The one year put option price of $\boldsymbol{Put\ Option\ Price = 34.86}$ calculated with the variables above, with an upper bound of 9%. The following step will determine the probability of this strike price being achieved over the course of the holding period. To calculate the put value,w have used Black-Scholes - Option Pricing model, with Stock Price - S0 as $271.55, Stock Std – σ 18.96%, Risk Free Rate – Rf-0.46% ,Maturity-1 Years and Strike Price – X $295.99. Pls refer table no -3 for detailed calculation.

Step 2 – Probability
The second step after determining the price of the put option is to calculate the probability that the asset will increase more than the strike price of 295.99m (disciplined investor sell price), over the one-year time horizon. To determine this probability, it is necessary to introduce two new variables: the cost of equity used across the asset acquisition and ongoing price valuation models, along with a z-score. The Z-score will be taken from a standard normal Z-Table distribution.

It may be noted that, to conduct another experiment, we have calculated standard deviation of the asset returns from the Hydro power- primary data. (data presented in table 1.1,)

(CoE) - Cost of Equity
The fund manager responsible for the case data determined a discount rate without an illiquidity premium attached to the risk premium. The cost of capital in this case data is an extended version of the capital asset pricing model (CAPM). A discount rate with an illiquidity premium figure could be used if the CAPM model used to derive the rate was transparent, and the ability to remove it and recalculate valuations would be possible To arrive at Ke (Cost of capital) , we have considered the Rf (Risk Free) rate of 0.46 % & Equity Risk premium of Germany 5.2% and Country Risk Premium of 0% [Source: Damodaran Database updated as on Jan, 2020] and Beta (Power-Germany) of 0.86 [Source: Capital IQ- calculation shown below]. We arrived at a Ke of $\boldsymbol{4.94}\%$ per year in this experiment.

(Z) – Z-Score

The probability of valuation crossing a strike price higher than the price at time zero in this experiment takes focus on only the positive right tail of a normal distribution Z-Table. The Z -score is calculated is calculated by taking the difference between the investor sell price and the cost of capital divided by the standard deviation. From this we can derive the probability that the price does exercise the strike price of the option in one year.

Experiment- I (Damodaran Power Industry -standard deviation)		
	Year 1	Year II
Probability	0.5847	0.6269
Z- Score	0.2140	0.3237

Experiment- II (Primary data Hydro Power -standard deviation)		
	Year 1	Year II
Probability	0.7004	0.7867
Z- Score	0.524	0.7949

Step 3: Value of Liquidity

After stepping through the Damodaran illustration of Longstaff's methodology, the focus shifts to the main hypothesis of this paper, pricing the illiquidity premium. The results will conclude whether the 1% rule of thumb is a close proxy and fair compensation to investors holding illiquid assets. Based on the constructions above, determining the value of the liquidity is made by multiplying the value of the put option by the probability the asset will rise above the strike price multiplied by the inverse of the Z-Score (see figure 1.1). The results conclude that the value of liquidity $\boldsymbol{Value\ of\ Liquidty = 14.48\ mn}$ for Year 1 by Experiment- I (by using Damodaran Volatility)

Finally, dividing the value of this liquidity by the value at time zero will represent a value to be the $\boldsymbol{iqudity\ Premium = 5.39}\%$.

Experiment- I (Damodaran Power Industry -standard deviation)		
	Year 1	**Year II**
Value of liquidity	14.48	22.04
Liquidity premium	5.33%	8.11%

Experiment- II (Primary data Hydro Power -standard deviation)		
	Year 1	**Year II**
Value of liquidity	7.40	7.06
Liquidity premium	2.73%	2.60%

Conclusion & Comparisons between Experiment- I & Experiment- II

The paper has conducted two experiments and compared the analysis/calculations of both the experiment results. Experiment -I used standard deviation from Damodaran database and Experiment –II used standard deviation from the hydropower data. In both, the experiment we calculated the Value of liquidity for Year 1 & Year II by keeping all the other variables constant except the standard deviation.

The paper found that, Liquidity premium is coming 5.33% for Year 1 & 8.11% for Year II in Experiment –I. While in Experiment –II, the Liquidity premium is coming 2.73% in Year -1 & 2.60% in Year II. Hence, the results of the study conclude that the current illiquidity premium being paid to the investors are below the premium that we find using Damodaran- standard deviation approach. The reason being, since these hydropower assets are quite costly with stable pricing history thus contain less volatility. Therefore, due to less the volatility the illiquidity premium comes below in potential comparison to other assets.

After analysing both the experiment results we may conclude that 2.73%-5.3% is the liquidity premium for Year 1 & 2.6%-8.11% liquidity premium for Year –II.

Limitation

Throughout the experiment numerous hurdles were encountered ranging from quantitative to qualitative; each hurdle was considered, and an approach was devised to overcome each one of the hurdles. Among the said hurdles, two hurdles are worth mentioning as regards to potential scope of improvement in future research:

a) The volume of data for current experiment was simply low, which is not uncommon when measuring niche private market assets, but its results could differ with future measurements. Also, regarding data, this experiment was conducted within a self-contained environment from one asset managers viewpoint on valuation; an interesting future focus could be to expand this comparison across multiple managers.

b) A second hurdle was the construction of an upper bound for a disciplined investor. Although acknowledged as a main assumption, future experiments might benefit from a behavioral finance approach to improve the strike price construction, using an irrational investor’s methodology for selling.

Figures

Figure 1.1 – Experiment I (Calculation of Put Value- using Damodaran -standard deviation)

Year 1

Black-Scholes - Option Pricing	
Stock Price - S_0	$271.55
Stock Std - σ	18.96%
Risk Free Rate - R_f	0.460%
Maturity - T	1.00
Strike Price - X	$295.99

$$d_1 = \frac{\ln(S_0 / X) + (r_f + \sigma^2/2)T}{\sigma \sqrt{T}}$$

d_1	-0.3355
$N(d_1)$	0.3686

$$d_2 = d_1 - \sigma \sqrt{T}$$

d_2	-0.5251
$N(d_2)$	0.2998

$$C = S_0 N(d_1) - X N(d_2) e^{-r_f T}$$

$$P = C + Xe^{-r_f T} - S_0$$

Call Value	$11.779
Put Value	**$34.860**

Year 2

Black-Scholes - Option Pricing	
Stock Price - S_0	$295.99
Stock Std - σ	26.81%
Risk Free Rate - R_f	0.460%
Maturity - T	$2.00
Strike Price - X	$322.63

$$d_1 = \frac{\ln(S_0 / X) + (r_f + \sigma^2/2)T}{\sigma \sqrt{T}}$$

d_1	-0.0135
$N(d_1)$	0.4946

$$d_2 = d_1 - \sigma \sqrt{T}$$

d_2	-0.3926
$N(d_2)$	0.3473

$$C = S_0 N(d_1) - X N(d_2) e^{-r_f T}$$

$$P = C + Xe^{-r_f T} - S_0$$

Call Value	$35.380
Put Value	**$59.065**

Figure 1.2 – Experiment I (Calculation of & Z score & Illiquidity Premium- using Damodaran - standard deviation)

	First Year	Second Year
Z-Score		
Cost of Equity	4.94%	10.13%
Standard Deviation	18.96%	26.81%
Investor Upside	9.00%	18.81%
Z-score	0.2140	0.3237
Probability-N(Z)	0.5847	0.6269
	0.5847	0.6269
Liqudity Value		
Put Option Price	€ 34.86	59.06450236
Probability	41.53%	37.31%
Z-Inverse		
Value of liqudity	€ 14.48	€ 22.04
Illiqudity Premium		
Value of liqudity	€ 14.48	€ 22.04
Price at Time Zero	€ 271.55	€ 271.55
liqudity Premium	**5.33%**	**8.11%**

Figure 1.3 – Experiment II (Calculation of Put Value- using Hydro Power-standard deviation)

Year 1

Black-Scholes - Option Pricing	
Stock Price - S_0	$271.55
Stock Std - σ	7.72%
Risk Free Rate - R_f	0.460%
Maturity - T	1.00
Strike Price - X	$295.99

d_1 = ln(S_0 / X) + (r_f + $\sigma^2/2$)T / σ √ T

d_1	-1.0180
$N(d_1)$	0.1543

$d_2 = d_1 - \sigma \sqrt{T}$

d_2	-1.0952
$N(d_2)$	0.1367

C = $S_0N(d_1)$ -

P = C + Xe^{-rfT-} S$_0$

Call Value	$1.631
Put Value	**$24.712**

Year 2

ack-Scholes - Option Pric	
Stock Price - S_0	$295.99
Stock Std - σ	10.92%
Risk Free Rate - R_f	0.460%
Maturity - T	$2.00
Strike Price - X	$322.63

d_1 = ln(S_0 / X) + (r_f + $\sigma^2/2$)T / σ √ T

d_1	-0.4213
$N(d_1)$	0.3368

$d_2 = d_1 - \sigma \sqrt{T}$

d_2	-0.5757
$N(d_2)$	0.2824

C = $S_0N(d_1)$ -

P = C + Xe^{-rfT-} S$_0$

Call Value	$9.402
Put Value	**$33.087**

Figure 1.4 – Experiment II (Calculation of Z- score & Illiquidity Premium - using Hydro Power-standard deviation)

	First Year	Second Year
Z-Score		
Cost of Equity	4.94%	10.13%
Standard Deviation	7.72%	10.92%
Investor Upside	9.00%	18.81%
Z-score	0.5254	0.7949
Probability-N(Z)	0.7004	0.7867
Liqudity Value		
Put Option Price	€ 24.71	33.09
Probability	29.96%	21.33%
Z-Inverse		
Value of liqudity	€ 7.40	€ 7.06
Illiqudity Premium		
Value of liqudity	€ 7.40	€ 7.06
Price at Time Zero	€ 271.55	€ 271.55
iliqudity Premium	**2.73%**	**2.60%**

Figure 1.5 – Comparing both the Experiments

Experiment- I (Damodaran Power Industry -standard deviation)		
	Year 1	**Year II**
Value of liquidity	14.48	22.04
Liquidity premium	5.33%	8.11%

Experiment- II (Primary data Hydro Power -standard deviation)		
	Year 1	**Year II**
Value of liquidity	7.40	7.06
Liquidity premium	2.73%	2.60%

Table 1.1 – Hydropower Case Data

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Asset Information		Time-Zero - Q1 2017 (Acquisition)									
Fund	Asset	Date	Plants	Ownership	Capacity (MW	Annual Production (GWh	Quarter Production (GW	Fair Value (NAV)	Equity (m)	Debt	UPWV Value (m)
A	1	24/02/2017	39	15.05%	119.00	409.00	26.98	€ 18.20	€ 112.02	€ 46.99	€ 112.02
A	2	01-03-2017	48	15.05%	161.00	499.00	53.13	€ 23.80	€ 159.53	€ 126.01	€ 159.53
Fund 1 Totals		31/03/2017	87		280.00	908.00	80.11	€ 4,19,73,881.18	€ 271.55	€ 173.00	€ 271.55

Asset Information		Time-One - Q2 2017									
Fund	Asset	Date	Plants	Ownership	Capacity (MW	Annual Production (GWh	Quarter Production (GW	Fair Value (NAV)	Equity (m)	Debt (m)	UPWV Value (m)
A	1	30/06/2017	40	15.05%	124.00	421.00	58.93	€ 18.83	€ 116.26	€ 48.99	€ 112.95
A	2	30/06/2017	49	15.05%	164.00	506.00	142.00	€ 24.19	€ 161.86	€ 132.77	€ 159.62
Fund 1 Totals		30/06/2017	89		288.00	927.00	200.93	€ 4,30,16,634.03	€ 278.12	€ 181.76	€ 272.57

Asset Information		Time-Two - Q3 2017 (*Merger)									
Fund	Asset	Date	Plants	Ownership	Capacity (MW	Annual Production (GWh	Quarter Production (GW	Fair Value (NAV)	Equity (m)	Debt (m)	UPWV Value (m)
A	1										
A	2	30/09/2017	98	15.48%	320.00	986.00	149.09	€ 4,98,48,562.65	€ 316.73	€ 191.62	€ 291.67
Fund 1 Totals		30/09/2017			320.00			€ 4,98,48,562.65	€ 316.73	€ 191.62	€ 291.67

Asset Information		Time-Three - Q4 2017									
Fund	Asset	Date	Plants	Ownership	Capacity (MW	Annual Production (GWh	Quarter Production (GW	Fair Value (NAV)	Equity (m)	Debt (m)	UPWV Value (m)
A	1										
A	2	31/12/2017	100	15.59%	333.57	1022.22	132.71	€ 49.30	€ 316.22	€ 192.14	€ 280.89
Fund 1 Totals		31/12/2017						€ 4,93,09,476.72	€ 316.22	€ 192.14	€ 280.89

Asset Information		Time-Four - Q1 2018									
Fund	Asset	Date	Plants	Ownership	Capacity (MW	Annual Production (GWh	Quarter Production (GW	Fair Value (NAV)	Equity (m)	Debt (m)	UPWV Value (m)
A	1										
A	2	31/03/2018	100	15.59%	333.57	1022.22	37.37	€ 48.90	€ 313.64	€ 243.31	€ 278.59
Fund 1 Totals		31/03/2018						€ 4,88,99,573.76	€ 313.64	€ 243.31	€ 278.59

Asset Information		Time-five - Q2 2018									
Fund	Asset	Date	Plants	Ownership	Capacity (MW	Annual Production (GWh	Quarter Production (GW	Fair Value (NAV)	Equity (m)	Debt (m)	UPWV Value (m)
A	1										
A	2	30/06/2018	100	15.59%	333.57	1022.22	316.64	€ 48.90	€ 313.64	€ 253.60	€ 278.59
Fund 1 Totals		30/06/2018						€ 4,88,99,573.74	€ 313.64	€ 253.60	€ 278.59

The data was gathered from eighty-seven (87) hydropower plants, all geographically located in Western Europe. The data set has installed capacity of 235.20 MW with annual production of 747.48 Gwh. Post data collection and upon further review, it was established that close proximity of the identified plants also offered an added advantage of all the plants operating under similar market conditions with transparent energy prices, especially financial schemes, making it possible to extrapolate data. The selected 87 plants, with corresponding energy generation capacities and values, are shown in table 1.1

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