

Triangular Fin

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Abstract:

Fins are used in a large number of applications to increase the heat transfer from surfaces. Typically, the fin material has a high thermal conductivity. The fin is exposed to a flowing fluid, which cools or heats it, with the high thermal conductivity allowing increased heat being conducted from the wall through the fin. The design of cooling fins is encountered in many situations and we thus examine heat transfer in a fin as a way of defining some criteria for design

CHAPTER1 INTRODUCTION

Fins are used in a large number of applications to increase the heat transfer from surfaces. Typically, the fin material has a high thermal conductivity. The fin is exposed to a flowing fluid, which cools or heats it, with the high thermal conductivity allowing increased heat being conducted from the wall through the fin. The design of cooling fins is encountered in many situations and we thus examine heat transfer in a fin as a way of defining some criteria for design.

Fins can be of a variety of geometry rectangular, triangular, parabolic, and hyperbolic and can be attached to the inside, outside or to both sides of circular, flat plate. Fins are most commonly used in heat exchanging devices such as radiators in cars, computer, CPU, heat sinks, and heat exchangers in power plants [1].

- Heat transfer through triangular fin array per unit mass is more than that of heat transfer through rectangular fin array. Therefore, the triangular fins are preferred for automobiles, central processing units, aero-planes, space vehicles etc. where, weight is the main criteria.
- At wider spacing, shorter fins are more preferred than longer fins.
- The aspect ratio for an optimized fin array is more than that of a single fin for both rectangular and triangular profiles [4].

TYPES OFFIN:

Fins can be broadly classified as:

1. Longitudinal fin-Rectangular, Trapezoidal and Concave profile.
2. Radial fin-Rectangular and Triangular profile.
3. Pin fin-Cylindrical.

Triangular Fin:

Distance=BC=CA=2cm

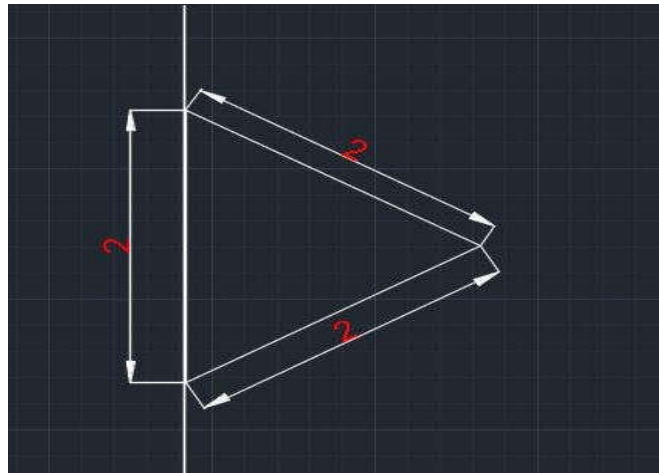


Figure1.1:Dimensions of Triangular Fin(Equilateral)

The triangular fin as shown in Figure-2, with the dimension of AB=BC=CA=2cm having equal distance from all the 3 sides.

Base temperature,

$T_b=40^{\circ}\text{C}$

Ambient temperature,

$T_{\infty}=20^{\circ}\text{C}$

Distance=5cm,BC=10cm,CA=7cm

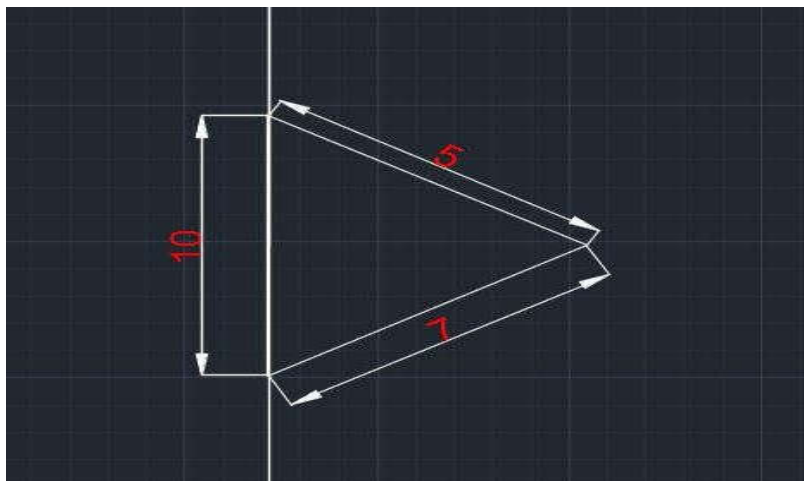


Figure1.2:Dimensions of Triangular Fin(Scalene).

The triangular fin as shown in Figure-3, with the dimension of AB=5cm, BC=10cm and CA=7cm having unequal distance from all the 3 sides.

Base temperature,

$$T_b=40^{\circ}\text{C}$$

Ambient temperature,

$$T_{\infty}=20^{\circ}\text{C}$$

Distance=0.14cm,BC = 0.10cm,CA =0.14cm

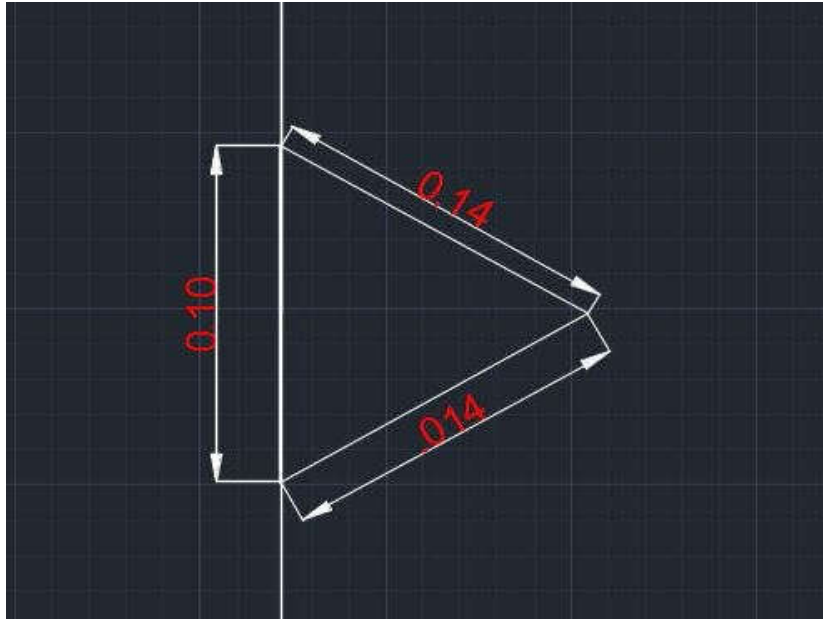


Figure1.3: Dimensions of Triangular Fin (Isosceles).

The triangular fin as shown in Figure-3, with the dimension of AB=0.14cm, BC=0.10cm and CA=0.14cm having 2 equal distance and an unequal distance from all the 3 sides.

Base temperature,

$$T_b=40^{\circ}\text{C}$$

Ambient temperature,

$$T_{\infty}=20^{\circ}\text{C}$$

CHAPTER2

LITERATUREREVIEW

2.1 Introduction:

The literature regarding the numerical and analytical solution for fins are presented below.

2.2 LiteratureSurveys:

The following are the few literature regarding the heat transfer from the surface of the fin.

Giulio Lorenzini and Simone Moretti[15] solved the problem of heat removal is amajor issue in modern industry. They faced the problem of optimizing fins to enhanceheat removal. Initially, they investigated the system numerically using a CFD code. They investigated Y-shaped profiles which consequently examined, obtained by varying the angle between the two arms of the original T. They observed that width reduction, typical of Y-shaped profiles with respect to T-shapedones, enhance

efficiency significantly. This new approach to heat removal optimization suggested the realization of arrays with multiple Y-shaped fins. Each array had the same width of the corresponding optimized T-shaped fin. This choice allowed immediate comparisons, so as to evaluate the actual performance enhancements typical of multiple-fin configurations, with respect to previous configurations.

Jacobet al.[6] have analyzed that natural convection heat transfer from triangular fin arrays experimentally and theoretically. They found that fin optimization is useful to go through the exercise of optimizing a fin in order to achieve the high rate of heat transfer per volume of fin material. The result of this optimization provides general guidelines relative to the dimensionless characteristics of a well designed fin.

Lokesh Aurangabadkaret al. [4] analyzed the thermal heat dissipation of fins by varying its geometry. Parametric models of fins have been developed to predict the transient thermal behavior. Thereafter models are created by varying the geometry such as rectangular, circular, triangular and fins with extension. After determining the material the final step is to increase the heat transfer rate of the system by varying geometrical parameters such as cross sectional area, parameter, length, thickness, etc which ultimately leads to fins of varying shape and geometries.

R.K.Rajput [11] considered three types of fins. He considered analytical method for solving the fin problems. He has given analytical formulation for solving heat transfer rate along the length.

HyungSukKang[14] considered about the optimization of a triangular fin with variable fin base thickness. There, he used two dimensional analytical methods for analysis. He observed that the optimal heat loss increased whereas the corresponding optimum fin effectiveness decreases with increase in fin volume.

2.3 Limitations from the Literature Surveys:

The literature survey shows the calculations relating to limiting the maximum heat transfer rate from the fin surface but remains silent which method is most effective for computation.

2.4 Objective of the research:

1. To calculate the temperature profile along the length of the fin with variable cross sectional area for different geometrical sections.
2. To calculate the efficiency, effectiveness and maximum heat transfer from the varying finned surface.
3. To compare the performance of fins.

2.5 Conclusion:

The present chapter provides the detail information to the fin problem. The literature have been studied related to the problem. Accordingly, the objective of the present work has been formulated. The necessary formulation for the fin is presented here for computation.

CHAPTER 3

METHODOLOGY

3.1 Introduction

The shooting method is used to solve initial value problems. An ordinary differential equation is converted to many first order differential equations. The known boundary values are converted to initial values of the solution. With guess value of unknown boundary value; using trial and error method or some other scientific approach, these one dimensional equations are solved simultaneously.

3.2 Formulation:

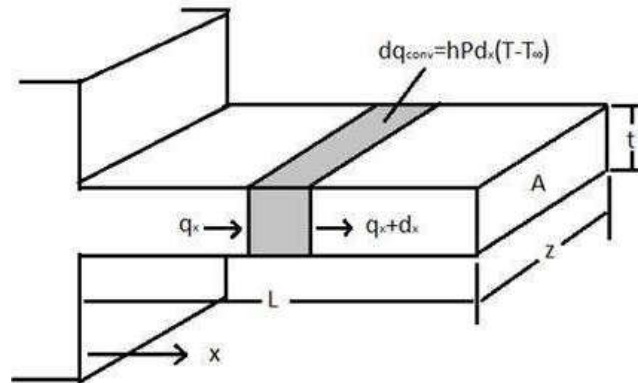


Figure 3.1 Geometry of rectangular fin.

Applying the energy balance on the rectangular fin:

$$Q_x = Q_{x+dx} + Q_c$$

Where:

Q_{x+dx} = Energy leaving the control volume.

Q_x = Energy entering the control volume.

Q_c = Heat transfer due to convection.

$$Q(x) = -kA(x) \frac{dT}{dx} \text{-----(3.1)}$$

By Using Taylor Series Expansion

$$Q(x) dx = Q_x + d/dx(Q(x))xi + Q_c \dots \dots \dots (3.2)$$

$$Q_c = hP(x)xi(T-T_\infty) = 0 \dots \dots \dots (3.3)$$

Using equation (3.1) and (3.3) in (3.2) we get,

$$d/dx \left(-kA(x) \frac{dT}{dx} \right) xi + hP(x)xi(T-T_\infty) = 0$$

d/dx

$$(A(x) \frac{dT}{dx})_{xi} - \frac{hP(x)}{k} (T - T_{\infty}) = 0 \text{-----(3.4)}$$

This is the governing differential equation for an extended surface. For constant area of cross-section:

$$d^2T/dx^2 - Hp/kA_c(T - T_{\infty}) = 0 \text{-----(3.5)}$$

where, $m^2 = hP/kA$

Let $\theta = T - T_{\infty} \Rightarrow dT = d\theta$

$$d^2\theta/dx^2 - m^2\theta = 0 \text{-----(3.6)}$$

General solution to the differential equation is:

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx} = C_1 \cosh(mx) + C_2 \sinh(mx) \dots (3.7)$$

Where C_1 and C_2 are two arbitrary unknown constants. The boundary conditions are:

$x=0, T=T_b$ (Base temperature) (3.8) Boundary conditions at fin tip:

1. Heat dissipation from an infinitely long fin. ($x \rightarrow \infty, T = T_\infty$) (3.8)a
2. Heat dissipation from a fin insulated at the tip. ($x=L, -k \frac{dT}{dx} = ht(T - T_\infty)$) (3.8)b
3. Heat dissipation from a fin losing heat at the tip. ($x=L, \frac{dT}{dx} = 0$) (3.8)c

Heat lost by rectangular fin,
 $Q = k A_c m \theta_0 [\cosh mL + \frac{h}{k m} \sinh mL]$

Where,
 θ_0 = Temperature difference, k = thermal conductivity, W/MK.
 m = Fin parameter, $\sqrt{hL/kA_c}$
 A_c = cross sectional area of fin, m^2 .
 h = heat transfer coefficient, W/m^2K .

$$\text{Rate of heat flow per unit mass, } (q_r) = \frac{kA_c m \theta_0 [h \cosh mL + k \sinh mL]}{2\delta p L [k \cosh mL + h \sinh mL]}$$

Efficiency of rectangular fin:

$$\eta_r = \frac{-kAc \ m \theta_0 \frac{h \cosh mL + k \sinh mL}{k \cosh mL + h \sinh mL}}{2WLh\theta_0}$$

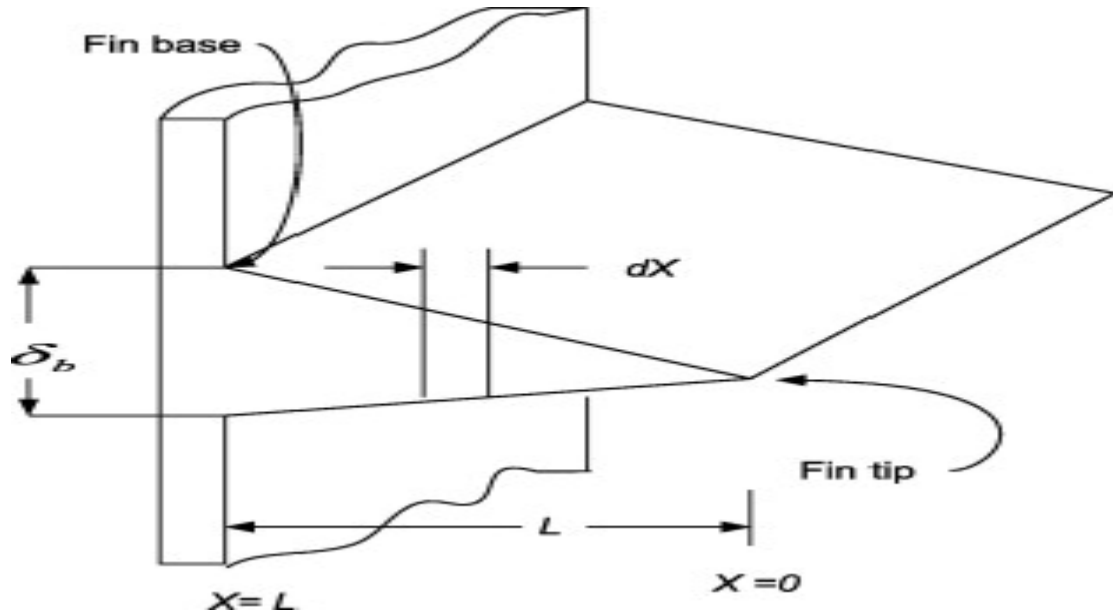


Figure 3.2 Geometry of triangular fin.

For a triangular fin representing length of fin L , thickness 2δ , and width of fin, W and assuming the heat flow is unidirectional and it is along length and the heat transfer coefficient (h) on the surface of the fin is constant.

Heat lost by triangular fin,

$$Q=1$$

I_1 =Bessel function of first kind.

$A_b = \text{Base Area,}$

3.3 Solution of the differential equation:

Solution of the Eq. (3.6) is obtained using analytical method and numerical method.

3.3.1 Analytical method:

Using the boundary condition Eq. (3.8) and (3.8) in Eq. (3.7); the solution obtained is:

$$\theta(x) = \theta_b e^{-mx} \dots \dots \dots (3.9)$$

Using the boundary condition Eq.(3.8) and (3.8) in Eq.(3.7); the solution obtained is:

$$\theta(x) = \theta_b \frac{e^{m(l-x)} + e^{-m(l-x)}}{e^{ml} + e^{-ml}} \dots \dots \dots (3.10)$$

The solution obtained is:

$$\theta(x) = \theta_b \frac{\cosh m(l-x) + \frac{h/km}{\cosh ml} [\sinh \{m(l-x)\}]}{\cosh hml + \frac{h/km}{\cosh ml} [\sinh(ml)]} \dots \dots \dots (3.11)$$

Fin efficiency: The efficiency of a fin is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at the base temperature.

$$\eta = Q_{fin} / Q_{mas} \dots \dots \dots (3.12)$$

3.3.2 Numerical method

For solution of the Eq.(3.6), the shooting technique is used. From Eq.(3.6), Let $z = \theta - \theta_b$ (3.13)

$$\text{and } y = z' \dots \dots \dots (3.14)$$

The boundary conditions are: At $x=0, \theta = \theta_b$ and at $x=L, \theta(L) = 0$. The solution of Eq.(3.13) is:

$$z_{i+1} = z_i + f_1(x_i, \theta_i, z_i) h$$

$$\text{or } z_{i+1} = z_i + m^2 \theta h \dots \dots \dots (3.15)$$

And the solution of Eq.(3.12)is:

$$\theta = \theta_0 + f_2(x_i, \theta_i, z_i)h \text{ or } \theta = \theta_0 + zh$$

$$\text{or } \theta_{i+1} = \theta_i + zh \dots\dots\dots (3.16)$$

The solution starts from $x=0$, $\theta = \theta_b$ and guess value of z and ends at $x=L$, $\theta = 0$ using Shooting techniques. Here h is the step size. That is calculated according to the number of points used along the fin surface. So $h = L/n$, where n is the number of points along L .

3.4 Shooting Technique:

Shooting technique is applied for the Eq.(3.15) and (3.16). Using the value of θ_0 and guessed value of z as initial value, these values are calculated at different points along the length of the fin till $x=L$. The solution is said to be converged when for a particular guess value of z ; θ becomes 0. The initial guess value is obtained using heat and trial method.

3.5 Conclusion:

The differential equation for the fin has been solved using analytical and numerical method. So, the solution procedure has been discussed. Then both the solution must be matched to get the accuracy of the solution.

CHAPTER4

<RESULTSANDSIMULATION>

4.1. INTRODUCTION:

The formulation to the fin for varying cross sectional area has been derived in the previous chapter. The Eq.(3.6) has been solved using analytical and numerical method. The numerical method of solution consists of Shooting techniques. The results and the required graphs for the said equation are presented below. .

4.2. RESULTS:

4.2.1. Using Eq.(3.13), the results obtained for different values of x are presented in table.4.1.

1. For Isosceles fin surface:

Fig4.1. Graphical representation of temperature profile along the length of the triangular fin (isosceles).

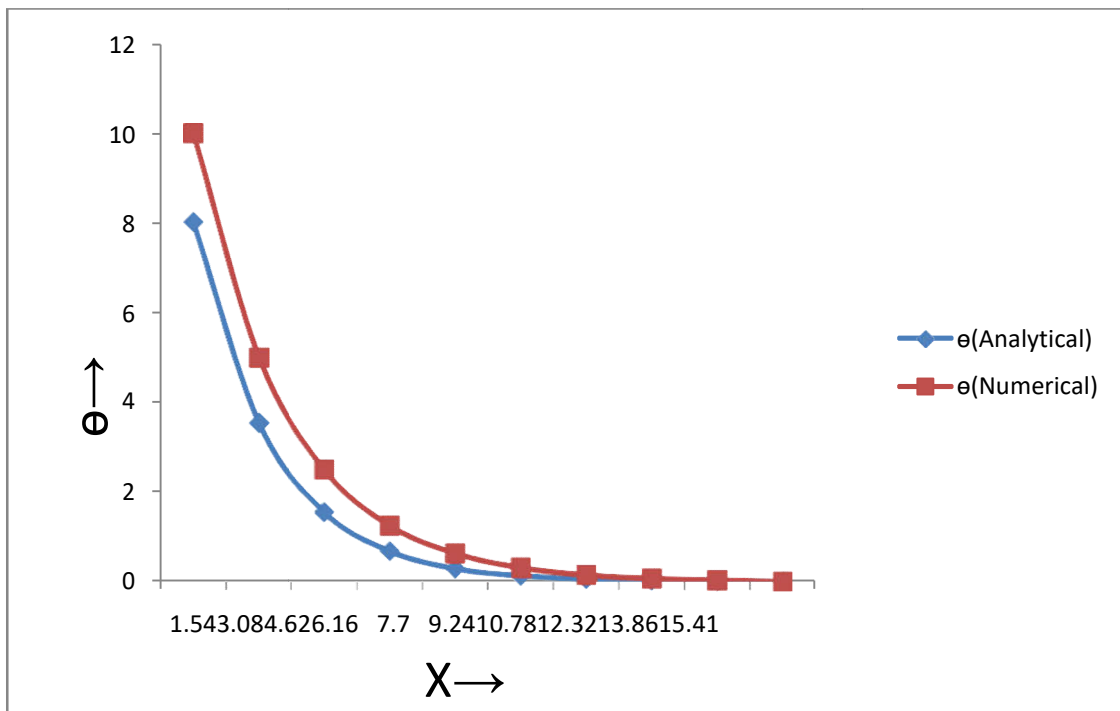


Table:4.1.Temperature profile along the length of the triangular fin(isosceles).

X	θ(Analytical)	θ(Numerical)
1.54	8.03	10.01
3.08	3.55	5.01
4.62	1.55	2.5
6.16	0.68	1.25
7.70	0.29	0.63
9.24	0.13	0.31
10.78	0.06	0.15
12.32	0.03	0.07
13.86	0.01	0.03
15.41	0	0

2. For Scalene fin surface:

Fig4.2. Graphical representation of temperature profile along the length of the triangular fin (Scalene).

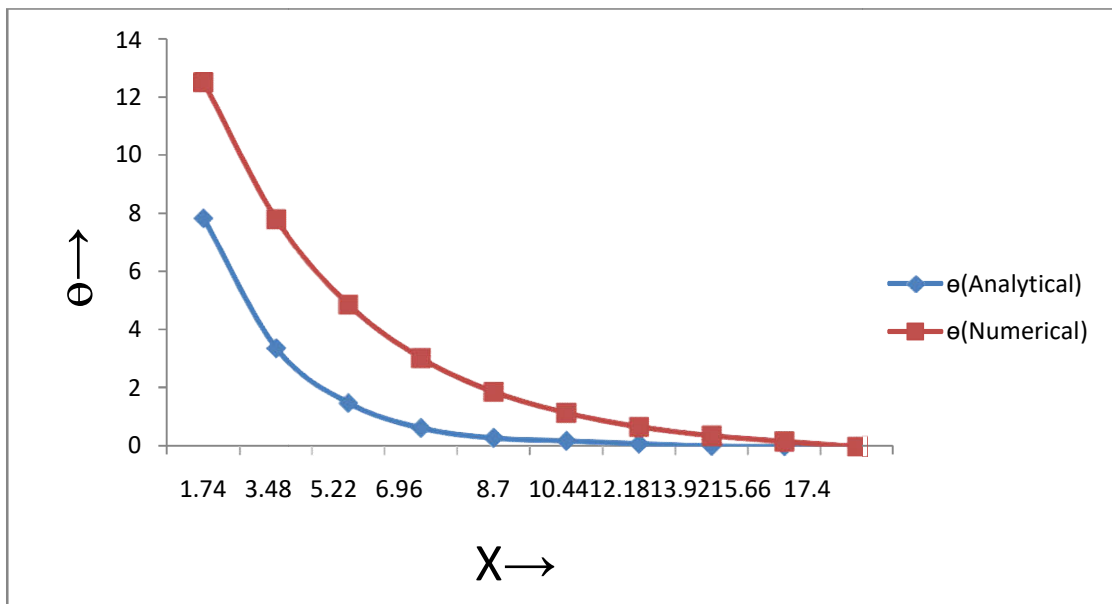


Table:4.2. Temperature profile along the length of the triangular fin(Scalene).

X	θ(Analytical)	θ(Numerical)
1.74	7.85	12.51
3.48	3.4	7.88
5.22	1.5	4.88
6.96	0.64	3.04
8.70	0.3	1.88
10.44	0.2	1.16
12.18	0.1	0.68

13.92	0.02	0.38
15.66	0.01	0.18
17.4	0	0

3. For Equilateral fin surface:

Fig4.3. Graphical representation of temperature profile along the length of the triangular fin (Equilateral).

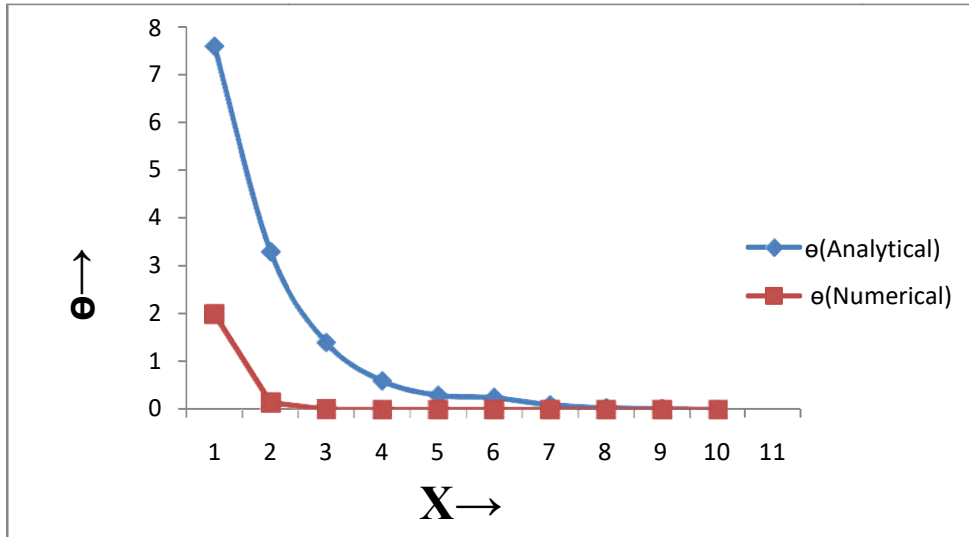


Table:4.3. Temperature profile along the length of the triangular fin(Equilateral).

X	$\theta(\text{Analytical})$	$\theta(\text{Numerical})$
2.138	7.6	1.74
4.276	3.3	0.15
6.414	1.4	0.01
8.552	0.6	0.001
10.690	0.3	0.0001
12.828	0.25	0.0001
14.966	0.10	0.0001
17.104	0.04	0.0003
19.242	0.02	0.0006
21.380	0	0

4.3. CONCLUSIONS:

From the results and simulation we conclude that in numerical method it is more easy and convenient for Isosceles and Scalene triangular fin surface as we can get exact

accuracy of solution that is 0. But in the Equilateral triangular fin and rectangular fin it is more complex and time taking to get the exact solution of 0.

In analytical method, both isosceles and scalene triangular fin surface the solution is close to 0 but the range is very less as compared to the numerical method. However in the case of equilateral triangular fin and rectangular fin surface we can get the exact solution of 0.

In rectangular and equilateral triangular fin there is fluctuation in numerical method but in the analytical method the temperature distribution of the fin surface is in the decreasing order.

CHAPTER 5**<CONCLUSIONS AND FUTURE WORK>**

5.1. CONCLUSION:

1. Temperature of rectangular fin increases along the length from the tip of the base of the fin.
2. Temperature of triangular fin decreases along the length from the tip to the base of the fin.
3. Heat transfer rate increases along the length from the tip to the base for all fin profiles.

5.2. FUTURE SCOPE OF WORK:

1. Fin analysis can be carried out by applying heat transfer rate due to radiation from the finned surface.
 2. Fin dimension can be calculated by considering the effect of radiation and without radiation process.
 3. Non-homogeneous and anisotropic materials can be used for analysis of different fin material.
-

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APPENDIX<A>
<SOFTWAREDETAILS>

1. For calculating temperature distribution of triangular fin (Isosceles) using numerical (shooting) method:

```
#include<stdio.h>#include<math.h>int main()
{
int count,n;
float m,h,p,k,A,a,x,y,yi,L,b,c,thetai,xi,theta,h1;count = 0;
n = 10;
L=15.40;
h = L/n;k=380;
h1=20;
a =0.14;b=0.1;c=0.14;A=0.19;
p =2.0*(a+b+c);x = 0;
printf("Enter the value of xi, yi, thetai:\n");scanf("%f%f%f\n",&xi,&yi,&thetai);while(count<=n-1)
{
m = sqrt( (h1*p)/(k*A) );y = yi + m*m*thetai*h;theta =thetai+ yi*h;
x = xi + h;yi=y;
xi=x;
thetai = theta;
printf("value of x,theta,y is: x=%f y=%f
theta=%fm=%f\n",x,y,theta,m);

count++;
}
return0;
}
```

2. For calculating temperature distribution of triangular fin (Equilateral) using numerical (shooting) method:

```
#include<stdio.h>#include<math.h>int main()
{
int count,n;
float m,h,p,k,A,a,x,y,yi,L,thetai,xi,theta,h1;count = 0;
n = 10;
L= 21.38;
a =2;
h = L/n;k=380;
h1 = 20;A=0.1732;
p =3.0*a;x = 0;
printf("Enter the value of xi, yi, thetai:\n");scanf("%f%f%f\n",&xi,&yi,&thetai);while(count<=n-1)
{
m = sqrt( (h1*p)/(k*A) );y = yi + m*m*thetai*h;theta =thetai+ yi*h;
x = xi + h;yi=y;
xi=x;
thetai = theta;
printf("value of x,theta,y is: x=%f y=%f theta=%fm=%f\n",x,y,theta,m);

count++;
}
return0;
}
```

3. For calculating temperature distribution of triangular fin (Scalene) using numerical (shooting) method:

```
#include<stdio.h>#include<math.h>int main()
{
int count,n;
float m,h,p,k,A,a,x,y,yi,L,b,c,thetai,xi,theta,h1;count = 0;
n = 10;
L=17.40;
h = L/n;k=380;
h1=20;
a =0.05;b=0.10;c=0.07;A=0.25;p=0.22;
x = 0;
printf("Enter the value of xi, yi, thetai:\n");scanf("%f%f%f\n",&xi,&yi,&thetai);while(count<=n-1)
{
m = sqrt( (h1*p)/(k*A) );y = yi + m*m*thetai*h;theta =thetai+ yi*h;
x = xi + h;yi=y;
xi=x;
thetai = theta;
printf("value of x,theta,y is: x=%f y=%f
theta=%fm=%f\n",x,y,theta,m);

count++;
}
return0;
}
```


4. For calculating temperature distribution of triangular fin (Isosceles) using analytical method:

```
#include<stdio.h>#include<math.h>int main()
{
intcount,n;
float m,h,p,k,A,a,x,L,b,c,thetai,xi,theta,h1,m1,m2,m3,m4;count = 0;
n = 10;
L=15.40;
a = 0.14;
b= 0.1;
c = 0.14;
h = L/n;k=380;
h1=20;
A=0.19;
p = 2.0*(a+b+c);x = 0;
printf("Enter the value of xi, thetai:\n");scanf("%f%f\n",&xi,&thetai);while(count<=n)
{
x = xi + count*h;
m=sqrt((h1*p)/(k*a));m1=thetai*cosh(m*(L-x));
m2 = (h1/(k*m))*sinh(m*(L-x));m3 = cosh(m*L);
m4 = (h1/(k*m))*sinh(m*L);theta=(m1+m2)/(m3+m4);
printf("value of x, theta is:x=%f theta=%f\n",x,theta);count++;
m1= m2= m3 =m4 =0.0;
}
return0;
}
```

5. For calculating temperature distribution of triangular fin (Equilateral) using analytical method:

```
#include<stdio.h>#include<math.h>int main()
{
int count,n;
float m,h,p,k,A,a,x,L,thetai,xi,theta,h1,m1,m2,m3,m4;count = 0;
n = 10;
L=21.38;
a = 2;
h = L/n;k=380;
h1=20;
A= 1.732;
p = 3.0*a;x = 0;
printf("Enter the value of xi, thetai:\n");scanf("%f%f\n",&xi,&thetai);while(count<=n)
{
x = xi + count*h;
m=sqrt((h1*p)/(k*a));m1=thetai*cosh(m*(L-x));
m2 = (h1/(k*m))*sinh(m*(L-x));m3 = cosh(m*L);
m4 = (h1/(k*m))*sinh(m*L);theta=(m1+m2)/(m3+m4);
printf("value of x, theta is:x=%f theta=%f\n",x,theta);count++;
m1= m2= m3 =m4 =0.0;
}
return0;
}
```

6. For calculating temperature distribution of triangular fin (Scalene) using analytical method:

```
#include<stdio.h>#include<math.h>int main()
{
int count,n;
float m,h,p,k,A,a,x,L,b,c,thetai,xi,theta,h1,m1,m2,m3,m4;count = 0;
n = 10;
L=17.40;
a =0.05;
b= 0.10;
c =0.07;
h = L/n;k=380;
h1=20;
A=0.25;
p= 0.22;
x = 0;
printf("Enter the value of xi, thetai:\n");scanf("%f%f\n",&xi,&thetai);while(count<=n)
{
x = xi + count*h;
m=sqrt((h1*p)/(k*a));m1=thetai*cosh(m*(L-x));
m2 = (h1/(k*m))*sinh(m*(L-x));m3 = cosh(m*L);
m4 = (h1/(k*m))*sinh(m*L);theta=(m1+m2)/(m3+m4);
printf("value of x, theta is:x=%f theta=%f\n",x,theta);count++;
m1= m2= m3 =m4 =0.0;
}
return0;
}
```