

# Assessing Betweenness Centrality For Multimodal Transport Network – A Case Study of Delhi Metropolitan Area

Jagannath Das<sup>1</sup>, Dr. Sewa Ram<sup>2</sup>

<sup>1,2</sup>Department of Transport Planning, School of Planning and Architecture, New Delhi.

## Abstract

This study introduces an approach for analysis of network centrality, with a focus on betweenness centrality in complex urban transport systems. Central to this research is the development of a Path Evaluation Function (PEF), a tool that enriches traditional centrality assessments for transport networks by factoring in critical network attributes like link capacity and travel time. This approach is particularly adept at unraveling the intricacies of betweenness centrality in multimodal transportation networks. Using this approach, the study delves into the dynamics of betweenness centrality within the Delhi Metropolitan transport network, an evolving urban landscape characterized by diverse transit modes. The analysis reveals a marked concentration of high betweenness centrality along key transport corridors, highlighting their role in facilitating urban mobility. This refined understanding of betweenness centrality, obtained through the PEF, offers deeper insights into how crucial nodes and links function within the network, thereby enabling more informed decision-making in urban planning and policy. Furthermore, the versatility of the PEF extends beyond betweenness centrality, showing promise as a tool for evaluating other centrality measures within varied network contexts.

**Keywords:** Betweenness centrality, path evaluation function, urban network analysis, multi modal transport network

## 1 Introduction

Human or goods movement involves two key steps: The first step is about deciding where to go, while the second is about how to get there. Therefore, every journey consists of a starting and ending point, known as 'to-movement' nodes, and several points along the way, referred to as 'through-movement' nodes (Hillier & Iida, 2005). Movements within a network and the network's inherent structure play a pivotal role in comprehending the dynamics of how various entities — such as people, or goods — traverse from one location to another. The study of networks has evolved significantly, shedding light on how nodes (points) and edges (paths) interact within these systems (Siew et al., 2019). Network structures can vary greatly, ranging from simple linear paths to complex, interconnected multimodal systems, each exhibiting unique properties and behaviors (Barabási & Oltvai, 2004). Graph theory offers a framework for studying these networks. It enables the representation of networks as a collection of nodes (vertices) and edges (connections), facilitating the analysis of their structure and dynamics (Newman, 2003). Centrality measures form the back bone of such assessments. Centrality measures, rooted in graph theory, provide a quantitative approach to determine the significance or influence of specific nodes or links in a network. When applied to transportation networks, these nodes can

symbolize entities ranging from intersections to transit stops, or even entire districts (Freeman, 1978). The utility of network centrality measures in transportation analysis has only recently begun to be fully appreciated. Various studies have demonstrated that these metrics can offer a more detailed and holistic evaluation of a transportation network's characteristics and their impact on landuse (see: (Wang et al., 2011, Liu et al., 2016 and Rui & Ban, 2014)) and traffic flow, vulnerability assessment (see: (Jayasinghe et al., 2017), (Furno et al., 2017) and (Gao et al., 2013)). At the heart of calculating centrality lies the estimation of the shortest path<sup>1</sup>(Porta et al., 2006). In an unweighted graph, this path is determined by the least number of edges traversed. Yet, when the graph is weighted with network-specific attributes, such as distance or average travel time, the calculations can be refined to better suit urban environments(Henry et al., 2019). Although the majority of research employing centrality measures in transportation networks has focused on weighted networks, a significant limitation in these studies is their predominant emphasis on urban streets and roads. This focus primarily revolves around the arrangement of nodes within the network, considering only their topological aspects and distances (Cheng et al., 2015). This approach often overlooks essential supply factors like the network's capacity and speed. As urban areas grow in complexity and multimodal transportation systems become increasingly common, a singular focus on one mode of transport can lead to a neglect of the interactions between various modes and their collective impact on network centrality.

This research aims to address this gap by adopting a weighted centrality approach that fully accounts for the multimodal characteristics of contemporary transport networks. Our focus lies in developing a methodology to assess the betweenness centrality of such networks. By employing path evaluation functions, this study delves into the interplay between road and rail systems within a multimodal transport framework and explores the dynamics of betweenness centrality in the rapidly evolving Delhi Metropolitan area.

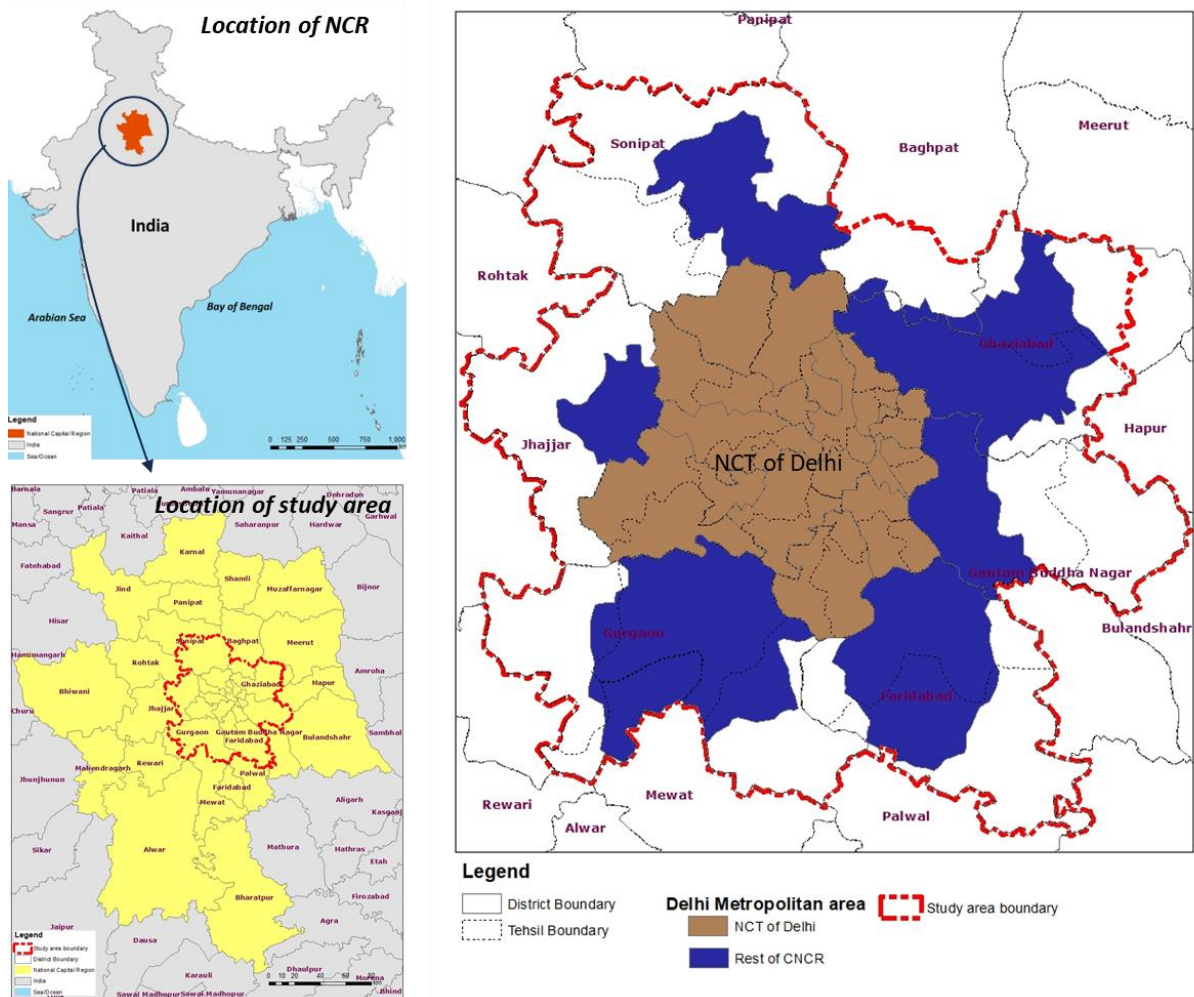
## 2 Materials and Methods

### 2.1 Study area

The Delhi National Capital Region (NCR), an expansive area that spans 35 districts across four states, is anchored by the National Capital Territory (NCT) of Delhi. As per the 2011 Census of India, this region is home to a bustling population of over 58.1 million people(Census of India, 2011). Predictions indicate that this number will surge to approximately 110 million by 2041, with the urbanization rate escalating from the present 54% to about 67%, according to NCRPB (2021). The focus of this study is the Delhi Metropolitan Area within the NCR. This specific area extends across both the NCT of Delhi and the Central National Capital Region. It is defined by the overlap of the Delhi Metropolitan Area's geographic extent and the administrative boundaries of the region. **Figure 1** illustrates the exact location of the Delhi Metropolitan Area within the broader context of the National Capital Region.

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<sup>1</sup> Exception in case of degree centrality where number of intermediaries are assessed rather than shortest path.



**Figure 1: Geographical Positioning of Delhi Metropolitan Area in India's National Capital Region**

Although the study area constitutes merely 13% of the NCR's total land area, it accommodates around 48% of its population. This considerable proportion of residents plays a pivotal role in shaping the economic and urban development of the NCR. Furthermore, the area is marked by its unique characteristics, blending historical elements with contemporary urban features.

## 2.2 Data sources and preparation

This research predominantly utilizes transport network data from the Delhi Metropolitan area. The data was gathered from several sources, including Open Street Maps, the National Capital Region Planning Board Master Plan (NCRPB, 2021), and the Delhi Development Authority (DDA) Master Plan (DDA, 2021). The synthesis of this data from diverse sources was essential for a comprehensive depiction of the transportation network within the study's geographic confines.

To create a detailed map of the transport infrastructure, the study harnessed Geographic Information Systems (GIS). This process entailed a digital representation of the area's roads, railways, and other transport facilities, based on the collected data.

Next, the following steps were undertaken to enhance the data quality as shown in **Figure 2**:

- **Linking Unconnected Segments:** this included identification and connection of disjointed segments within the network, ensuring continuity and functionality.

- Removal of Redundant Elements: Any superfluous or erroneous elements, such as shooting and dangling links in the GIS data, were detected and removed for a more coherent network.
- Elimination of duplicate links: Redundancies in the network data were located and removed, thereby preventing data duplication and potential analytical biases.

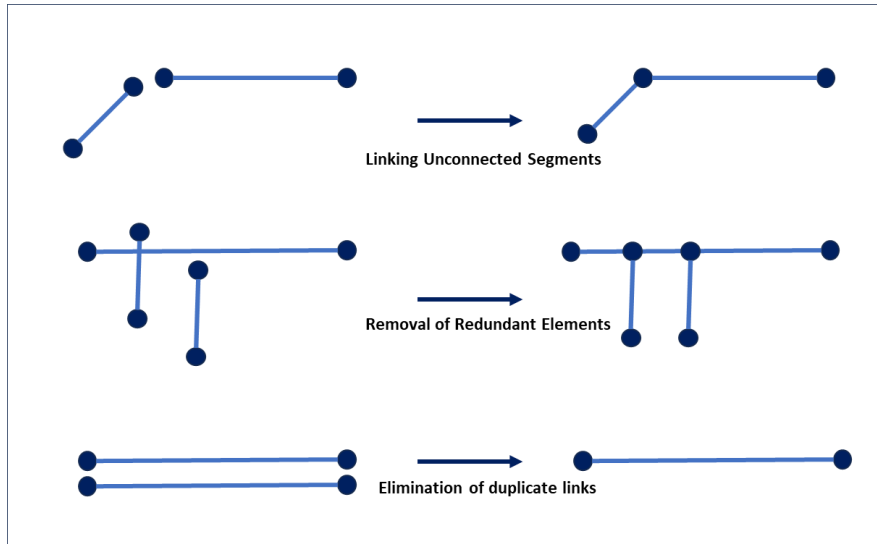
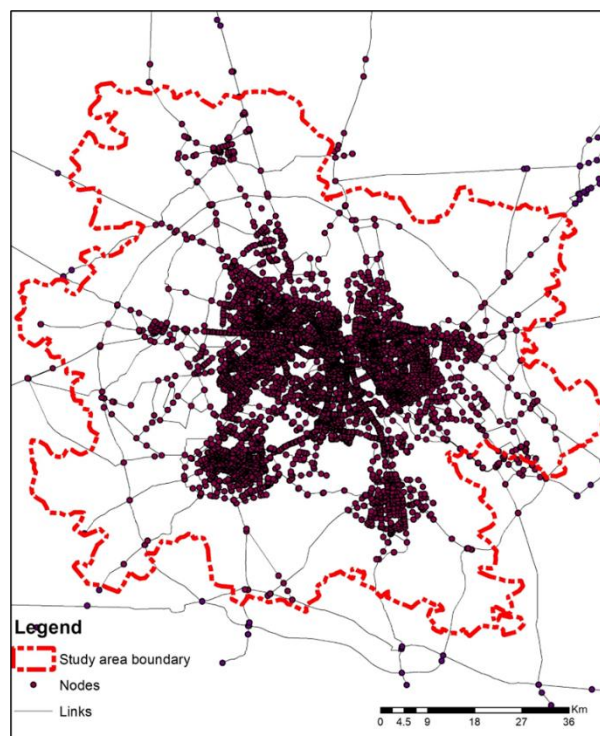


Figure 2: Network cleaning



With the foundational structure of the transport network established in GIS, additional data pertaining to speed and capacity were coded into the network. This supplementary data, derived from Bing API and Google Maps (specifically for lane information), provided vital insights into traffic conditions across various network segments.

The research adopts a primal approach for depicting the transport network. In this model, intersections or stations are considered as **nodes** and roads/ metro line are considered as **links**. This setup facilitates

the integration of key factors like traffic density, speed, and actual distances (Oberoi et al., 2018). This method is preferred over the dual graph approach, which assumes uniform distances and fails to accurately reflect the diverse nature of a transportation network (Porta et al., 2006). The primal graph's emphasis on geographical dimensions and real-world distances is particularly advantageous for evaluating network accessibility. **Figure 3**, shows the network representation (links and nodes) for the study area.

To enhance efficiency and reduce computational load, the study excludes local roads. This decision is in line with urban planning practices in Indian cities, where local roads are often not the primary focus in master plans (MOUD-Govt. of India, 2014). Consequently, the final network comprises 6,721 nodes and 8,989 links, each characterized by attributes such as Link ID, length, travel time, end node ID, and their respective coordinates.

### 2.3 Weighted centrality assessment-Betweenness centrality

Weighted centrality analysis is a method used in network analysis to evaluate the significance of each node. This approach not only takes into account the connectivity of each node but also factors in the specific weights assigned to the edges connecting them (Singh et al., 2020). Originally, centrality metrics were developed for binary networks, as outlined by Freeman (Freeman, 1978). These networks are characterized by connections that are either present or absent, lacking any nuanced representation of actual distances or connection strengths. In binary networks, centrality metrics focus solely on the structural layout of nodes and their links, a method that overlooks significant variations within the network (Alves et al., 2022). However, weighted centrality assessment takes into account the weights of the network connections. This leads to the application of various measures such as degree centrality, closeness centrality, betweenness centrality, straightness centrality, and PageRank centrality, as discussed by Porta et al. (2006). Adopting this approach enables a more nuanced and accurate analysis of fully weighted networks, capturing the distinctive features of nodes in densely interconnected, weighted networks. In centrality measures, the relative importance of a node is based on its relative positioning in the network (Disney, 2020). Access to all nodes is valued at each node represents an equal potential opportunity.

#### 2.3.1 Betweenness centrality:

Betweenness centrality is a concept focusing on nodes that serve as intermediaries within a network, acting neither as origin nor destination points. The interaction of two non-adjacent nodes depends on intermediate nodes that have strategic control and influence on them. The betweenness centrality of a node is a measure of how frequently it appears on the shortest path linking a specific pair of nodes (Freeman, 1977). The betweenness centrality is defined as (Porta et al., 2006):

$$C^B = \frac{2}{(N-1)(N-2)} \sum_{j=1; j \neq i} \frac{n_{jk}(i)}{n_{jk}} \quad \dots(1)$$

Where  $N$  is the number of nodes,  $n_{jk}$  is the number of shortest paths between node  $j$  and  $k$  and  $n_{jk}(i)$  is the number of shortest paths between  $j$  and  $k$  that contains node  $i$ . primary benefit of betweenness centrality lies in its ability to evaluate link/ vertex/ edge betweenness (see: (Yoon et al., 2006), (Brandes, 2008). Link/ vertex/edge betweenness centrality is characterized as the count of shortest paths that pass through a particular edge within a graph or network (Girvan & Newman, 2002).

As discussed in section 1, traditional centrality measures have primarily emphasized the spatial arrangement of nodes, focusing on their distances. However, this method tends to homogenize all network features, overlooking vital physical aspects such as the intensity, strength, and capacity of both links and nodes. For instance, in this setting, the centrality ( $C^B$ ) of a node in a Mass Rapid Transit System is considered similar to that of a node on a local street, provided their distances to other nodes are same. This simplification presents a major limitation, as it disregards the unique attributes of different transportation systems (Kazerani & Winter, 2009) . Consequently, there is a need for a more refined approach that considers these varying system characteristics, offering a deeper and more complete understanding of network dynamics, their influence on traffic flow, and the identification of critical nodes. The introduction of a path evaluation function represents a significant advancement in addressing this issue.

### 2.3.2 Path evaluation function

The transport network's architecture presents a complex interplay of interconnected systems, characterized not only by their topological structure but also by factors like capacity, travel time, and cost. The path evaluation function, as defined by Sosnowska & Skibski, (2018) assesses the path between two nodes based on two key aspects: the cumulative weight of the edges and the number of intermediary nodes. For each node, critical parameters include the number of connecting edges and the total sum of edge weights. Each aspect is quantified using a constant, termed ' $\alpha$ '. Opsahl et al. (2010), referred to this constant as the tuning parameter. The advantage of the tuning parameter ' $\alpha$ ' lies in its ability to be adjusted or iterated, allowing for a more tailored fit to specific applications.

The assessment of betweenness centrality hinges on identifying and measuring the shortest paths linking nodes within a network. To tailor these measures for networks with diverse and multifaceted factor weights, the initial step involves redefining how shortest paths are identified and how their lengths are calculated in the context of weighted networks. This study seeks to integrate two essential factors — capacity and travel time — to refine the accuracy and applicability of the path evaluation function. To this end, we propose a function that concurrently considers both these elements in determining the value of a path between any two nodes in the network. Herein, let's assume that  $C_{ij}$  and  $T_{ij}$  be the capacity and time taken to traverse the link between Node i and Node j, then the Path Evaluation Function (PEF) can be denoted as:

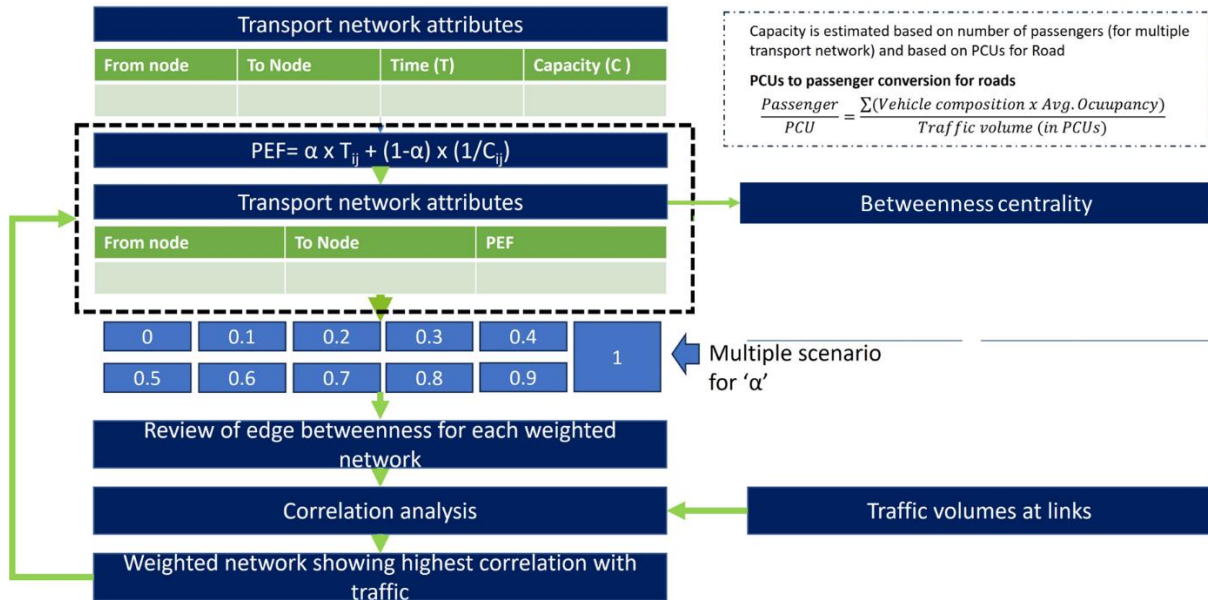
$PEF = \alpha \times T_{ij} + (1 - \alpha) \times \frac{1}{C_{ij}}$	...(2)
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In the proposed formula, the turning parameter  $\alpha$  plays a crucial role in modulating the relative significance of travel time and capacity. By varying  $\alpha$ , the focus can be shifted between these two factors. It's essential to understand that the path evaluation function itself is not a weight but rather acts as a tool to identify the most suitable or efficient path between two nodes in a network. Once a path is selected based on this function, the actual metrics of the path, such as distance, travel time, or other relevant factors, can subsequently be computed.

#### 2.3.2.1 Calibrating the turning parameter ( $\alpha$ ) in the path evaluation function

The tuning parameter, represented as ' $\alpha$ ', is a key element in the path evaluation function and is integral to network analysis. Precise calibration of ' $\alpha$ ' is vital to ensure that the path evaluation function is in

harmony with the specific objectives of network analysis and the realities of the network's conditions. The study adopts the following methodology to determine the appropriate value of 'α'.



**Figure 4: Framework to assess path evaluation function and centrality measures**

The process began with a comprehensive preparation of the transport network, where each link was assigned two essential attributes:

- **Capacity** (in passengers/hour): This attribute indicates the maximum number of passengers that a link can accommodate per hour. Considering the mix of road and rail systems in the study area, it was crucial to standardize these measurements. For roads, this involved converting the capacity measure from Passenger Car Units (PCU) per hour to passengers per hour.
- **Travel Time**: This measures the time needed to traverse a specific link.

Utilizing these attributes, we calculated the Path Evaluation Function (PEF) for each network link, as outlined in equation (2). This approach effectively captures both the connectivity and intensity aspects of the transport network. We then explored various scenarios by adjusting the turning parameter (α). Subsequently, we calculated the edge betweenness for each scenario with different PEF values (varying α). To pinpoint the most optimal 'α', we correlated these betweenness centrality values with the actual traffic flow data on the links. The configuration that exhibited the highest correlation was employed to assess the betweenness centrality, offering a comprehensive view of the network's dynamics under varying conditions.

### 3 Results

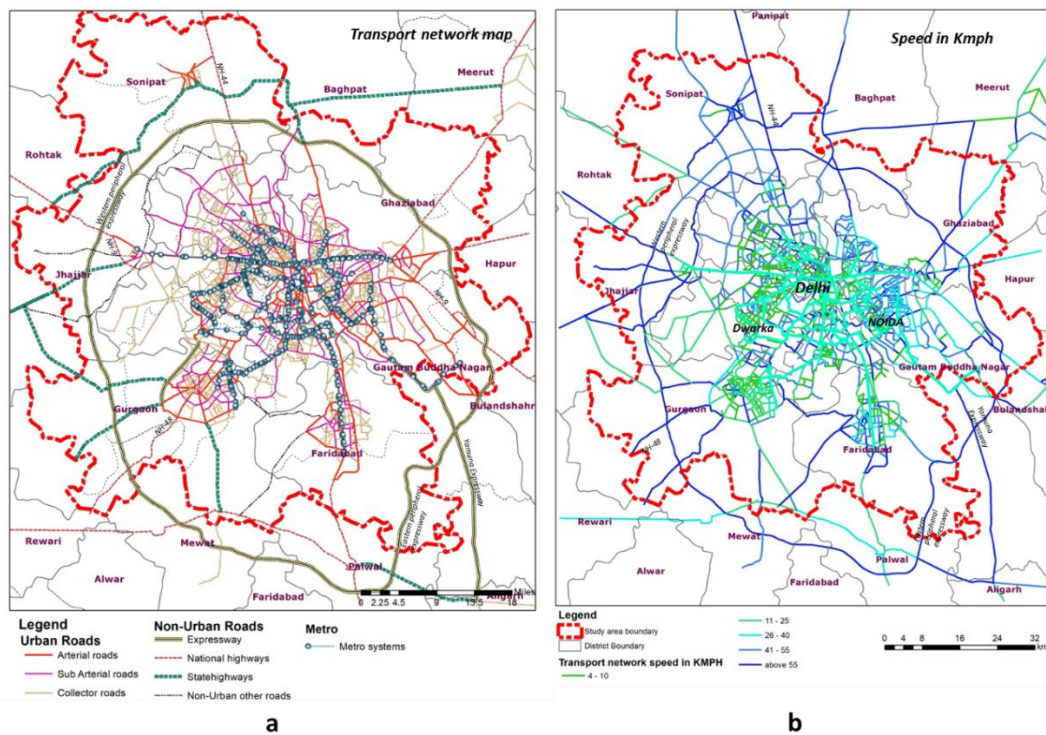
#### 3.1 Network Characteristics

The transport network within the study area covers a total of 3,842 kilometers, featuring a diverse mix of urban roads, non-urban/interurban roads, and a metro system. **Figure 5** illustrates the specific characteristics of the transport network in this area. The urban roads, which constitute 70.3% of the network, with traffic flowing at an average speed of 31.2 km/h. In contrast, the non-urban and interurban roads account for 19.5% of the network and, due to their less congested nature, support a higher average speed of 54.1 km/h. The metro rail system, an essential component for daily commuting, makes up 10.2% of the network and operates at an average speed of 36.7 km/h.

Table 1 provides a comprehensive breakdown of the transport network’s composition and respective speeds.

*Table 1: Transport network characteristics in the study area*

Classification	Transport network	Total length (in km)	Average speed (kmph)
Urban roads	Arterial roads	706.7	35.34
	Sub-arterial roads	645.6	34.22
	Collector roads	1350.0	27.59
Non-Urban roads	Expressway	208.90	63.70
	National highway	151.51	53.30
	State highway	121.33	51.64
	Other Non-Urban roads	266.22	48.21
Metro system	Metro system	392.21	36.70



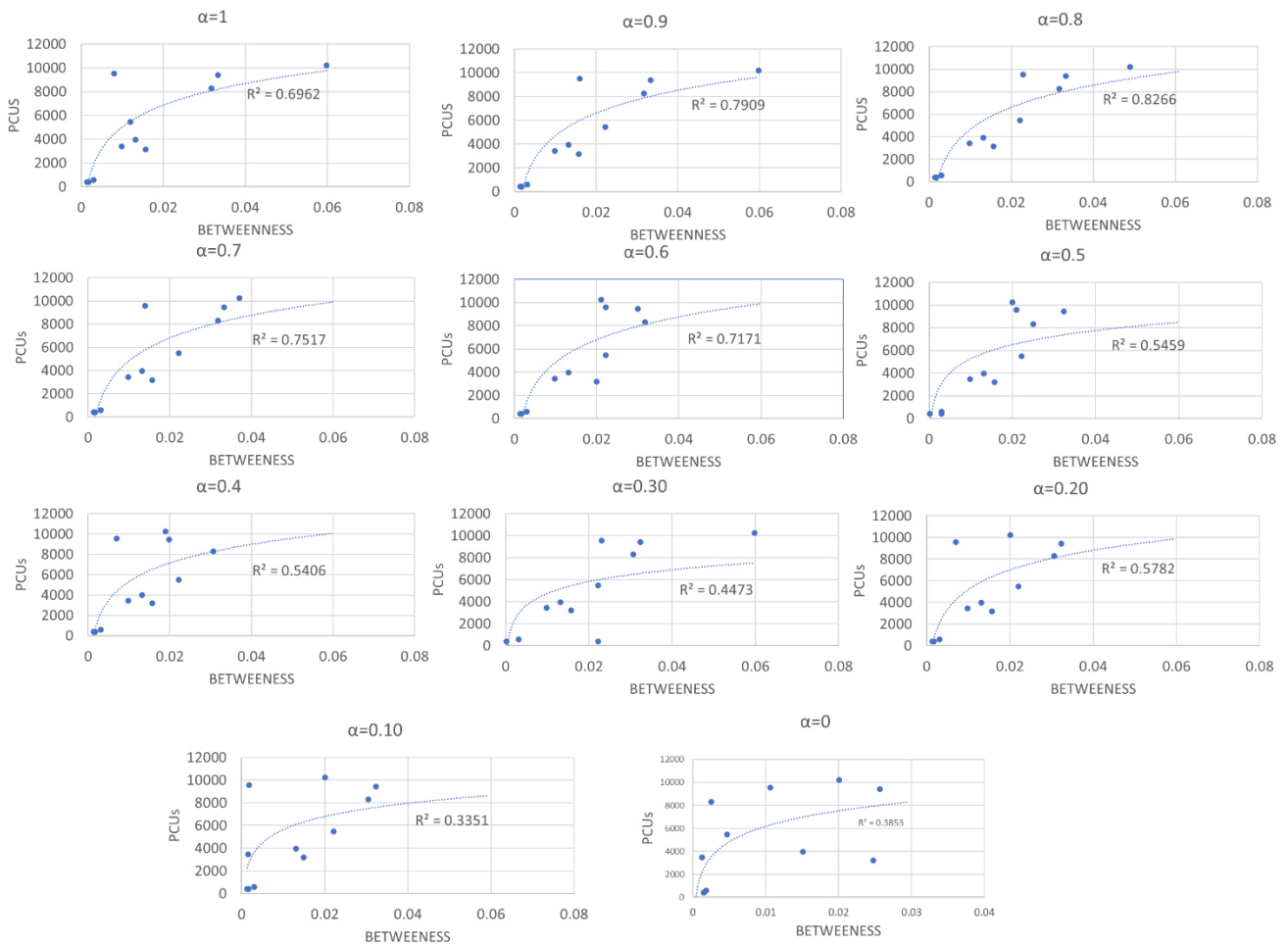
*Figure 5: Transport network characteristics for the study area. a) represents the typology of transport network in the study area; b) represents the peak hour speeds on the identified transport network i.e. Roads and Metro system*

### 3.2 Derivation of Path Evaluation Function

The analysis focuses on identifying the most suitable turning parameter ( $\alpha$ ) for the transport network. This is achieved by evaluating the correlation between the computed link betweenness and the actual traffic volumes. Figure 6 illustrates these results, displaying the correlation between varying levels of

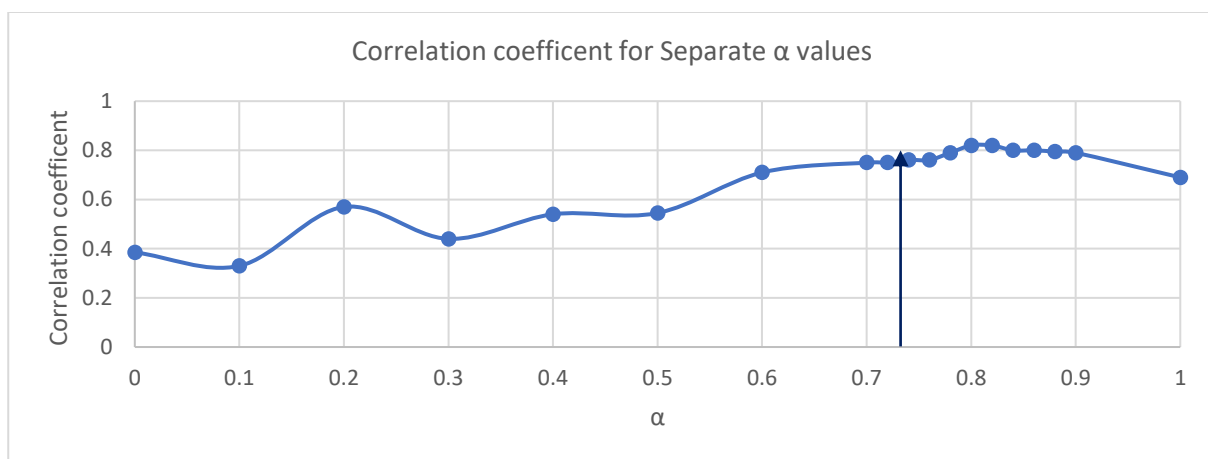


betweenness centrality (adjusted according to the Path Evaluation Function (PEF)) and traffic flow. This illustration effectively highlights how the network dynamics change under different scenarios.



**Figure 6: Correlation assessment for varying betweenness centrality (with varying Path Evaluation Function (PEF)) and traffic flow**

The following figure shows the variation of correlation coefficient of different betweenness centralities with different  $\alpha$  values and traffic volume.



**Figure 7: Correlation coefficients for separate  $\alpha$  values**

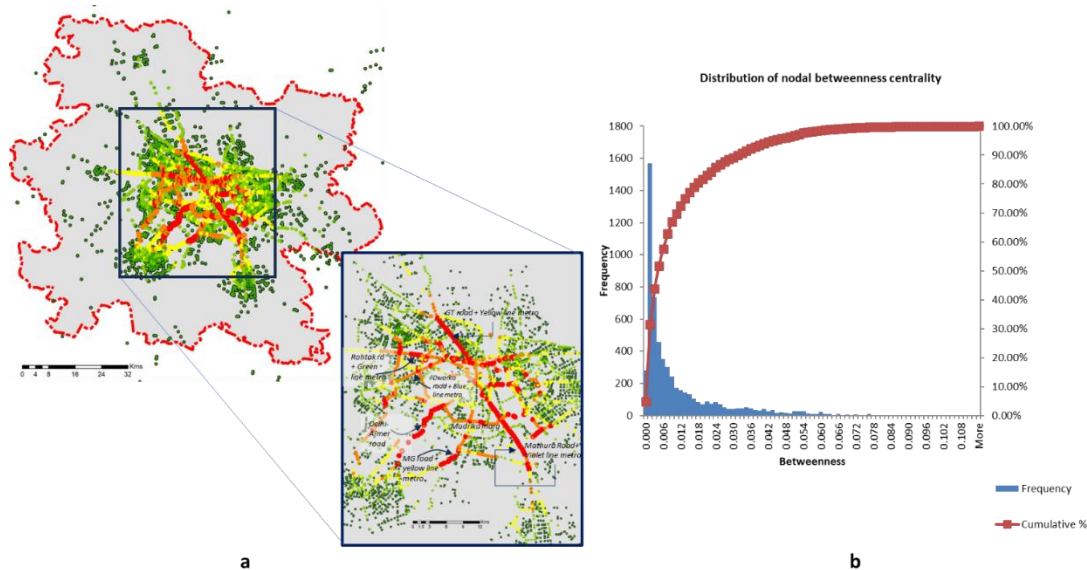
Based on **Figure 7**, the path evaluation function for the study area was chosen to be:

$$PEF = 0.80 \times T_{ij} + (0.20) \times \frac{1}{C_{ij}} \quad \dots(3)$$

Where,  $T_{ij}$  is the time travel to traverse a link and  $C_{ij}$  is its capacity. This indicates that although capacity is an important element in path evaluation, it is deemed to be of lesser significance compared to travel time. The inverse relationship ( $\frac{1}{C_{ij}}$ ) implies that higher capacity links will result in a lower PEF value, which can be interpreted as more desirable.

### 3.3 Betweenness centrality

The study computed the betweenness centrality utilizing the derived Path Evaluation Function (PEF). The analysis of betweenness centrality shed light on significant patterns within the transport network of the study area. A prominent feature observed was the concentrated high betweenness centrality along principal transportation routes, with a marked emphasis on arterial roads and metro lines. This was followed by a lesser but still significant concentration on sub-arterial and collector roads, and to a smaller extent, on non-urban roads. An additional key observation was the high level of betweenness centrality found specifically in Delhi. This points to the critical role of these major transport routes, particularly in Delhi, in dictating the flow and dynamics of movement within the network, highlighting their significance in the overall connectivity and efficiency of the transport system. **Figure 8**, shows the betweenness centrality map of the study area along with its distribution.



**Figure 8: Centrality maps for the study area; a) shows betweenness centrality of the study area, b) shows distribution of betweenness centrality in the study area**

## 4 Discussions and conclusion

The study embarked on an in-depth exploration of the betweenness centrality dynamics within the Delhi Metropolitan transport network, employing a new approach that integrates the complexities of multimodal transit systems. It has successfully demonstrated that the traditional methods of using only one network attribute (time/ distance) for centrality estimation, while informative, often fall short in capturing the multifaceted nature of modern urban transport. By leveraging a weighted centrality

framework and the Path Evaluation Function, this research has provided a better way to estimate betweenness centrality in urban/ metropolitan transport ecosystem. The same framework of assessment can be applied in the exploration of other centrality measures such as closeness, straightness and others. However, the study also highlights the need for further research in this area. The introduction of the Path Evaluation Function, which adeptly considers both capacity and travel time, marks a significant step forward in centrality analysis. However due to data limitations more network attributes such as cost couldn't be factored in this assessment. Furthermore, the application of the tuning parameter  $\alpha$ , while effective in the context of this study, may require adjustment when applied to different urban settings or network scales. Overall, this study contributes a significant advancement in network analysis, providing a more comprehensive perspective on the role of centrality in shaping the dynamics of urban transportation systems.

### Author Contributions

Dr. Sewa Ram led the design of the study, provided overarching guidance, and authored key sections of the manuscript. Jagannath Das was instrumental in collecting data, analysing it, and contributing to the manuscript's writing. All authors have reviewed and approved the final version of the manuscript for publication.

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