A Uniform Temperature Distribution Prevails During Couette Flow Within a Narrow Channel Gap

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Abstract:
The flow field used is a generalization of the well known Couette flow solution of steady flow, in which one wall is at rest and the other wall oscillates in its own plane about a constant mean velocity. The solutions is subject to two boundary conditions that corresponds to the heat transfer and thermometer problems. The exact solutions of the Couette flow are compared with the approximate solution of the flat plate boundary layer flow in terms of the wall characteristic values at high frequencies.

Keywords: Couette Flow, Adiabatic, Thermometer Problems.

Introduction:
The effect of plate temperature oscillation on free convection boundary layer has been studied by Singh et al. [7]. The exact solution of Couette flow and heat transfer of conducting liquid in a rotating system has been obtained by Jana and Datta [3]. It has been found that at the moving plate, the resultant shear stress increases and the rate of heat transfer decreases with increase in rotation parameter. An exact solution of the unsteady energy equation for an incompressible fluid with constant properties has been derived by Ishigaki [1, 2] and the effect of oscillation through the viscous dissipation on temperature field is discussed. The flow field used is a generalization of the well known Couette flow solution of steady flow, in which one wall is at rest and the other wall oscillates in its own plane about a constant mean velocity. The solutions is subject to two boundary conditions that corresponds to the heat transfer and thermometer problems. The exact solutions of the Couette flow are compared with the approximate solution of the flat plate boundary layer flow in terms of the wall characteristic values at high frequencies. Fluctuating flow problems past horizontal, vertical flat plate and through parallel plates have been studied by Jeschbe and Hans Beer [4], Nag et al. [5], Sharma et al. [6], Syam Babu and Jayaraj [8] and Zhi Gang - Gang and Efstathios [9].

Mathematical Analysis:
A particular simple exact solution of Couette flow between two parallel flat walls of which one is at rest and the other moving with a constant velocity Uo in its own plane (Figure - 1.1).

The boundary conditions of the flow are

$$y = 0, u = 0, y = h, u = U_0 \qquad (1.1)$$

In Couette flow steady equation of motion in x-direction takes form

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \qquad (1.2)$$

Solution:

The solution of equation (1.2) under boundary condition (1. 1) is

$$u = \frac{y}{h}U_0 - \frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}\left(1 - \frac{y}{h}\right) \qquad (1.3)$$

In a particular case for vanishing pressure gradient we have

$$u = \frac{y}{h}U_0 \qquad (1.4)$$

The flow given by equation (1.4) is known as simple shear flow. The general case of Couette flow is a superposition of shear flow over the flow between two flat walls. The shape of the superimposed flow is determined by the dimensionless pressure gradient

$$P = \frac{h^2}{2\mu U_0}\left(-\frac{dp}{dx}\right) \qquad (1.5)$$

Discussions:

For P > 0 i.e. for a pressure gradient decreasing in the direction of motion, the velocity is positive over the whole width of the channel. For negative values of P, the velocity over the portion of the channel width can become negative, that is back flow may occur near the wall which is at rest. It is seen from the figure that this happen when P< -1. In this case the dragging action of the faster layer exerted on fluid particles in the neighbourhood of the wall is insufficient to overcome the influence of the adverse pressure gradient.

We have plotted the Graph for superimposed Couette flow between two parallel flat walls when
P > 0, pressure decreases in direction of wall motion

P < 0 , pressure increases

P = 0 , zero pressure gradient.

Temperature field :

Equation for steady temperature distribution in the medium is given by

$$\rho c\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (1.6)$$

It is postulated that temperature is constant along the wall and so the boundary conditions are

y = 0, T = T_0; : y = h ; T = T_h (1.7)

In view of boundary Conditions (1.7), the equation (1.6) has a solution which is independent of x. Since with v = 0, the term $v\frac{\partial T}{\partial y}$ on the left hand side also vanishes. All the convective terms are equal to zero. The resulting temperature distribution, therefore, is solely due to the generation of heat through conduction and to friction in the y direction. Under this hypothesis equation (1.6) becomes

$$0 = k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 \quad (1.8)$$

Put $\frac{du}{dy}$ from equation (1.4) and get

$$k\frac{d^2T}{dy^2} = -\mu\frac{{U_0}^2}{h^2} \quad (1.9)$$

Solution of equation (1.9) satisfying conditions (1.7) is

$$\frac{T-T_0}{T_h-T_0} = \frac{y}{h} + \mu\frac{{U_0}^2}{2k(T_h-T_0)}\ \frac{y}{h}\left(1-\frac{y}{h}\right) \quad (1.10)$$

Using dimensionless parameter η,P_r and E as

$$\eta = \frac{y}{h},\ P_r = \frac{\mu c}{k}\ \text{(Prandtl Number)},\ E = \frac{{U_0}^2}{c\,(T_h-T_0)}$$

We write the equation (1.10) as

$$\frac{T-T_0}{T_h-T_0} = \eta + \frac{1}{2}P_r\ E\,\eta(1-\eta) \quad (1.11)$$

with zero heat generation by friction, we have

$$\frac{T-T_0}{T_h-T_0} = \frac{y}{h} \quad (1.12)$$

which shows that temperature distribution is linear.
When heat generation through friction is considered in temperature field equation, then distribution of temperature field is parabolic.
For various values of the product P_r, E we have plotted $\frac{T-T_0}{T_h-T_0}$ verses η in graph 1.2.

Discussion:

When $T_h > T_0$, then heat flows from the upper wall to the fluid only as long as the velocity U_0 of the upper wall does not exceed a certain value.
From equation (1.1)

$$\left[\frac{dT}{dy}\right]_{y=0} = \left\{\frac{1}{h} + \mu\frac{{U_0}^2}{2k(T_h-T_0)}\ \left(\frac{1}{h}-\frac{2y}{h^2}\right)\right\}(T_h-T_0) \quad (1.13)$$

Or $$\left[\frac{dT}{dy}\right]_{y=0} = \left\{(T_h-T_0) + \mu\frac{{U_0}^2}{2k}\ \right\}\frac{1}{h}$$

$$\left[\frac{dT}{dy}\right]_{y=0} = 0 \quad \text{when } (T_h-T_0) = -\mu\frac{{U_0}^2}{2k}$$

$$\mu\frac{{U_0}^2}{2k(T_h-T_0)} < 1$$

Or P_r. E < 2.

This causes the cooling of upper wall because the heat flows from the upper wall to the fluid when $\mu \frac{{U_0}^2}{2k(T_h - T_0)} > 1$ or P_r. E > 2.

Then heat flows from fluid to the upper wall causing heating of the upper plate,

when, both the walls have equal temperature, i.e. $T_h = T_0$

$$T(y) - T_0 = \mu \frac{{U_0}^2}{2k} \; \frac{y}{h}\left(1 - \frac{y}{h}\right) \qquad (1.14)$$

which is a simple parabolic temperature distribution symmetrical with respect to the mean axis.

From the equation (1.14), highest temperature T_{max} generated by frictional heating occurs at the centre and has the values

$$T_{max} - T_0 = \mu \frac{{U_0}^2}{8k} \qquad (1.15)$$

A further important distribution of temperature in the medium occur when it is postulated that all the heat due to friction is transferred to one of the walls only, and no heat transfer takes place at the other wall. In this case we assume that the lower wall is insulated adiabatically A. Barletta et.al [10], Jiji LM, Danesh-Yazdi AH.[11].

Thus, we solve the equation (1.6) under the boundary condition

$$y = 0; \frac{dT}{dy} = 0 \; ; \; y = h, T = T_0 \qquad (1.16)$$

on integrating equation (1.9) we have

$$\frac{dT}{dy} = -\mu \frac{{U_0}^2}{k} \; \frac{y}{h^2} + A$$

Thus constant A= 0 is obtained by boundary condition(1.16).

Hence $\frac{dT}{dy} = -\mu \frac{{U_0}^2}{k} \; \frac{y}{h^2}$ (1.17)

Further integrate equation (1.17) and use boundary condition (1.16) for calculating arbitrary constant, we have

$$T(y) - T_0 = \mu \frac{{U_0}^2}{2k} \; \left(1 - \frac{y^2}{h^2}\right) \qquad (1.18)$$

We can plot (1.18) as

Temperature increase of the lower wall is given by

$$T(0) - T_0 = \mu \frac{U_0}{2k} = T_{ad} - T_0 \qquad (1.19)$$

T_{ad} is called the adiabatic wall temperature and it is equal to the reading on a thermometer in the form of a flat plate: on comparing equation (1.19) with (1.15), it is seen that the highest temperature rise in the

centre of the channel for the case of equal wall temperatures is equal to one-forth of the adiabatic wall temperature rise :
thus,

$$T_{ad} - T_0 = u(T_{max} - T_0)$$

For cooling and heating of the upper wall under adiabatic wall conditions can be given as below:

$T_{ad} - T_0 < T_1 - T_0$: cooling of the upper wall

$T_{ad} - T_0 > T_1 - T_0$: heating of the upper wall.

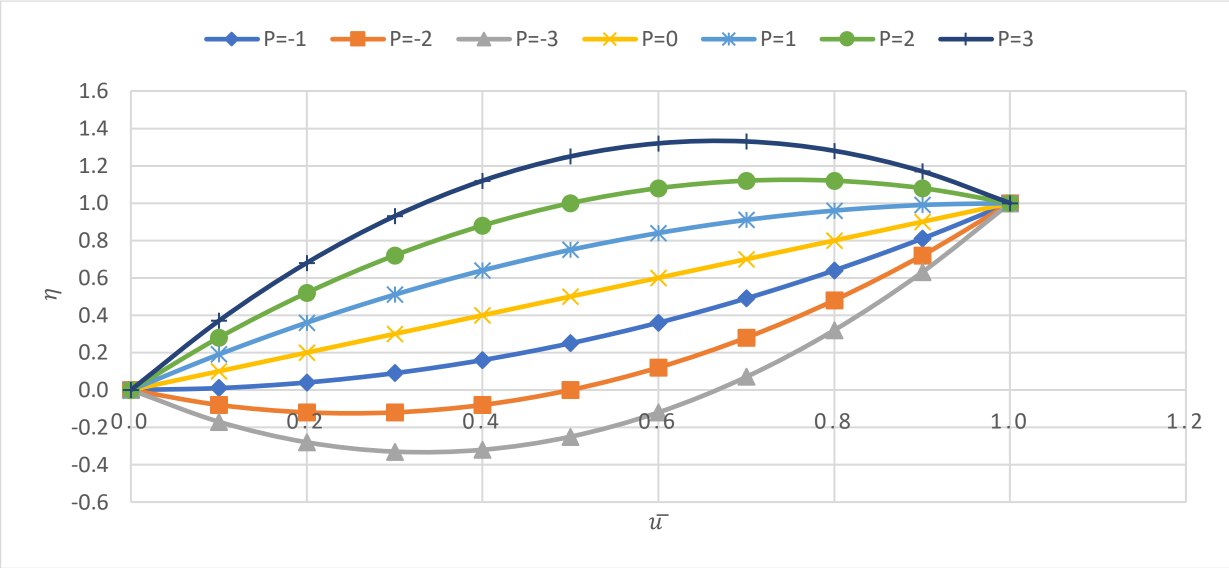
Superimpose Couette Flow Velocity Variation for Different Values of Pressure Gradient

η	$\bar{u}$						
	P=-1	P=-2	P=-3	P=0	P=1	P=2	P=3
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.01	-0.08	-0.17	0.1	0.19	0.28	0.37
0.2	0.04	-0.12	-0.28	0.2	0.36	0.52	0.68
0.3	0.09	-0.12	-0.33	0.3	0.51	0.72	0.93
0.4	0.16	-0.08	-0.32	0.4	0.64	0.88	1.12
0.5	0.25	0	-0.25	0.5	0.75	1.00	1.25
0.6	0.36	0.12	-0.12	0.6	0.84	1.08	1.32
0.7	0.49	0.28	0.07	0.7	0.91	1.12	1.33
0.8	0.64	0.48	0.32	0.8	0.96	1.12	1.28
0.9	0.81	0.72	0.63	0.9	0.99	1.08	1.17
1.00	1.00	.00	1.00	1.00	1.00	1.00	1.00

Table – 1.1 Temperature field variation with respect to different Values of P_r. E, for Heat generation by Friction

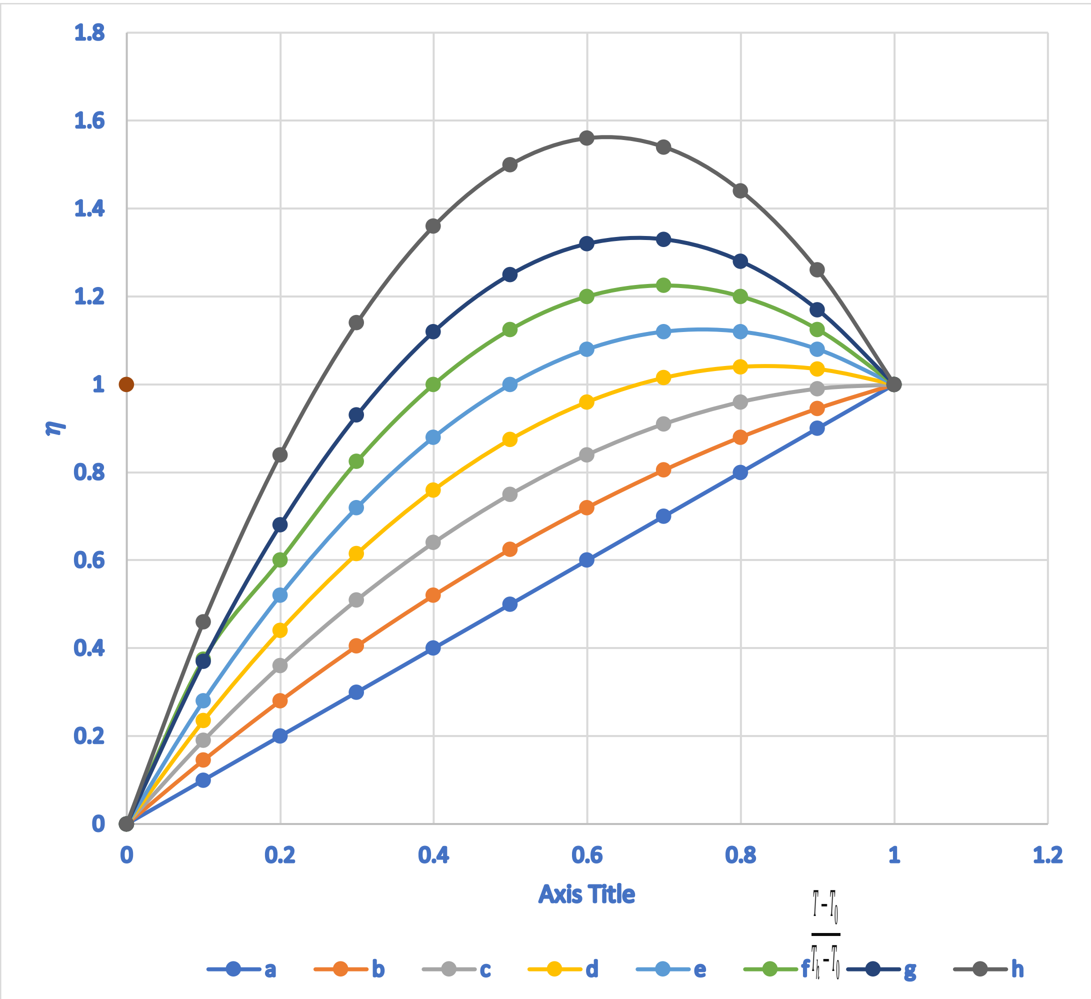
η	$\frac{T - T_0}{T_h - T_0}$								
	P_r. E=0	P_r. E=1	P_r. E=2	P_r. E=3	P_r. E=4	P_r. E=5	P_r. E=6	P_r. E=7	P_r. E=8
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.10	0.145	0.190	0.235	0.280	0.375	0.370	0.830	0.460
0.2	0.20	0.280	0.360	0.440	0.520	0.600	0.680	0.760	0.840
0.3	0.30	0.405	0.510	0.615	0.720	0.825	0.930	1.035	1.140
0.4	0.40	0.520	0.640	0.760	0.880	1.000	1.120	1.240	1.360
0.5	0.50	0.625	0.750	0.875	1.000	1.125	1.125	1.375	1.500
0.6	0.60	0.720	0.840	0.960	1.080	1.200	1.320	1.440	1.560
0.7	0.70	0.805	0.910	1.015	1.120	1.225	1.330	1.435	1.540
0.8	0.80	0.880	0.960	1.040	1.120	1.200	1.280	1.360	1.440
0.9	0.90	0.945	0.990	1.035	1.080	1.125	1.170	1.215	1.260
1.00	1.00	1.00	1.00	1.00	1.000	1.000	1.000	1.000	1.000

Table 1.2 Superimpose Couette flow velocity variation for different values of Pressure gradient



Graph-1.1

Temperature Field variation with respect to different values of P_r. E for heat generated by Friction



P_r. E=0	P_r. E=1	P_r. E=2	P_r. E=3	P_r. E=4	P_r. E=5	P_r. E=6	P_r. E=7
a	b	c	d	e	f	g	h

Graph-1.2

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