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Pedagogical Processes to Help Children Learn Algebraic Identities in Mathematics

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Abstract
This paper focuses on refining the pedagogical processes instrumental in the mastering of algebraic identities, employing a meticulous approach to instruction. Emphasizing the utility of resources specifically tailored for visualization, the paper primarily seeks to deepen understanding. Its aim is to instill and foster algebraic thinking among children, encouraging active engagement in the exploration of patterns and problem-solving within the realm of algebra. A significant facet of the research lies in its nuanced acknowledgment and targeted resolution of common challenges inherent in comprehending patterns and algebraic thinking, providing a thoughtful intervention in educational practices. Moreover, the research advocates for the exploration of diverse representations of identical algebraic concepts, acknowledging the multifaceted nature of learning, drawing from other subjects, the idea of memorization of patterns in order to internalize the concepts such that their application can be made irrespective of circumstances such as in grammar where the base rules for the constructions of sentences have to memorized after which their rules can be applied in varied circumstances. The outlined learning objectives for students extend beyond rote memorization, encompassing the identification and recall of common algebraic identities, graphical representation of their meanings, application in simplifying expressions, and the cultivation of critical thinking and problem-solving skills. The paper's holistic approach is further highlighted by its emphasis on effective communication in mathematics, stressing the importance of utilizing appropriate language. This well-rounded strategy, integrating theoretical understanding, practical skills, and critical thinking, is meticulously designed to optimize learning outcomes in the intricate domain of algebraic identities, offering a comprehensive foundation for students to build upon.

Keywords: Algebra, Identities, Visualization, Memorization, Pattern.

1. Introduction

Algebraic identities play an important role in mathematics curriculum and in mathematics in general. In Indian secondary school curriculum, eight types of identities are introduced and taught to students when solving equations and polynomials. Knowing and recognizing these identities helps students to learn otherwise complex mathematical procedures. It will also enable them to develop fluency when applying these procedures in algebraic manipulations and problem solving. It is important to be able to spot variations in the algebraic identities if one is to use the identities in varied problems. The main issue when learning and applying identities is that, for most students, the topic is purely memorisation and regurgitation. This paper will explore different approaches and exemplify them with visual

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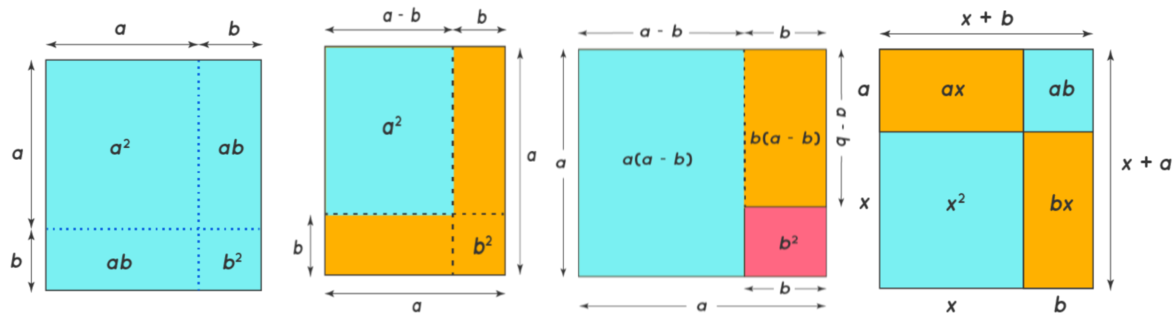
representations that can help students learn algebraic identities more efficiently. These approaches rely less on memorisation and instead build on understanding and internalizing the concepts of identities.

2. Pedagogical Processes

2.1 Visualization for Developing Area Concept

Visualization is the process of interpreting verbal or non verbal objects into visual form. It is an extremely important skill for humans as most of teaching (passing down of information) has to be done through making use of visualization due to the fact that concrete objects may not always be available to use as an example. Initially, visualization is difficult for children because they are taught through memorization since their childhood but as they grow up, the subjects rely more and more on visualization of abstract concepts. Visualization means seeing an image of something in your mind and it is more effective compared to base memorization. Different people will not always ‘see’ things in the same way, but ‘visual thinking’ is very important to the construction of understanding by students (Dörfler, 1991). Visualization can be introduced with relatively simple operations. For example, multiplication can be represented as a product table that is similar to area. Visualization can be further enhanced by making children engage in educational games which will engage the children and incite motivation among them to understand the concept at a deeper level (Kayan &Aydin).



“Figure 1” “Figure 2” “Figure 3” “Figure 4”

The algebraic expression $(a+b)^2$ is nothing but $(a+b) \times (a+b)$. This can be visualized as a square “Figure 1” whose sides are $(a+b)$ and the area is $(a+b)^2$.The square with a side of $(a + b)$ can be visualized as four areas of a^2, ab, ab, and b^2. The sum of these areas $a^2 + ab + ab + b^2$ gives the area of the big square $(a+b)^2$. Hence, $(a+b)^2 = a^2 + ab + ab + b^2$. $(a+b)^2$ is one of the basic algebraic identities and finds usage in a number of mathematical operations particularly while finding the squares of large numbers where the large numbers can be broken down into two smaller numbers whose calculations are a bit easier to conduct.

Let us consider $(a - b)^2$ as the area of a square “Figure 2”, with length $(a - b)$.In the above figure, the biggest square is shown with area a^2.To prove that $(a - b)^2 = a^2 - 2ab + b^2$, consider reducing the length of all sides by factor b, and it forms a new square of side length a - b. In the figure above, $(a - b)^2$ is shown by the blue area. Now subtract the vertical and horizontal strips that have the area $a \times b$. Removing $a \times b$ twice will also remove the overlapping square at the bottom right corner twice hence add b^2. On rearranging the data we have $(a - b)^2 = a^2 - ab - ab + b^2$. This identity too can be used for inferring the square of a a large number that contain 9 or 8 in their tens and unit places such as 999 which can be rewritten as (1000-1).

The objective is to find the value $a^2 - b^2$, which can be taken as the difference of the area of two squares "Figure 3", of sides **a** units and **b** units respectively. This is equal to the sum of areas of two rectangles as presented in the below figure. One rectangle has a length of a units and a breadth of (a - b) units. Another rectangle is taken with a length of (a - b) and a breadth of b units. Further, we take the areas of the two rectangles and sum the areas to obtain the resultant values. The respective areas of the two rectangles are $(a - b) \times a = a(a - b)$,and$(a - b) \times b = b(a - b)$. Finally, we take the sum of these areas to obtain the resultant expression**:** $a(a - b) + b(a - b) = (a - b)(a + b)$.This formula is especially useful in its application in solving sums of the Pythagoras theorem.

(x+a)(x+b) is nothing but the area of a rectangle "Figure 4", whose sides are (x+a) and (x+b) respectively. The area of a rectangle with sides (x+a) and (x+b) in terms of the individual areas of the rectangles and the squareis x^2, ax, bx, and ab. Summing all these areas we have $x^2 + ax + bx + ab$. This gives us the proof for the algebraic identity**:** $(x + a)(x + b) = x^2 + ax + bx + ab = x^2 + x(a + b) + ab$.

2.2 Learning Through Memorization

Learning through memorization, or rote learning, is a learning technique based on repetition.

There are several arguments in favour of this learning approach: one is that having a rapid recall of certain facts in mathematics is necessary to become fluent in other mathematics topics.

Many students are encouraged to learn their 'multiplication tables' by rote. This is so that when they are solving problems they do not spend too much time and effort working out relatively simple calculations such as 6 x 7, especially when they have no access to calculators. Knowing multiplication tables by heart also gives them a better number sense of the numbers' magnitude, of how numbers are related or of multiples and fractions. Drawing an example from another subject, I.e., Grammar, the rules for forming particular sentences are also meant to be memorized such that patterns may be recognized in any type of sentence one encounters and then an understanding made on the previously memorized rules/syntaxes. Thus, similar arguments could be made for learning algebraic identities through memorization.

However, there are also many counter-arguments to using memorization as a learning technique (De Morgan,1865; Marton and Booth,1997). One is about accessibility; not all students benefit from memorization due to their poor school attendance, their lack of time or opportunity for the required practice, or just their poor ability to recall. Students with special educational needs such as dyslexia, for example, are enormously disadvantaged.

Another argument concerns the kind of learning that memorization affords. Memorization does not focus on comprehension or building understanding; nor does it support any exploration of what concepts could mean, or how they are connected to other areas of mathematics. It focuses on memorizing and accurate reproduction, which can become problematic when studying more complex aspects of a subject (such as formulae and algorithms) that entail many steps. Memorization does not lead to understanding of meaning, which means that elements get missed out, details get muddled up, stress increases and exams can be failed.

The learning experience when using memorization is often not exciting; it can even be considered boring because of its repetitive nature and lack of focus on understanding and making connections. Students mechanically 'go through' the exercises, engaging their brains as little as possible. This is problematic for all students, including high achievers. Boredom when learning mathematics, little demand for thinking and a lack of opportunity to work on making connections and giving meaning to mathematics makes it hard for learners to understand and enjoy the subject.

2.3 Patterns

Patterns are regularities that we can perceive. We need to understand not just the individual elements within this pattern unit, but also how the pattern unit is repeated.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(p+q)^2 = p^2 + 2pq + q^2$$

In daily life, the accuracy of any judgement is usually measured by how consistent it is. Consistency is regularity in results and thus is of vital importance in any situation where the accuracy of something has to be evaluated. In algebra, we usually start teaching identities by using variable (commonly used alphabets are a,b ; x,y) but the actual purpose is to then transfer that pattern of solving problems into the other fields of mathematics such as the squaring a very large number

2.4 Spotting Patterns

Mathematics has sometimes been called a science of patterns (Resnik, 1981). We think of mathematics as having structure, and that structure enables us to solve problems. The structure is built around looking for and manipulating patterns.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$a^2 + b^2 = (a+b)^2 - 2ab$$
$$ab = \frac{1}{2}[(a+b)^2 - (a^2 + b^2)]$$

For example, the identities are taught in one way but the student can then take any part of the equation as the starting point and then readjust the whole equation according to their needs as shown below where $\boldsymbol{a^2 + b^2}$ as well as $\boldsymbol{ab}$ can be calculated from the same formula by readjusting the position of the variables and constants.

2.5 Pattern recognition

In general, the ability of the human brain to recognize information from a stimulus to information stored in one's memory is known as pattern recognition. As has been discussed above, patterns do emerge in daily life and in mathematics as well and students that are able to recognize patterns faster are able to solve problems more efficiently. Algebra in specific requires the proactive use of pattern recognition as many questions have to be sorted and categorized into certain pattern of formulae in order to solve them. Thus, developing the ability to spot patterns is paramount to ensuring that algebraic identities are properly internalized by the students.

3. Conclusion

In conclusion, this paper underscores the significance of innovative pedagogical approaches to instill a profound understanding of algebraic identities among students. By emphasizing visualization techniques, educators can guide children in translating abstract mathematical concepts into tangible images, fostering a more intuitive grasp of algebraic expressions. Furthermore, the paper advocates for a balanced approach that acknowledges the utility of memorization for certain foundational aspects of mathematics, such as multiplication tables, while cautioning against over-reliance on this method. The integration of visual aids, educational games, and the cultivation of pattern recognition skills emerges as a potent combination to transcend mere memorization, offering students a more engaging and holistic learning experience.

The paper's insights are particularly valuable in addressing the common challenge of algebraic identities being perceived as mere memorization tasks. By encouraging students to explore patterns and visualize mathematical expressions, educators can enhance the students' ability to apply these identities in problem-solving contexts. This holistic approach not only promotes a deeper understanding of algebraic concepts but also aims to make the learning process more enjoyable, stimulating intellectual curiosity and paving the way for a more proficient and confident mathematical journey for students.

4. References

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