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A Study on Intuitionistic Multi-Fuzzy Ideals in BH-Algebra

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Abstract:
The purpose of this paper is to implement the concept of intuitionistic multi-fuzzy sets to ideals in BH-algebra. In this paper, we introduce the notion of intuitionistic multi-fuzzy ideals, intuitionistic multi-fuzzy closed ideals in BH-algebra and investigate some of their related properties. Also we discuss the relation between intuitionistic multi-fuzzy ideals and intuitionistic multi-fuzzy closed ideals of BH-algebra. Finally we define the upper level subset of intuitionistic multi-fuzzy ideals of BH-algebra and study some of its properties based on (α,β)-cut.

Key words: BH-algebra BH-ideal Multi-fuzzy set Intuitionistic multi-fuzzy set Intuitionistic multi-fuzzy BH-ideal Intuitionistic multi-fuzzy closed ideal Level subset Homomorphism.

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INTRODUCTION

BCK and BCI-algebras, two classes of algebras of logic, were introduced by Imai and Iseki [5,6], It is known that the class of BCK-algebras is a proper subclass of the class of BCI -algebras.Zadeh, L.A [1] introduced Fuzzy Sets, Atanassov, K.T [2] are introduced Intuitionistic fuzzy sets, FuzzySets. Sabu, S. and T.V. Ramakrishnan,[3] are introducedMulti-fuzzy sets. Imai, Y. and K. Iseki [4] are introducedon axiom system of Subalgebras of BG-Algebra. Kim, C.B. and H.S. Kim [7] are introduced On BG-algebras. Ahn, S.S. and D. Lee[8] are introduced Fuzzy subalgebras of BG algebras. Y. B. Jun, E. H. Roh and H. S. Kim [11] are introduced On BH-algebras.R.Muthuraj and S.Devi [9] are introduced Intuitionistic Multi-fuzzy BG-subalgebra.Shinoj, T.K. and J.J. Sunil[10] are introduced Intuitionistic Fuzzy Multi sets. In this paper, we define a new algebraic structure of Intuitionistic Fuzzy Multi ideals and Intuitionistic Fuzzy Multi-fuzzy closed ideals of BH-algebra and discuss some of their related properties based on level subsets.

Preliminaries:

In this section, the basic definitions of a BG-algebra, BG-ideal, multi-fuzzy sets are recalled. We start with

Definition 2.1: [5, 6]
Let X be a set with binary operation $*$ and a constant 0. Then (X, $*$, 0) is called a BCK-algebra if it satisfies the following axioms:
(1) $((x * y) * (x * z)) * (z * y) = 0$

(2) $(x * (x * y)) * y = 0$
(3) $x * x = 0$
(4) $0 * x = 0$
(5) $x * y = 0$ *and* $y * x = 0$ *imply* $x = y$ *for all* $x, y, z \in X$

Definition 2.2 [1, 3]:
Let X be a non-empty set. A multi-fuzzy set A in X is defined as a set of ordered sequences: A = {(x , μ_1(x), μ_2(x),.....,μ_i(x),….) : x ∈ X}, where μ_i : X ⟶ [0,1] for all i . Here k is called the dimension of A.

Definition 2.3 [2]:
Let X be a non-empty set. Anintuitionistic fuzzy set(IFS) A in X is a set of the form A ={ (x, μ(x), (x)): x∈ X }, where μ: X →[0, 1] and v: X→ [0,1]define the degree of membership and the degree ofnon-membership of the element x ∈X respectively, with$0 \le \mu(x) + v(x) \le 1$.

Definition 2.4 [3]:
Let X be a non-empty set. A multi-fuzzy set Ain X is a set of the form$A = \{(x, \mu_A(x),\ v_A(x))$: x ∈ X },
Where $\mu_A(x) = (\mu_1(x), \mu_2(x), \ldots, \mu_k(x))$, $v_A(x) = (v_1(x), v_2(x), \ldots, v_k(x))$
Andeach μ_i: X →[0, 1],v_i:X →[0, 1]with$0 \le \mu_i(x) + v_i(x) \le 1$, x ∈ X for all i=1,2,…..k
Here $\mu_1(x) \ge \mu_2(x) \ge \ldots \ge \mu_k(x)$, x ∈ X

Remark 2.5
Note that although we arrange the membership sequence in decreasing order, the corresponding non membershipsequence need not be in decreasing or increasing order
A= {(x, μ (x), μ (x)... μ (x)): x X}, where μ: X [0, 1]

Definition 2.6 [7]:
A non-empty set X with a constant 0 and a binary operation " ∗ " is called a BG-algebra if it satisfies the following axioms:
1. $x * x = 0$
2. $x * 0 = x$
3. $(x * y) * (0 * y) = x \quad \forall x, y \in X.$

Example 2.7.
Let X= {0, 1, 2} be a set with the following cayley table

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then (X, ∗, 0) is a BG-algebra.

Definition 2.8[11]
A nonempty set X with a constant 0 and a binary operation ∗ is called a BH-algebra, if it satisfies the following axioms
(BH1) $x * x = 0$
(BH2) $x * 0 = 0$
(BH3) $x * y = 0$ *and* $y * x = 0 \Rightarrow x = y$ for all $x, y \in X$

Definition 2.9[8]

Let S be a non-empty subset of a BG-algebra X, then S is called a subalgebra of X if $x * y \in S$ for all x, y ∈ S.

Definition 2.10[7]

Let X be a BH-algebra and I be a subset of X. Then I is called a BH ideal of X if it satisfies the following conditions:

(i). $0 \in I$

(ii). $x * y \in I$ *and* $y \in I \Rightarrow x \in I$

(iii). $x \in I$ *and* $y \in X \Rightarrow x * y \in I$

Definition 2.11[8]

Let μ be a fuzzy set in a BG-algebra X. Then μ is called a fuzzy subalgebra of X if$\mu(x * y) \geq min\{ \mu(x) , \mu(y) \}, \forall x , y \in X$

Definition 2.12 [9, 10]: An intuitionistic multi-fuzzy subset A = $\{(x, \mu_A(x), v_A(x)) : x \in X\}$in X is called an intuitionistic multifuzzy subalgebra of X if it satisfies:

1. $\mu_A(x * y) \geq min\{ \mu_A(x) , \mu_A(y) \}$
2. $v_A(x * y) \leq max\{ v_A(x) , v_A(y) \}, \forall x , y \in X$

Definition 2.13 [1]

A mapping $f : X \to Y$ of a BH-algebra is called a homomorphism if $f(x * y) = f(x) * f(y) \forall x , y \in X.$

Remark 2.14 [1]

If $f : X \to Y$ is a homomorphism of BH-algebra then $f(0) = 0$.

Definition 2.15: [9]

Let X and Y be any two non-empty sets and $f : X \to Y$ be a mapping. Let A and B be any two IMF subsets of X and Y respectively having the same dimension K. Then the pre-image of B (⊆Y) under the map f is denoted by f^{-1}(B)=$(\mu_B(f(x)), v_B(f(x))), x \in X$

Definition 2.16: [9,10]

Let A= $\{(x, \mu_A(x), v_A(x)) : x \in X\}$ and B =$\{(x, \mu_B(x), v_B(x)) : x \in X\}$be any two IMFS's having the same dimension k of X. Then,

i). A ⊆ B if and only if $\mu_A(x) \leq \mu_B(x), v_A(x) \leq v_B(x)$for all $x \in X$

ii). A = B if and only if$\mu_A(x) = \mu_B(x), v_A(x) = v_B(x)$, for all $x \in X$

iii). $A \cup B = \{(x, \mu_{A\cup B}(x), v_{A\cup B}(x)) : x \in X\}$ *where* $\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(y)\} = (max\{\mu_{iA}(x), \mu_{iB}(y)\})^k{}_{i=1}$ and

$v_{A\cup B}(x) = min\{v_A(x), v_A(y)\} = (\min\{v_{iA}(x), v_{iB}(x)\})^k{}_{i=1}$.

iv). $A \cap B = \{(x, \mu_{A\cap B}(x), v_{A\cap B}(x)) : x \in X\}$ *where* $\mu_{A\cap B}(x * y) = min\{ \mu_A(x) , \mu_B(y) \} = (min\{ \mu_{iA}(x) , \mu_{iA}(y) \})^k{}_{i=1}$ and

$v_{A\cap B}(x * y) = max\{v_A(x), v_A(y)\} = (\max\{v_{iA}(x), v_{iB}(x)\})^k{}_{i=1}$.

Definition 2.17: [10]

An intuitionistic multi-fuzzy subset A ={(x, μ_A (x), A(x)): x ∈X} in X is called an intuitionistic Multi-fuzzy ideal of X if it satisfies:

1. $\mu_A(0) \geq \mu_A$ (x) and μ_A (0) ≤ μ_A (x)
2. μ_A (x)≥ min {μ_A (x ∗ y), μ_A (y)}
3. v_A (x)≤ max { v_A (x ∗ y), v_A (y)} x, y∈ X

3. Intuitionistic Multi-Fuzzy Ideals of BH-Algebra:

In this section, we define the new notion of intuitionistic multi-fuzzy ideals and intuitionistic multi-fuzzy closed ideals of BH-algebra and discuss some of its properties.

Definition 3.1:

An intuitionistic multi-fuzzy subset A ={(x, μ_A(x), A(x)): x ∈X} in X is called an intuitionistic Multi-fuzzy ideal of X if it satisfies:

1. μ_A(0)≥ μ_A(x) and μ_A (0) ≤ μ_A (x)
2. μ_A (x)≥ min {μ_A(x ∗ y), μ_A(y)}
3. v_A(x)≤ max { v_A (x ∗ y), v_A (y)} x, y∈ X

Example 3.2

Consider a BH-algebra X= {0, 1, 2, 3} be a set with the following cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let A = (μ_A, v_A) be an intuitionistic multi-fuzzy subset in X defined as follows:

μ_A(0) = μ_A (1) = (1, 1), μ_A (2) = μ_A(3) = (0.7, 0.5) and v_A(0)= v_A(1) = (0, 0), v_A(2) = v_A(3) = (0.2, 0.3)

Then A is an intuitionistic multi-fuzzy ideal of X.

Theorem 3.3: Let A = (μ_A, v_A) be an intuitionistic multi-fuzzy ideal of a BH-algebra X.

If $x * y$ ≤ z then μ_A (x) ≥min {μ_A(y), μ_A (z)}, v_A(x)≤ max {A(y), A(z) } for all x, y, z∈ X.

Proof:

Let $x, y, z \in X$ such that $x * y = z$.

Then (x * y) * z = 0

μ_A(x) ≥min { μ_A $(x * y)$, μ_A (y) }

≥min{min{$\mu_A((x * y) * z)$, μ_A(z) }, μ_A (y) }

= min{min {μ_A(0), μ_A(z) }, μ_A (y) }

= min { μ_A(y) }, μ_A (z) }

A (x) max { $v_A(x * y), v_A$ (y) }

= max{ max{((v_A (x * y) * z), v_A (z)}, v_A (y)}

= max {max { v_A (0), v_A (z) }, v_A (y) }

= max { v_A (y), v_A (z) }

Theorem 3.4: Let A = (μ_A, v_A) be an intuitionistic multi-fuzzy ideal of a BH-algebra X. If x y then μ_A(x) ≥ μ_A(y),v_A(x) v_A(y) for all x, y X.

Proof:

Let x, y∈ X such that x ≤y . Then $x * y = 0$.

μ_A(x) ≥ min{$\mu_A(x * y), \mu_A$ (y) }

≥min{μ_A(0), μ_A (y) }

= μ_A (y)

And v_A (x) ≤max { v_A $(x * y)$, A(y) }
≤ max { v_A (0), v_A(y) }
= v_A(y)

Theorem 3.5: Let A = (μ_A, v_A) and B = (μ_B, v_B) be two intuitionistic multi-fuzzy ideals of a BH-algebra X. Then the intersection A∩ B is also an intuitionistic multi-fuzzy ideal of X.

Proof:

Let x, y∈ A ∩B
Then x, y ∈A and x, y ∈B
μ_{AB}(0) = μ_{AB} (x * x)
= min { μ_A(x * x), μ_B (x * x) }
min{min{μ_A(x), μ_A(x)}, min{μ_B(x), μ_B (x)} }
= min { μ_A(x), μ_B (x) }
= $\mu_{A\cap B}$ (x)
$v_{A\cap B}$(0) = $v_{A\cap B}$(x ∗ x)
= max { v_A(x * x), v_B(x ∗ x) }
≤max{max{ v_A(x), v_A(x)}, max{ v_B(x), v_B(x) } }
= max{v_A(x), v_B(x) }
= $v_{A\cap B}$(x)
$\mu_{A\cap B}$ (x) = min { μ_A(x), μ_B (x) }
= min { min { μ_A(x * y), μ_A(y) }, min { μ_B (x ∗ y), μ_B (y)}}
≥min{min{μ_A(x ∗ y), μ_B(x ∗ y)}, min{μ_A (y), μ_B (y)}}
= min { $\mu_{A\cap B}$ (x ∗ y), $\mu_{A\cap B}$ (y) }
$v_{A\cap B}$(x) = max { v_A(x), v_B(x) }
= max{max{ v_A(x ∗ y), v_A(y)}, max{ v_B(x ∗ y), v_B(y) }}
= max{max{ v_A(x ∗ y), v_B($x * y$)}, max{ v_A(y), v_B(y)}}
≤max { $v_{A\cap B}$(x ∗ y), $v_{A\cap B}$(y) }

Theorem 3.6: Let X be a BH-algebra. Then an intuitionistic multi-fuzzy set A is an intuitionistic multi-fuzzy ideal of X if and only if A is an intuitionisticmulti-fuzzy subalgebra of X.

Proof:

Every intuitionistic multi-fuzzy ideal of X is an intuitionistic multi-fuzzy subalgebra of X.
Conversely, let A bean intuitionistic multi-fuzzy subalgebra of X.
Let x, y∈ X
μ_A(0) = μ_A(x ∗ x)
≥min { μ_A(x), μ_A(x) } = μ_A(x)
v_A(0) = v_A(x ∗ x)
≤max { v_A(x), v_A(x) }

= ν_A(x)
μ_A(x) = μ_A((x ∗y) ∗ (0 ∗ y))
≥min { μ_A(x ∗ y), μ_A(0 ∗ y) }
≥ min { μ_A(x ∗y), min { μ_A(0), μ_A(y) }}
= min { μ_A(x ∗ y), μ_A(y) }
ν_A(x) = ν_A((x ∗ y) * (0 ∗ y))
≤max { ν_A(x ∗y), ν_A(0 ∗ y) }
≤ max { ν_A(x ∗ y), max { ν_A (0), ν_A (y) }}
= max { ν_A(x ∗ y), ν_A (y) }

Definition 3.7: An intuitionistic multi-fuzzy subset A ={ (x, μ_A(x), ν_A(x)): x∈ X } in X is called an intuitionistic multi-fuzzy closed ideal of X if it satisfies:
μ_A (0∗ x) ≥ μ_A(x) andν_A(0∗x) ≤ ν_A(x)
μ_A (x)≥min { μ_A(x ∗ y), μ_A(y) }
ν_A (x) ≤max { ν_A(x ∗ y), ν_A(y) } x, y∈ X

Example 3.8: Consider a BH-algebra X = {0, 1, 2, 3} with the following table:

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Let A = (μ_A, ν_A) be an intuitionistic multi-fuzzy subset in X defined as follows:
μ_A(0) = μ_A(1) = (0.8, 0.5), μ_A(2) = μ_A(3) = (0.4, 0.3) and ν_A(0) = ν_A(1) = (0.2, 0.3), ν_A(2) = ν_A(3) = (0.5, 0.4)
Then A is an intuitionistic multi-fuzzy closed ideal of X.

Theorem 3.9: Every intuitionistic multi-fuzzy closed ideal of a BH-algebra X is an intuitionistic multi-fuzzy ideal of X.
Proof:
Let A = (μ_A, ν_A) be an intuitionistic multi-fuzzy closed ideal of X.
It is enough to prove that μ_A(0) ≥ (x) and ν_A(0)≤ ν_A(x)
Now, μ_A(0) ≥min { μ_A(0∗x), v(x) }
min≥{ μ_A(x), μ_A(x) } = μ_A (x)
ν_A(0) ≤max { ν_A(0∗x), ν_A(x) }
≤max { ν_A(x), ν_A(x) }
= ν_A(x)

Remark: 3.10 The converse of the above theorem is not true. Let us prove this by the following example.

Example 3.11 Let Consider a BH-algebra X= {0, 1, 2, 3} with the following table

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

μ_A (2) = μ_A (3) = (0.5, 0.3)and $\upsilon_A(0)$ = (0.1, 0.2), $\upsilon_A(1)$ = (0.2, 0.4),$\upsilon_A(2)$ = $\upsilon_A(3)$ = (0.4, 0.6)
Then A is an intuitionistic multi-fuzzy ideal of X but it is not an intuitionistic multi-fuzzy closed ideal of X.

Corollary 3.12: Every intuitionistic multi-fuzzy subalgebra satisfying the conditions μ_A(x) ≥min { μ_A(x ∗y), μ_A(y)}, υ_A (x) ≤max { υ_A(x ∗ y), υ_A(y) }, is an intuitionistic multi-fuzzy closed ideal.

Theorem 3.13: Every intuitionistic multi-fuzzy closed ideal of a BH-algebra X is an intuitionistic multi-fuzzy subalgebra of X.

Proof:

Let A = (μ_A, υ_A) be an intuitionistic multi-fuzzy closed ideal of X.
Then μ_A(x∗y)≥min{μ_A((x∗y)*(0∗y), μ_A(0 ∗y)}
= min μ_A(x), μ_A(0 ∗y)}
≥min{μ_A(x), μ_A(y) }
υ_A(x∗y) ≤max { υ_A ((x∗y) ∗ (0∗y)), υ_A(0∗y)}
=max{υ_A (x), υ_A(0 ∗y) }
≤max{υ_A (x), υ_A (y) }

Preposition 3.14: If an intuitionistic multi-fuzzy set A =(μ_A, υ_A) in X is an intuitionistic multi-fuzzy closed ideal,then for all x ∈X, μ_A(0)≥ μ_A(x) and $\upsilon_A(0) \leq \upsilon_A$(x)

Proof: Straight forward.

Definition 3.15:

Let A = { (x, μ_A(x), υ_A(x)): x∈ X } be an intuitionistic multi-fuzzy subset in X. Then the (α, β)-cut of A is denoted by $[A]_{(\alpha,\beta)}$and is defined by $[A]_{(\alpha,\beta)}$= { x∈X: μ_A(x)≥ α and $\upsilon_A(x) \leq \beta$} where α= (α_1, α_2, …. α_k) and β= (β_1, β_2, …. β_k) where each α_i, β_i ∈[0, 1] with 0≤ $\alpha_i + \beta_i$ ≤1 for all i = 1, 2, ….k such that μ_i(x) ≥ $\alpha_i$with the corresponding $v_i(x)$ ≤ i for all i = 1, 2, …..k.

Theorem 3.16: If A is an intuitionistic multi-fuzzy ideal of X, then the subset $[A]_{(\alpha,\beta)}$is an BH-ideal in X.

Proof

Since A = (μ_A, υ_A) is an intuitionistic multi-fuzzy ideal
in X, μ_A(0)≥ μ_A(x)≥ α
and$\upsilon_A(0) \leq \upsilon_A$(x)≤ β
Then 0 ∈ $[A]_{(\alpha,\beta)}$
Let x * y ∈ $[A]_{(\alpha,\beta)}$ and y ∈ $[A]_{(\alpha,\beta)}$
Then μ_A (x * y) ≥ α, υ_A(x * y) ≤ β and
μ_A (y)≥ α, υ_A(y)≤ β
μ_A(x)≥min { μ_A(x * y), μ_A (y) }

$\geq$min { α , α} =α
andv_A(x)$\leq$ max { v_A(x * y), v_A(y) }
$\geq$min { β ,β } =β
This implies that x$\in$ $[A]_{(\alpha,\beta)}$
Let x$\in$ $[A]_{(\alpha,\beta)}$and y$\in$ X
Choose y in X such that μ_A (y) $\geq$ α , v_A(y) $\leq$ β
μ_A (x*y) $\geq$min { μ_A(x), μ_A (y) }
$\geq$min {α , α} =α
andv_A(x*y) $\leq$max { v_A(x), v_A(y) }
$\leq$max {β ,β } =β
This implies that x * y $\in$ $[A]_{(\alpha,\beta)}$
Hence the subset$[A]_{(\alpha,\beta)}$ is a BG-ideal in X.

Theorem 3.17: Let X be a BH-algebra . If the set $[A]_{(\alpha,\beta)}$is a BH-ideal in X then an intuitionistic multi-fuzzy set A =(μ_A, v_A) is an intuitionistic multi-fuzzy ideal in X .

Proof:

Let $[A]_{(\alpha,\beta)}$ be a BG-ideal in X.
Assume that A = (μ_A, v_A) is not an intuitionistic multi-fuzzy ideal in X.
Then there exists a, b$\in$ X such that μ_A(a) < min { μ_A (a*b),
μ_A (b) } and v_A(a) > max { v_A(a*b), v_A(b) } hold.
Let α= [μ_A (a) + min {μ_A(a*b),μ_A(b)}] / 2, β= [v_A(a) +max {v_A(a*b), v_A(b)}] / 2
Then μ_A(a) <α< min { μ_A(a*b), μ_A (b) } and
v_A(a) >β>max{v_A(a*b), v(b) }
This implies that a*b, b$\in$ $[A]_{(\alpha,\beta)}$
butμ_A (a)<α and v_A(a) >β
That is, a$\notin$ $[A]_{(\alpha,\beta)}$
which is a contradiction that $[A]_{(\alpha,\beta)}$is a BG-ideal of X.
Therefore μ_A(x) $\geq$min { μ_A(x * y), μ_A (y) } and v_A(x)$\leq$ max{ v_A (x * y), v_A(y) }
Hence A = (μ_A, v_A) is an intuitionistic multi-fuzzy ideal in X.

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