International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

What Do Mathematicians Do and Why is Mathematics Important?

A Chandrashekaran

Department of Mathematics, Central University of Tamil Nadu, Thiruvarur

Abstract

This article is a summary of the author's presentation in an orientation programme to general audience about about the effectiveness of Mathematics in the real world. In this article, we address the following questions for a layman who is not a professional Mathematician: What is Mathematics?; What is the meaning of research in Mathematics?; Why is research in Mathematics necessary? And How to teach Mathematics?

Keywords: Mathematics Education, Mathematicians, research in Mathematics

1. INTRODUCTION

“The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.” –Eugene Wigner [1]

It is a common perception that Mathematics is a subject with many tedious or complex calculations or a subject with theorems and proofs. One time, there was a doctor who asked me the following question.

“Does research in Mathematics mean finding new formulae?”

Before looking at the answer to the above question, let us observe the implied ignorance or the common understanding of Mathematics and research in Mathematics by non-professional Mathematicians. Ironically, Mathematics teachers at every level spread this rumour. With this understanding, we are going to discuss the following questions.

A. What is Mathematics?
B. What is the meaning of research in Mathematics?
C. Why is research in Mathematics necessary?
D. How to teach Mathematics?

The above questions or the discussions here cannot be exhaustive to provide a complete understanding of the subject. However, we provide substantial information and illustrations to help readers understand the subject.

2. WHAT IS MATHEMATICS?

Mathematics is abstraction and the study of abstract objects. Like the integers 1, 2, 3, … are abstraction of the concept of counting, we Mathematicians abstract many things like measure, physical properties like smoothness, curvature etc.,.We also model economic structures, dynamical systems etc.,. A simple example for such an abstraction would be the concept of metric spaces, which includes any thing that you measure like length, volume, density, sound intensity, angular momentum, etc.,. Although the concept of abstraction is difficult to explain, we attempt to do here. Abstraction of any concept relies on observation of simple examples to identify the defining properties of the concept in consideration. Once we have listed the defining properties we have to ensure that the chosen such properties are irredundant. Now, if possible, we need to test whether these defining properties are available in other examples too. On satisfactory verification of these properties, we translate these properties from plain English to the language of Mathematics. Let us see the above concept in the situation of abstracting distance. The defining properties of distance are as follows.

1. Distance cannot be negative.
2. Distance between two points are same irrespective where you start measuring.
3. Distance is zero when the points are same and when the points are same, distance will be zero.
4. Distance between two points is always smaller or equal to the distance measured between those two points via a given third point.

The above are some defining properties observed about distance. Let us now see the analogue for area of a bounded surface.

1. Area cannot be negative.
2. Area of the bounded surface is same irrespective of how you measure or where you start measuring from.
3. Area will be zero only when the boundary is just a single point or a piece of line/curve with different end points.
4. Lastly area of the bounded region is always smaller when it is a flat surface.

With this analogue we shall define what is called a metric in an abstract way that generalizes the above mentioned four properties in both the examples above. Mathematically a metric space is a set of objects and between any two of those objects there is a metric or distance defined with the following properties. Before writing the properties let us give names for some of the necessary things. Let us name the set of objects as M. We shall use small letters like x,y,z for naming arbitrary objects in the set M. The distance between two objects or points x and y in M shall be denoted by d(x,y). Now the above discussed defining properties translate as follows:

1. $d(x,y) >= 0$ for all pairs x, y from M.
2. $d(x,y) = d(y,x)$ for all pairs x,y from M.
3. $d(x,x) = 0$ for all x in M.
4. $d(x,y) <= d(x,z) + d(z,y)$ for all x,y,z in M.

The above four might look justified with respect to the example of distance. However, to understand these properties with reference to area we should understand the boundary as two pieces of lines/curves with same end points and these two lines /curves together form a bounded surface. Now the area bounded by the two such lines/curves is defined as the distance between those two lines/curves forming a bounded surface. This observation makes the second example as a collection of lines/curves with some fixed end points and the distance is calculated between two such lines/curves.

With the above examples we shall move to answer the second question.

3. WHAT IS THE MEANING OF RESEARCH IN MATHEMATICS?

Research in Mathematics can be broadly for the following reasons.

- To appreciate the beauty of abstraction and the outcomes of such abstraction.
- To address a real-life problem.
- To answer a question out of curiosity.

We shall illustrate the first point by considering the example of abstracting the concept of distance in the previous Section. Now, to see an example of the type of research mentioned in the second point, we shall consider an example of the paper industry and the problems arising there. The complete process of making paper is explained as follows.

Wood (variety of Eucalyptus trees) harvesting - De-barking and chipping - Mechanical Pulping - Chemical Pulping - Cleaning - Paper making - Drying Coating Calendering - Finishing for end users - Recycling the used paper - Pulping - De-inking - Cleaning and Paper making.

In this process, a mathematical understanding at various levels can give profits in crores of rupees for the company. Some of the problems that mathematical formulations can address are as follows.

- Inventory of wood to reduce wastage and storage costs.
- Checking the quality of the paper in terms of even thickness of the produced paper.
- Reducing the trim loss in paper cutting and hence reducing the recycling cost.

The following example is a case of answering a curious mind, leading to a very useful subject of Mathematics called Graph Theory.

"The Königsberg bridge problem asks if the seven bridges of the city of Königsberg, formerly in Germany but now known as Kaliningrad and part of Russia, over the river, Preger can all be traversed in a single trip without doubling back, with the additional requirement that the trip ends in the same place it began. This is equivalent to asking if the multigraph on four nodes, see Figure 1 and seven edges has an Eulerian cycle. This problem was answered in the negative by Euler (1736) and represented the beginning of graph theory.", see [3].

Figure 1: Königsberg Bridges

The vertices (dots) represent the land mass and the edges (lines connecting the dots) represent the seven bridges of the city Königsberg. The reader can start at a vertex and try to traverse all the edges with doubling back and reach back to the starting vertex without removing the pen from the paper.

The above problem helps to understand and study various problems of computer networks and industrial optimization problems, etc. Now, we shall move to the third question of our discussion.

4. WHAT IS RESEARCH IN MATHEMATICS?

To address this, we shall restrict ourselves to just five points, just a drop of what to see from an ocean of applications.

1. Life-Saving applications: This includes the application of Mathematics to determine the expiry date of drugs, designing heart stents, designing the airbags of modern cars etc.,
2. Day-to-day applications: This includes assigning railway platforms to different trains, controlling traffic flow in metropolitan cities, etc., using the knowledge of graph theory.
3. Economics: This includes the application of game theory in studying the economic behaviour of different types of markets.
4. Engineering and Physics: This includes the application of calculus in rocket science or to any physical phenomenon. Further, all the laws of Physics are written in the language of Mathematics.
5. Banking: Modern security systems available for any money transaction are powered by the theory of Algebra, particularly Cryptography.

With this sample of applications, we shall move forward to the next question.

5. HOW TO TEACH MATHEMATICS?

The following points will also address how to approach and learn Mathematics. Teaching mathematics is very difficult. To show the students how we learn mathematics as teachers is better than a detailed lecture to ensure students' understanding. Hence, the best way is to do the following.

1. Ask and train the students to ask the right questions in a given situation.
2. Answer the questions and train the students to answer themselves. In this process, we shall also show them how to think by thinking in front of the students.
3. Further, help them to verify their answers.
4. Now, look to refine the original question and repeat the above process until a satisfactory understanding is achieved.

Creating examples and hunching possible generalizations of the original question is also very important to repeat the above process. Further, it is important to stress that it is impossible to learn without doing Mathematics at a personal level. An interested reader may look at the book by Pólya [2] for interesting discussions about learning Mathematics.

CONCLUDING REMARKS

Although the above discussion highlights a few points to ponder, unless someone really gets into Mathematics, it's nearly impossible to know what mathematics is or what Mathematicians do.

REFERENCES

1. Eugene Wigner (1960), “The unreasonable effectiveness of Mathematics in the natural sciences.” Communications in Pure and Applied Mathematics, 13(1), 1-14.
2. Pólya, George (1957). How to Solve It. Garden City, NY: Doubleday. p. 253
3. Weisstein, Eric W. "Königsberg Bridge Problem." From MathWorld--A Wolfram Web Resource.
4. https://mathworld.wolfram.com/KoenigsbergBridgeProblem.html