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Anisotropic Bianchi Type-1 Cosmological Model Containing Dark Energy

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Abstract
Considering the LRS Bianchi type I metric here we studied the spatial homogeneous and anisotropic dark energy cosmological models in the frame work of B-D scalar tensor theory of gravity along with variable equation of parameter and deceleration parameter. Exact solutions of field equations are obtained with certain physical assumptions in different cases. The physical behavior of the model is also discussed in details.

Keywords: Anisotropic; Bianchi; Type-I; Cosmological; Model; Dark Energy.

1. Introduction

In the recent years many prominent researchers have investigated and proposed different cosmological models and ideas of the universe in different context. Brans and Dicke have given an interesting alternative to general theory of relativity based on Mach's principle. This theory is the best and suited for a frame work in which the gravitational constant G arise from the structure of the universe. It means a changing gravitational constant may be looked upon as the Machian consequence of a dynamic universe. Later, Sciama had given general argument leading to a relation between gravitational constant (G) and the large scale structure of the universe. According to Brans and Dicke postulate G behave as the reciprocal of a scalar field means $G\approx\phi-1$, where is expected to satisfy a scalar wave equation which is spread all over the matter in the universe. Since this theory contain a scalar field in addition to the metric tensor $g\mu\nu$, the Brans and Dicke theory [1] is known as scalar-tensor theory of gravitation.

The Brans-Dicke field equations for combined scalar and tensor field may be written as

$G\mu\nu=R\mu\nu-Rg\mu\nu$=8π/c4ϕ T_μν-ω/ϕ^2 (ϕ_μ ϕ_ν-1/2 g_μν ϕ_η ϕ^(η))-1/(ϕ)(〖ϕ;〗 _μν-g_(μν)⊡ϕ)(1)

Where ⊡=1/c^2 ∂^2/〖∂t〗^2 -∂^2/〖∂x〗^2 -∂^2/〖∂y〗^2 -∂^2/〖∂z〗^2 =1/c^2 ∂^2/〖∂t〗^2 −∇2, ⊡−Wave operator and ∇−is Laplace's operator.

⊡ϕ=8π/ ((2ω+3)) T, (2)

Here $G\mu\nu=R\mu\nu-1/2Rg\mu\nu$ is Einstein tensor, R is a scalar curvature, ω is a dimension-less coupling constant, $T\mu\nu$ is stress-energy tensor, T is the trace of $T\mu\nu$, comma and semicolon denote partial and co-variant differential respectively. Also the law of conservation of energy may be written as

T_ (μϑ; ϑ) =0(3)

Several researchers have discussed the various aspects of Brans-Dicke cosmology in their research work as cited in their corresponding papers [2-6]. In recent research findings the discovery of the accelerated expansion of the universe provides a very good platform for observational Cosmology. Recent research data from various sources indicates the presence of a new un-accounted candidate for DE [7, 8] that opposes the self attraction of the matter and cause the expansion of the universe to accelerate. This acceleration is due to negative pressure and positive energy density. The current observations of large scale cosmic micro wave background radiation (CMB) suggest that our physical universe is expanding, isotropic and homogeneous with a positive cosmological constant. Analysis of WMAP data indicates that the universe could have a preferred direction [9, 10]. In recent work of Mishra et al. [11] have studied LRS Bianchi type-I models in-scalar tensor theory with energy momentum tensor for a bulk viscous fluid and concluded that at present the universe has a phase transition from early decelerating phase to accelerating phase at present epoch, which is also a good agreement with the recent observation. While that in a separate communication [12] the same authors have obtained Bianchi type -II by considering variable deceleration parameter. The motive of this study is to investigate the cosmological implications of the Bianchi type -I DE cosmological models in Branse-Dicke theory of gravity with variable deceleration parameter and dynamic ω term. The outline of the paper is as follows: In Sect. 2, the metric and field equations are described. Section 3 deals with the solutions of field equations by considering scale factors and power law relation. In Sect. 4, we have summarized the results, discussion and conclusion of the study.

2. Some important terms

2.1 Cosmology

Cosmology is a branch of science which deals with matter and its motion at large; it deals with the study of the universe on a large scale. The word cosmology is derived from Greek word Kosmos which has been revealed in the beauty of the sky.

Instead of confining ourselves to the study of our galaxy, we try to study, on a large scale, further space as far as the limits of the observational universe, some ten thousand million light year away on the very large time scale we look back into the past of the very fast moments of the initial expansion about twelve thousand million year ago. In prevailing theory about the origin and evolution of our universe is the Big-Bang theory. The central theme of modern cosmology is the idea that universe is expanding and that this implies that at some time in the distant part it incredibly dense and hot.

2.2 Cosmological Principle:

The hypothesis that the universe is homogeneous and isotropic is known as cosmological principle. The Cosmological principle consists of the basic belief that the fundamental laws of science are universal and two other simplifying assumptions about the nature of the universe when considered as a single entity. In case of Homogeneity of space, it is the belief that over the largest distance scales, matter and energy are distributed approximately uniformly. There is no preferred location in space. Also in case of Isotropy of space, it states that there is no preferred direction in space. So the hypothesis that the universe is homogeneous and isotropic is known as the Cosmological principle.

2.3 Cosmological Model:

Cosmological model of the universe is the model of the Universe with which we try to see to what extent this model resembles with the actual universe. One of the principal goals of cosmological models is to describe the different phases of evolution of the Universe. The first epoch is that of rapid expansion of the Universe, also known as inflationary period. Most of the theories describe this phase by means of a scalar field related to the hypothetic inflation. The next phase corresponds to the declaration when the matter and radiation dominate over the scalar field. The present era is characterized by the accelerated mode of expansion where dark matter and dark energy play the dominating role. By this acceleration we understand the acceleration that we observe at present time.

2.4 Big-Bang:

The Big Bang theory is the prevailing cosmological model for the universes from earliest known periods through its subsequent large-scale evolution. The model accounts for the fact that the universe expanded from a very high density and high temperature state, and offers a comprehensive explanation for the broad range of phenomena, including the abundance of light elements, the cosmic microwave background, large scale structure and Hubble's Law.

2.5 Dark Energy:

A hypothetical form of Energy, it is supposed, is spread uniformly throughout the space (and time) and has anti gravitational properties. It represents a possible mechanism for the Cosmological constant, and thus is one of the possible explanations for the current accelerating rate of expansion of the universe. And it is estimated to account for about 74 percent of the mass-energy of the universe. In the early 20th century the common world-view held that the Universe is static-more or less the same throughout eternity. Even Albert Einstein supported this long-standing idea, and in order to get the steady state Universe he introduced cosmological constant in his famous system. So, when in 1922 the Russian meteorologist and mathematician Alexander Friedmann had published a set of possible mathematical solutions that gave a non static Universe, Einstein rejected it nothing that this model was indeed a mathematically possible solution to the field equations. This model has gained big popularity only after the works of Robertson and Walker and became known as FRW model. This model describes a homogeneous and isotropy Universe. By homogeneity we mean that space has the same metric properties (measure) in all points, whereas by isotropy we mean that the space has the same measure in all directions. This idea of expanding Universe suggested the presence of an initial singularity, which means the finiteness of time. Though the idea of an expanding Universe was supported both theoretically and experimentally, it was strongly believed that the Universe is expanding with deceleration. So, in 1998, when it was found that the Universe is expanding with acceleration, it comes like a bolt from the blue. The observation of type Iasu-pernova (SNeIa) in 1998 established that our Universe is currently accelerating and recent observation of SNeIA of high confidence level have further confirmed this. In addition, measurements of the cosmic microwave background (CMB) and large scale structure (LSS) strongly indicate that our Universe is dominated by a component with negative pressure, dubbed as dark energy.

2.6 Cosmological Constant (Λ):

In the context of cosmological constant is a homogenous energy density that causes the expansion of the Universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the Universe was found to be expanding. Now the cosmological constant is invoked to explain the observed acceleration of the expansion of the Universe. The cosmological constant is the simplest realization of dark energy, which the more generic name is given to the unknown cause of the acceleration of the Universe.

2.7 Line Element:

A formula which express the distance between adjacent points is called a metric or line element. For example $ds^2 = dx^2 + dy^2 + dz^2$ is a line element. It expresses the distance between adjacent points (x, y, z) and (x+dx, y+dy, z+dz). More generally curvilinear co-ordinates u, v, w $ds^2 = adu^2 + bdv^2 + cdw^2 + 2fdvdw + 2gdwdu + 2hdudv$

Where a, b, c, ………, h are functions of co-ordinates u, v, w.

This idea was generalized and extended to a space of n dimensions by Riemann who defined the infinitesimal distance ds between adjacent points whose co-ordinates in a system are $x^i and\ x^i + dx^i$ by the formula $ds^2 = g_{ij}dx^i dx^j$ (where i, j =1, 2,….., n)

Where the co-efficient g_{ij} are functions of co-ordinates x^i.

2.8 Ricci Tensor:

In relativity theory, the Ricci tensor is the part of the curvature of space-time that determines the degree to which will tend to converge or diverge in time (via the Raychaudhuri equation). It is related to the matter content of the universe by means of the Einstein field equation. In differential geometry, lower bounds on the Ricci tensor on a Riemannian manifold allow one to extract global geometric and topological information by comparison with the geometry of a constant curvature space form. If the Ricci tensor satisfies the vacuum Einstein equation, then the manifold is an Einstein manifold, which have been extensively studied (cf, Besse 1987). In this connection, the Ricci flow equation governs the evolution of a given metric to an Einstein metric; the precise manner in which this occurs ultimately leads to the solution of the Poincare conjecture.

The Ricci tensor is defined as the contraction of the Riemann tensor in the following manner $R_{ij} = g^{\alpha\beta} R_{\alpha ij\beta} = R^{\beta}_{ij\beta}$.

2.9 Deceleration parameter:

The deceleration parameter q in cosmology is a dimensionless measure of the cosmic acceleration of the expansion of space in a Friedmann Lematre Robertson Walker universe.

It is defined by:

$$q = -\frac{\ddot{a}a}{\dot{a}^2}$$

Where a is the scale factor of the universe and the dots indicate derivatives by proper time. The expansion of the universe is said to be accelerating if $\ddot{a}$ is positive (recent measurement suggest it is), and in this case the deceleration parameter will be negative.

2.10 Hubble's Law:

Hubble's law is the name observation in physical cosmology that:

1. Object observed in deep space (extragalactic space, 10 mega par sec (Mpc or more) are found to have a Doppler shift interpretable as relative velocity away from Earth.

2. This Doppler-shift measured velocity, of various galaxies receding from the Earth, is approximately proportional to their distance from the Earth for galaxies up to a few hundred mega per sec away.
3. Hubble's law is considered the first observational basis for the expansion of the universe and today serves as one of the pieces of evidence most often cited in support of Big Bang model.

3. Field equations and their solutions

Here we consider the spatially homogenous anisotropic LRS Bianchi-I metric expressedas

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)(dy^2 + dz^2) \qquad (4)$$

Where A(t) and B(t) are the potential functions of cosmic time only along the direction of x and y-axes respectively.

The energy momentum tensor of fluid may be given as

$$T_\mu^v = diag[T_0^0, T_1^1, T_2^2, T_3^3] \qquad (5)$$

We can parameterize as

$$T_\vartheta^\mu = diag[\rho, -p_x, -p_y, -p_z]$$
$$= diag[1, -w_x, -w_y, -w_z]$$
$$= diag[1 - w, -(w + \delta), -(w + \delta)] \qquad (6)$$

Here p_x, p_y, p_z $and w_x, w_y, w_z$ are the directional pressure components and directional EoS parameters in the directional of x,y and z respectively. The EoS parameter $w = \frac{p}{\rho}$ is the deviation free parameter of the fluid and for isotropy we choose $w_x = w$. Also introducing the skewness parameter δ, which is the deviation from w along both y and z direction.

$$+\frac{\dot{B}^2}{B^2} + \frac{\omega\dot{\emptyset}^2}{2\emptyset^2} + \frac{2\dot{B}\dot{\emptyset}}{B\emptyset} + \frac{\dot{\emptyset}}{\emptyset} = -\frac{8\pi w\rho}{\emptyset} \qquad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\omega\dot{\emptyset}^2}{2\emptyset^2} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\frac{\dot{\emptyset}}{\emptyset} + \frac{\ddot{\emptyset}}{\emptyset} = -\frac{8\pi(W+\delta)\rho}{\emptyset} \qquad (8)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{\omega\dot{\emptyset}^2}{2\emptyset^2} + \left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right)\frac{\dot{\emptyset}}{\emptyset} = \frac{8\pi\rho}{\emptyset} (9)$$

$$\frac{\ddot{\emptyset}}{\emptyset} + \left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right)\frac{\dot{\emptyset}}{\emptyset} = \frac{8\pi}{(3+2\omega)}(1 - 3W - 2\delta)\rho (10)$$

By the law of conservation of energy-momentum tensor $T_{\mu v}$ for perfect fluid, we have $\dot{\rho} + \rho(1 + w)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) + 2\delta\rho\frac{\dot{B}}{B} = 0 \qquad (11)$

Here dot denotes the partial differential with respect to t. Now here we wish to define some other cosmological parameters as mention below.

The Hubble's parameter (H) may be expressed as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_x + H_y + H_z) = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \qquad (12)$$

Here, $H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{\boldsymbol{B}} = H_z$ are the directional Hubble's parameters along x,y and z axis. The expansion scalar θ, anisotropy parameter, shear scalar are define as

$$\theta = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \quad (13)$$

$$\sigma^2 = \frac{1}{2}\left(\sum H_i^2 - 3H^2\right) \quad (14)$$

$$A_m = \sum_{i=3}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \quad (15)$$

In next section we have obtain the solution of field equations and also calculate the values of above mention parameters by applying suitable assumptions.
As stated above the field equations (7-11) are having six unknowns i.e. A, B, ρ, δ, $w$and. For the explicit solution of this problem we are required more additional constraints associated to these parameters, for this purpose here we wish to calculate the variable deceleration parameter q is a function of 't' as suggested by Mishra et al.[12] i.e.

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = (t) \quad (16)$$

With a suitable assumption we have

$$a(t) = (t^k e^t)^{\frac{1}{n}} (17)$$

where k and n are positive constants. To avoid the complicacy of the problem we have decided to assume shear scalar σ is proportional to expansion parameter i.e. $\sigma \propto \theta$. This condition leads to

$$A = k_1 B^m \quad (18)$$

Here $k_1$is constant of proportionality.
Hence for simplicity and without loss generality we assumek_1= 1, therefore

$$A = B^m \quad (19)$$

The sources of motivation behind this assumption are the recent study done by several authors (Throne [13]).The observation of the velocity red-shift relation for extra galactic source indicates that the Hubble's expansion of the universe is isotropic at present within 30% (Kantowski et al. [14], Kristian et al. [15]).

$$A = (t^k e^t)^{\frac{3m}{n(m+2)}} \quad (20) \qquad B = (t^k e^t)^{\frac{3}{n(m+2)}} \quad (21)$$

Here m is the constant of proportionality. Now considering a well-accepted power law relation between scale factor (t) and scalar field, as suggested by Pimentel ([16], Johri & Kalyani [17], Ahmadi & Raizi [18];

$$\varphi(t) \alpha\, a(t) \quad (22)$$

$$\varphi(t) = \varphi_0 (a(t))^l \qquad (23)$$

Here $\emptyset_0$ is constant of proportionality.

$$\frac{\sigma}{H} = \frac{\sqrt{3}(m-1)}{m+2} \qquad (24)$$

Under motivation by recent observatory results regarding red-shift study we may place the limit $\frac{\sigma}{H} \leq 0.3$ on the ratio of shear scalar σ to H in the neighborhood of our galaxy at present time. Collins et al. [19] have suggested that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies the condition,

$$\frac{\sigma}{H} = \text{constant} \qquad (25)$$

4. Physical Interpretation of the Solutions

From the above solutions the expressions for physical parameters such as spatial volume parameter (V), Hubble's parameter (H), directional Hubble parameters (H_x, H_y) expansion scalar (θ), deceleration parameter (q), shear scalar (σ) and anisotropic parameter (A_m) are found as

$$V = (t^k e^t)^{\frac{3}{n}} \qquad (26)$$

$$H = \frac{1}{n}\left(1 + \frac{k}{t}\right) \qquad (27)$$

$$H_x = \frac{3m}{n(m+2)} . \left(1 + \frac{k}{t}\right) \qquad (28)$$

$$H_x = H_y = \frac{3m}{n(m+2)} . \left(1 + \frac{k}{t}\right) \qquad (29)$$

$$\theta = \frac{3}{n}\left(1 + \frac{k}{t}\right) \qquad (30)$$

$$q = -1 + \frac{nk}{(k+t)^2} \qquad (31)$$

$$\sigma^2 = \frac{3(m-1)^2}{n^2(m+2)^2}\left(1 + \frac{k}{t}\right)^2 \qquad (32)$$

$$A_m = 2\left(\frac{m-1}{m+2}\right)^2 \qquad (33)$$

The metric (1) becomes as

$$ds^2 = -dt^2 + (t^k e^t)^{\frac{6m}{n(m+2)}} dx^2 + (t^k e^t)^{\frac{6}{n(m+2)}} (dy^2 + dz^2) \qquad (34)$$

From equations (9), (21), (22), (23), we get the value of energy density parameter (ρ) EoS parameter (w) and skewness parameter , which are expressed in below

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$$\rho = \frac{\varphi_0}{16\pi}\left[\frac{18(2m+1) - b(\omega b - 6)(m+2)^2}{n^2(m+2)^2}\right] . (t^k e^t)^{\frac{b}{n}} \qquad (35)$$

$$\omega = \frac{-2\{b^2(m+2)^2(\omega+2) - 6b(m+2) + 54\}.(1+\frac{k}{t})^2 - kn\{(m+2)(b(m+2)+6)\}\frac{1}{t^2}}{18(2m+1) - b(\omega b - 6)(m+2)^2} \qquad (36)$$

$$\delta = \frac{1}{2} - \frac{(m+2)\{b^2(\omega+3)(m+2)-3b(2\omega+9)\}(1+\frac{k}{t})^2-\{(b^2+b)(m+2)+6\}\frac{1}{t^2}}{18(2m+1)-b(\omega b-6)(m+2)^2} \qquad (37)$$

From equation (26) we observed that the volume element V is zero at t = 0 and increasing with time. This show that the expansion of the universe that start from zero volume at t = 0 (i.e. singularity at t = 0). From eq.(31) we observed that $q>0$ if $t<\sqrt{nk}-k$ and $q<0$ if $t \geq \sqrt{nk}-k$. For better analysis we take $(t) = (te^t)^{\frac{1}{n}}$ when k = 1.Also for n > 1, q is positive, for small value of t and later on becomes negative (i.e at present), this type behavior of q show the transition phase i.e. early time deceleration and present time acceleration. So our model universe are very close resemblance with SNeIa supernova observational data suggested by (Riess et al. [8], Permuttter et al. [9]).

From the behavior of shear scalar σ with cosmic time t it seem that that the shear scalar is very large at t = 0 and converges to a small constant value at present time.

Also, from equation (35) the density of the fluid (ρ) is a decreasing function of time and it converges to a small positive value as $t \to \infty$ .Also from equation (36) the EoS parameter w is negative and increasing function of time and less than -1 so our model universe behaves like phantom energy type of dark energy, So our model universe supports and strengthen the recent observational findings.

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