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Secure Inverse Domination in the Corona and Lexicographic Product of Two Graphs

Jesica P. Dagodog[1], Enrico L. Enriquez[2], Grace M. Estrada[3], Marie Cris A. Bulay-og[4], Edward M. Kiunisala[5]

[1]MS Math, Department of Computer, Information Sciences, and Mathematics, School of Arts and Sciences, University of San Carlos, 6000 Cebu City, Philippines
[2]Full Professor, Department of Computer, Information Sciences, and Mathematics, School of Arts and Sciences, University of San Carlos, 6000 Cebu City, Philippines
[3]Associate Professor, Department of Computer, Information Sciences, and Mathematics, School of Arts and Sciences, University of San Carlos, 6000 Cebu City, Philippines
[4]Assistant Professor, Mathematics and Statistics Programs, University of the Philippines Cebu, 6000 Cebu City, Philippines
[5]Professor, Mathematics Department, College of Computing, Artificial Intelligence and Sciences, Cebu Normal University, 6000 Cebu City, Philippines

Abstract
As secure domination and inverse domination garnered attention from various researchers, the combination of the two also raised a certain amount of curiosity. This paper aimed to investigate the secure inverse domination in graphs which is defined as follows. Let G be a connected simple graph and let D be a minimum dominating set of G. A dominating set $S \subseteq V(G) \setminus D$ is an inverse dominating set of G with respect to D. The set S is called a secure inverse dominating set of G if for every $u \in V(G) \setminus S$, there exists $v \in S$ such that $uv \in E(G)$ and the set $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of G. The secure inverse domination number of G, denoted by $\gamma_s^{(-1)}(G)$, is the minimum cardinality of a secure inverse dominating set of G. A secure inverse dominating set of cardinality $\gamma_s^{(-1)}(G)$ is called $\gamma_s^{(-1)}-$set. Particularly, the researchers examined and provided the characterization of secure inverse dominating set in the corona and lexicographic product of two graphs in this study. Moreover, the secure inverse domination number of graphs under the binary operations corona and lexicographic product were determined.

Keywords: Dominating Sets, Corona of Two Graphs, Inverse Dominating Sets, Secure Dominating Sets, Lexicographic Product of Two Graphs

1. Introduction

A famous problem known as the "Königsberg Bridge Problem" challenged the Swiss mathematician, Leonhard Euler, in the 18th century which led to the development of graph theory [1]. Over the years, the study of graphs flourished and variations of the notions of graphs emerged. One of these notions was the domination in graphs. The term "domination" was first used by Oystein Ore in 1962 [2]. A nonempty subset S of a vertex set $V(G)$ is a dominating set of a graph G if every vertex $u \in (V(G) \setminus S)$ is adjacent to at least one vertex $v \in S$; that is, for every $u \in (V(G)\backslash S)$, there exists $v \in S$ such that $uv \in E(G)$. The smallest cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. Some studies on domination in graphs were found in the papers [3 - 20].

In 1991, a type of domination in graphs called inverse domination was introduced by Veerabhadrappa Kulli, et al. [21]. Let D be a minimum dominating set of G. A dominating set $S \subseteq V(G)\backslash D$ is called an inverse dominating set of G with respect to D. The smallest cardinality of an inverse dominating set in G is called the inverse domination number of G and is denoted by $\gamma^{(-1)}(G)$. Related studies on inverse domination in graphs can be read in some papers [22-32].

Another type of domination in graphs known as secure domination was established in 2003 by Ernest J Cockayne, et al. [33]. A dominating set $S \subseteq V(G)$ is a secure dominating set of a graph G if for each $u \in V(G)\backslash S$, there exists $v \in S$ such that $uv \in E(G)$ and the set $(S\backslash\{v\}) \cup \{u\}$ is a dominating set in G. The minimum cardinality of a secure dominating set of G, denoted by $\gamma_s(G)$, is called the secure domination number of G. Some related studies on secure domination in graphs can be found in papers [34-40].

This study, is an extension of the paper [41]. In this regard, the researchers explore the secure inverse domination in the corona and lexicographic product of two graphs. Let D be a minimum dominating set in a graph G. Then a dominating set $S \subseteq V(G)\backslash D$ is an inverse dominating set in G with respect to D. The set S is called a secure inverse dominating set in G if for every $u \in V(G)\backslash S$ there exists $v \in S$ such that $uv \in E(G)$ and the set $(S\backslash\{v\}) \cup \{u\}$ is a dominating set in G. The secure inverse domination number of G, denoted by ${\gamma_s}^{(-1)}(G)$, is the minimum cardinality of a secure inverse dominating set of G. A secure inverse dominating set of cardinality ${\gamma_s}^{(-1)}(G)$ is called a ${\gamma_s}^{(-1)}$-set. In this paper, the researchers determine the secure inverse domination number of the corona and lexicographic product of two graphs. For the general terminology in graph theory, readers may refer to [42].

2. Results

Remark 2.1 Let D be a minimum dominating set of G. Then $V(G) \setminus D$ is a secure dominating set of G, that is, $V(G) \setminus D$ is a secure inverse dominating set of G.

In Remark 2.1, $V(G) \setminus D$ can be an inverse secure dominating set of G if D is a secure dominating set of G. Hence, every inverse secure dominating set is a secure inverse dominating set, however, the converse is not always true. For example, in $P_5 = [x_1, x_2, \dots, x_5]$, the set $D = \{x_1, x_4\}$ is a minimum dominating set of P_5 and $S = V(P_5) \setminus D = \{x_2, x_3, x_5\}$ is an inverse dominating set with respect to D. Since S is a secure dominating set, it follows that S is a secure inverse dominating set of P_5. However, it is not an inverse secure dominating set of G because D is not a secure dominating set of G. The following definitions are needed for the subsequent results.

Definition 2.2 A nonempty subset S of $V(G)$, where G is any graph, is a clique in G if the graph $\langle S \rangle$ induced by S is complete.

Definition 2.3 The corona of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the i-th copy of H. For every $v \in V(G)$, we denote by H^v the copy of H whose vertices are joined or attached to the vertex v.

Remark 2.4 Let G and H be nontrivial connected graphs. Then $V(G)$ is a minimum dominating set of $G \circ H$.

Remark 2.5 Let G and H be nontrivial connected graphs. Then $\gamma_s(G \circ H) = |V(G)| \cdot \gamma_s(H)$.
The following result is the characterization of the secure inverse dominating set in the corona of two graphs.

Theorem 2.6 Let G and H be nontrivial connected graphs. Then a subset $S \subseteq V(G \circ H) \setminus D$, is a secure inverse dominating set of $G \circ H$ with respect to D if and only if one of the following statements holds.

1. $D = V(G)$ and $S = \bigcup_{v \in V(G)} S_v$ where S_v is a secure dominating set of H^v for each $v \in V(G)$.
2. $D = \bigcup_{v \in V(G)} D_v$ where $D_v = \{x\}$ is a dominating set of H^v for each $v \in V(G)$, and $S = V(G)$ with H is complete or $S = \bigcup_{v \in V(G)} S_v$ where $S = S_v \subseteq V(H^v) \setminus D_v$ is a secure dominating set of H^v for each $v \in V(G)$.

Proof: Suppose that a subset $S \subseteq V(G \circ H) \setminus D$ is a secure inverse dominating set of $G \circ H$ with respect to D. Consider the following cases.
Case 1. If $D = V(G)$, then $S \subseteq \bigcup_{v \in V(G)} V(H^v)$, since $S \subseteq V(G \circ H) \setminus D = \bigcup_{v \in V(G)} V(H^v)$. If $S = \bigcup_{v \in V(G)} V(H^v)$, then S is a secure inverse dominating of $G \circ H$ with respect to D (trivial). If $S = \bigcup_{v \in V(G)} S_v$, then let $S_v \subset V(H^v)$ for each $v \in V(G)$ such that $S = \bigcup_{v \in V(G)} S_v$. Suppose S_v is not a secure dominating set of H^v for each $v \in V(G)$. Then for each $v \in V(G)$, there exists $u \in V(H^V) \setminus S_v$ such that for all $x \in S_v$, $ux \notin E(H^v)$ or $(S_v \setminus \{x\}) \cup \{u\}$ is not a dominating set of H^v. This implies that S is not a secure dominating set of $G \circ H$, a contradiction. Thus, S_v must be a secure dominating set of H^v for each $v \in V(G)$. This satisfies statement (i).
Case 2. If $D \neq V(G)$, then $D \subset \bigcup_{v \in V(G)} V(H^v)$ and $D \neq \bigcup_{v \in V(G)} V(H^v)$ (since D is a minimum dominating set). Let $D_v \subset V(H^v)$ for each $v \in V(G)$ such that $D = \bigcup_{v \in V(G)} D_v$. Then $D_v = \{x\}$ is a dominating set of H^v for each $v \in V(G)$, otherwise, D is not a minimum dominating set of $G \circ H$, a contradiction of the definition of S. If $S = V(G)$ and suppose that H is not complete, then S is not a secure dominating set of $G \circ H$, a contradiction. Thus, H must be a complete graph, showing statement (ii). If $S \neq V(G)$, then $S \subseteq \bigcup_{v \in V(G)} V(H^v) \setminus D_v$. Let $S_v \subseteq V(H^v) \setminus D_v$ such that $S = \bigcup_{v \in V(G)} S_v$. Suppose S_v is not a secure dominating set of H^v for each $v \in V(G)$. Then for eachv $\in V(G)$, there exists $u \in V(H^v) \setminus S_v$ such that for all $x \in S_v$, $ux \notin E(H^v)$ or $(S_v \setminus \{x\}) \cup \{u\}$ is not a dominating set of H^v. This implies that S is not a secure dominating set of $G \circ H$, a contradiction. Thus, S_v must be a secure dominating set of H^v for each $v \in V(G)$. This satisfies statement (ii).
For the converse, suppose that statement (i) is satisfied. Then $D = V(G)$ is a minimum dominating set of $G \circ H$ by Remark 2.4 and $S \subseteq V(G \circ H) \setminus D = \bigcup_{v \in V(G)} V(H^v)$ is the inverse dominating set of $G \circ H$ with respect to D. Since $S = \bigcup_{v \in V(G)} S_v$ where S_v is a secure dominating set of H^v for each $v \in V(G)$, S is a secure dominating set of $G \circ H$. This implies that a subset $S \subseteq V(G \circ H) \setminus D$ is a secure inverse dominating set of $G \circ H$ with respect to D.
Now, suppose that statement (ii) is satisfied. Then $D = \bigcup_{v \in V(G)} D_v$ where $D_v = \{x\}$ is a dominating set of H^v for each $v \in V(G)$, that is,

$$|D| = \left|\bigcup_{v\in V(G)} D_v\right| = \sum_{v\in V(G)} |D_v| = |V(G)| \cdot |D_v| = |V(G)| \cdot 1 = |V(G)|.$$

Thus, $|D| = |V(G)|$ implies that D is a minimum dominating set of $G \circ H$ (Remark 2.4) and $S \subseteq V(G \circ H) \setminus D$ is an inverse dominating set of $G \circ H$ with respect to D. Now, $V(G \circ H) \setminus D = V(G \circ H) \setminus \left(\bigcup_{v\in V(G)} D_v\right) = V(G) \cup \bigcup_{v\in V(G)}(V(H^v) \setminus D_v)$. Let $S_v \subseteq V(H^v) \setminus D_v$ such that $S \subseteq V(G) \cup (\bigcup_{v\in V(G)} S_v)$. If $S = V(G)$ with H is complete then S is a secure dominating set of $G \circ H$ (for each $u \in V(G \circ H) \setminus S = \bigcup_{v\in V(G)} V(H^v)$, there exists $v \in S$ such that $uv \in E(G \circ H)$ and $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of $G \circ H$). Thus, a subset $S \subseteq V(G \circ H) \setminus D$ is a secure inverse dominating set of $G \circ H$ with respect to D. If $S = \bigcup_{v\in V(G)} S_v$ where $S_v \subseteq V(H^v) \setminus D_v$ is a secure dominating set of H^v for each $v \in V(G)$, then S is a secure dominating set of $G \circ H$. Thus, a subset $S \subseteq V(G \circ H) \setminus D$ is a secure inverse dominating set of $G \circ H$ with respect to D. □

The following result is a quick consequence of Theorem 2.6.

Corollary 2.7 Let G and H be nontrivial connected graphs with $m = |V(G)|$. Then ${\gamma_s}^{(-1)}(G \circ H) = m \cdot \gamma_s(H)$.

Proof: Suppose that S_v is a secure dominating set of H^v for each $v \in V(G)$. Let $D = V(G)$ and $S = \bigcup_{v\in V(G)} S_v$ for each $v \in V(G)$. By Theorem 2.6, a subset $S \subseteq V(G \circ H) \setminus D$ is a secure inverse dominating set of $G \circ H$ with respect to D. Thus,

$${\gamma_s}^{(-1)} \leq |S| = \left|\bigcup_{v\in V(G)} S_v\right| = \sum_{v\in V(G)} |S_v| = |V(G)| \cdot |S_v| = m \cdot |S_v|.$$

for all $S_v \in V(H^v)$, that is, ${\gamma_s}^{(-1)}(G \circ H) \leq m \cdot \gamma_s(H)$. Note that m is the order of G and $\gamma_s(H)$ is a minimum secure dominating set of H. Thus, $m \cdot \gamma_s(H) = \gamma_s(G \circ H)$ by Remark 2.5. Hence, $m \cdot \gamma_s(H) = \gamma_s(G \circ H) \leq {\gamma_s}^{(-1)}(G \circ H) \leq m \cdot \gamma_s(H)$ implies that ${\gamma_s}^{(-1)}(G \circ H) = m \cdot \gamma_s(H)$. □

Definition 2.8 The lexicographic product of two graphs G and H, denoted by G[H], is the graph with $V(G[H]) = V(G) \times V(H)$ and edge set $E(G[H])$ satisfying the following conditions: $(u_1v_1)(u_2v_2) \in E(G[H])$ if either $u_1u_2 \in E(G)$ or $u_1 = u_2$ and $v_1v_2 \in E(H)$.

Remark 2.9 Let $G = P_m$, $m \equiv 0 (\mathrm{mod}\ 2)$ and $H = K_2$. The nonempty set $X \times \{u\}$ is a minimum dominating set of G[H] if X is a minimum dominating set of G and $u \in V(H)$.

Proposition 2.10 Let $G = P_m$, $m \equiv 0 (\mathrm{mod}\ 2)$, $m \neq 0$, and $H = K_2$. Then

$${\gamma_s}^{(-1)}(G) = \frac{m}{2}\,.$$

Proof: Consider that $V(G) = \{v_1, v_2, \dots, v_m\}$ and $S = \{v_2, v_4, \dots, v_m\} \subset V(G)$. Let $v \in V(G) \setminus S = \{v_1, v_3, \dots, v_{m-1}\}$. Then there exists $v' \in S$ such that $vv' \in E(G)$ and $(S \setminus \{v'\})$ is a dominating set of G. Hence, S is a secure dominating set of G. Let $v' \in S$ and consider $S \setminus \{v'\}$. Clearly, $S \setminus \{v'\} = \{v_2, v_4, \dots, v_m\} \setminus \{v'\}$ is not a dominating set of G for any $v \in S$. Consider that $|S'| = |S \setminus \{v'\}|$. If $|V(G)| = 2$, then $|S'| = 0$, that is, S' is not a dominating set of G. If $|V(G)| = 4$, then $|S'| = 1$, that is, S' is not a dominating set of G. If $|V(G| = 6$, then $|S'| = 2$, that is, $S' = \{v_2, v_5\}$ is a minimum dominating set of G, but S' is not a secure dominating set of G since $(S' \setminus \{v_5\}) \cup \{v_6\}$ is not a dominating set of G. Similarly, for $|V(G)| \geq 8$ where m are all even integers, S' is not a secure dominating set of G. Since S is a secure dominating set of G and $|S| - 1$ is not a secure dominating set of G, it follows that $S = \{v_2, v_4, \dots, v_m\}$ is a minimum secure dominating set of G, that is, $\gamma_s{}^{(-1)}(G) = |S| = \frac{m}{2}$. □

The following result is the characterization of the secure inverse dominating set in the lexicographic products of two graphs.

Theorem 2.11 Let $G = P_m$, $m \geq 2$ and $H = K_2 = \{u_1, u_2\}$. Then $S \subseteq V(G[H]) \setminus D$ is a secure inverse dominating set of G[H] with respect to a minimum dominating set D of G[H], if $D = A \times \{u\}$ where A is a minimum dominating set of G, $u \in V(H)$ and one of the following is satisfied.

(i) $S = (V(G) \setminus A) \times \{u\}$.
(ii) $S = S' \times \{u\}$ where $S' \subset (V(G) \setminus A)$ and S' is a secure dominating set of G.
(iii) $S = [(V(G) \setminus A) \times \{u\}] \cup (S' \times \{u'\})$ where $u' = u$ and $\emptyset \subset S' \subseteq V(G)$.

Proof: If $D = A \times \{u\}$ where A is a minimum dominating set of G, $u \in V(H)$, then D is a minimum dominating set of G[H] by Remark 2.9. Suppose that statement (i) is satisfied. Then $S = (V(G) \setminus A) \times \{u\}$ and clearly, S is a dominating set of G[H]. Since

$$\begin{aligned} V(G[H]) \setminus D &= V(G[H]) \setminus (A \times \{u\}) \\ &= [(V(G) \setminus A) \times \{u\}] \cup [V(G) \times \{u'\}] \end{aligned}$$

where $H = [u, u']$, it follows that $S = (V(G) \setminus A) \times \{u\} \subset V(G[H]) \setminus D$ is an inverse dominating set of G[H] with respect to D. Since A is a minimum dominating set of G, $V(G) \setminus A$ is a secure dominating set of G by Remark 2.1. Thus, if $S = (V(G) \setminus A) \times \{u\}$, $u \in V(H)$, then $S \subseteq V(G[H]) \setminus D$ is a secure inverse dominating set of G[H] with respect to a minimum dominating set D of G[H].

Suppose that statement (ii) is satisfied. Then $S = S' \times \{u\}$ where $S' \subset (V(G) \setminus A)$ and S' is a secure dominating set of G. Now, if $S = S' \times \{u\}$ where $S' \subset (V(G) \setminus A)$, then $S \subset (V(G) \setminus A) \times \{u\} \subset V(G[H]) \setminus D$. Thus, S is an inverse dominating set of G[H] with respect to D. Given that S' is a secure dominating set of G and $S = S' \times \{u\}$, let $v \in V(G) \setminus S$. Then there exists $v' \in S'$ such that $vv' \in E(G)$ and $(S' \setminus \{v'\}) \cup \{v\}$ is a dominating set of G (since S' is a secure dominating set of G). Thus, if $(v, u) \in V(G[H]) \setminus S$, then there exists $(v', u) \in S$ such that $(v, u)(v', u) \in E(G[H])$ and $(S \setminus \{(v', u)\}) \cup \{(v, u)\}$ is a dominating set of G[H]. Hence, S is a secure dominating set of G[H]. Therefore, if $S = S' \times \{u\}$ where $S' \subset (V(G) \setminus A)$ and S' is a secure dominating set of G, then

$S \subseteq V(G[H]) \setminus D$ is a secure inverse dominating set of $G[H]$ with respect to a minimum dominating set D of $G[H]$.

Suppose that statement (iii) is satisfied. Since in statement (i), $(V(G) \setminus A) \times \{u\}$, $u \in V(H)$ is a secure inverse dominating set of $G[H]$, it follows that if $S = [(V(G\setminus) \setminus A) \times \{u\}] \cup (S' \times \{u'\})$ where $u = u'$ and $\emptyset \subset S' \subseteq V(G)$, then $S \subseteq V(G[H])$ is a secure inverse dominating set of $G[H]$ with respect to a minimum dominating set D of $G[H]$. □

The following result is an immediate consequence of Theorem 2.11.

Corollary 2.12 Let $G = P_m$, $m \equiv 0 (\text{mod } 2)$, $m \neq 0$, and $H = K_2 = \{u_1, u_2\}$. Then

$$\gamma_s{}^{(-1)}(G[H]) = \frac{m}{2}.$$

Proof: Let D' be a minimum dominating set of G. Then $D = D' \times u_1$ is a minimum dominating set of $G[H]$ by Remark 2.9. Suppose that $S = S' \times \{u_2\}$ and $S' \subset V(G) \setminus A$ is a secure dominating set of G where A is a minimum dominating set of G. Then by Theorem 2.11(ii), $S \subseteq V(G[H]) \setminus D$ is a secure inverse dominating set of $G[H]$ with respect to a minimum dominating set D of $G[H]$. Thus,

$$\gamma_s{}^{(-1)}(G[H]) \leq |S| = |S' \times \{u_2\}| = |S'| \cdot |\{u_2\}| = |S'| \cdot 1 = |S'|$$

for all secure dominating set S' of G. This implies that $\gamma_s{}^{(-1)}(G[H]) \leq \gamma_s(G)$. By Proposition 2.10, $\gamma_s{}^{(-1)}(G) = \frac{m}{2}$. Since $\frac{m}{2} = \gamma_s{}^{(-1)}(G) \leq \gamma_s{}^{(-1)}(G[H]) \leq \gamma_s{}^{(-1)}(G) = \frac{m}{2}$, it follows that $\gamma_s{}^{(-1)}(G[H]) = \frac{m}{2}$. □

3. Conclusion

An extension of the study of secure inverse domination in graphs was established in this paper. In particular, the secure inverse dominating sets in the corona and lexicographic product of two graphs were characterized. Moreover, the secure inverse domination number resulting from the corona and lexicographic product of two graphs were computed. This study will contribute to the development of more concepts in domination theory and lay foundation to new researches such as investigating the properties of secure inverse dominating sets in directed graphs. Identifying the characterization of secure inverse domination in graphs under other binary operations is also a good extension of this study. Moreover, combining secure inverse domination with other parameters can also be explored.

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