Numerical Analysis of MHD Casson Fluid Flow Over a Stretching Surface with Chemical Reaction and Variable Thermal Conductivity

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ABSTRACT
The motivation of the current article is to explore the Numerical analysis of MHD Casson fluid flow over a Stretching surface. Three dimensional, non-linear of an incompressible fluid flow in the presence of chemical reaction and variable thermal conductivity is considered. The mathematical model effectively describes the current flow analysis. The Governing equations are simplified with the help of boundary layer approximations. Non-linear coupled equations for momentum, energy and mass transfer are tackled with MATLAB bvp4c. The behavior of various non-dimensional parameters which are involved in the derived set of equations are described and explained in detailed through graph.

Keywords: Casson fluid, non-newtonian fluid, Magnetohydrodynamics (MHD), Casson parameter, Chemical reaction, Magnetic field, Variable thermal conductivity.

1. INTRODUCTION
In recent year, non-Newtonian liquids have become substantially more significant because of its modern and designing applications. Truth be told, the interest in boundary layer flows of non-Newtonian liquid is significantly because of its huge number of functional applications in Manufacturing process, natural liquids. A few models identified with applications are penetrating mud, hot rolling, cooling metallic plates, polymer plates and so on. Irfan and Farooq [1] researched the effects of variable viscosity and thermal conductivity over an exponentially stretching sheet in MHD free stream Newtonian fluid with internal heat generation/absorption in a porous medium. Boundary value problems for ODE in MATLAB using bvp4c was investigated by Shampine et al.,[2]. Ibrahim and Suneetha [3] reported MHD stagnation point flow on a linear stretching sheet in porous medium channel with thermal radiation, heat generation, variable thermal conductivity and mass transfer using Shooting technique. Dual solutions of MHD three dimensional Casson fluid flow over a stretching sheet was analyzed by Anantha Kumar et al., [4]. Krishnamurthy [5] observed three dimensional incompressible non-newtonian fluid flow over an unsteady exponentially stretching sheet. Similarity solution of three-dimensional boundary layer flow was elaborated by Mohan babu and Pandurangappa [6] using numerical technique. Nadeem et al., [7] has been investigated the three- dimensional Casson fluid flow due to a porous linearly stretching sheet utilizing Adomian decomposition method (ADM). A numerical study made for MHD Casson nanofluid flow with thermal radiation, convective and slip boundary condition was examined by Oyelakin et al., [8]. An analytical solution of Heat and Mass transfer Casson nanofluid

The main emphasis of this study is to discuss the numerical solutions of chemical reaction on MHD Non-Newtonian three dimensional Casson fluid flow over a stretching surface with heat and Mass transfer. MATLAB bvp4c solver has been used for solving the nonlinear differential equations. The flow fields for heat and mass transfer are significantly affected by the non-dimensional physical parameters.

2. MATHEMATICAL FORMULATION

The three-dimensional nonlinear hydromagnetic flow of an incompressible Casson fluid flow over a stretching surface with existence of variable thermal conductivity is considered. The cartesian coordinate axes are (x, y, z) considered with corresponding velocities (u, v, w) and the sheet has stretched along x, y
plane at $z = 0$. The uniform magnetic field $B_0$ is applied towards the positive direction of $z$ axis. Assume that the stretching velocities of the surface along $x$ and $y$ directions be $u_w = ax$ and $v_w = by$ respectively.

Under the above assumptions, the governing equations of continuity equation, momentum equation, energy equation and concentration equation for the boundary layer flow can be expressed as:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

Momentum Equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial z^2} - \frac{\sigma^* B_0^2}{\rho} u - \frac{u - u}{K} \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 v}{\partial z^2} - \frac{\sigma^* B_0^2}{\rho} v - \frac{v - v}{K} \quad (3)$$

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{1}{\rho c_p} \frac{\partial}{\partial z} \left(k'(T) \frac{\partial T}{\partial z}\right) + \frac{Q}{\rho c_p} (T - T_\infty) \quad (4)$$

Concentration Equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \frac{\partial^2 C}{\partial z^2} - K_r (C - C_\infty) \quad (5)$$

where $u$, $v$, and $w$ are the velocity components along the $x$, $y$, and $z$ directions, respectively. $\nu = \frac{\mu_B}{\rho}$ is the kinematic viscosity coefficient, $\beta = \mu_B \sqrt{\frac{2\pi \rho}{\nu}}$ is the Casson fluid parameter, $\rho$ is the density of the fluid, $\sigma^*$ is the electrical conductivity. Where $T$ is the fluid temperature, $T_\infty$ the ambient temperature, $T_w$ is the fluid temperature of the wall, $C_w$, $C_\infty$ are near and far away the fluid concentration. $Q$ is the volumetric heat generation/absorption coefficient.

Temperature dependent thermal conductivity is stated as,

$$k'(T) = k_\infty (1 + \epsilon \theta)$$

Where $k_\infty$ is the fluid free stream conductivity and $\epsilon$ is a small parameter.

The boundary conditions for the considered flow problem are

$$u = ax, v = by, w = 0, -k \frac{\partial T}{\partial z} = h_w (T_w - T), \quad \text{at} \quad z = 0$$

$$-D \frac{\partial C}{\partial z} = h_s (C_w - C)$$

$$u = 0, \quad v = 0, \quad T \to T_w, \quad C \to C_\infty \quad \text{as} \quad z \to \infty$$

The following similarity transformations have used to convert the above partial differential equations (1)- (5) into the set of ordinary differential equations by using the similarity variable $\eta$, dimensionless stream functions $f(\eta)$, $g(\eta)$, dimensionless temperature $\theta(\eta)$ and dimensionless concentration $\varphi(\eta)$.
\[ \eta = \sqrt[\beta]{u = ax f'(\eta)}, v = ay g'(\eta) \]  

\[ w = -\sqrt{a}(f(\eta) + g(\eta)), \theta(\eta) = \frac{T - T_{w}}{T_{w} - T_{c}}, \phi(\eta) = \frac{C - C_{w}}{C_{w} - C_{c}} \]  

The governing boundary layer equations (2)-(5) take the following forms:

\[ \left(1 + \frac{1}{\beta}\right)f'' + (f + g)f' - f' - \left(M^2 + \frac{1}{\lambda}\right)f' = 0 \]  

(8)

\[ \left(1 + \frac{1}{\beta}\right)g'' + (f + g)g' - g' - \left(M^2 + \frac{1}{\lambda}\right)g' = 0 \]  

(9)

\[ (1 + \varepsilon \theta)\theta' + \varepsilon \theta'' + Pr(f + g)\theta' + Pr B\theta = 0 \]  

(10)

\[ \phi' + Sc\left[(f + g)\phi' - \gamma\phi\right] = 0 \]  

(11)

The relevant boundary conditions are

\[ f(0) = 0, g(0) = 0, f'(0) = 0, g'(0) = 0, \theta(0) = 1, \phi(0) = 1 \text{ at } \eta = 0 \]  

\[ f'(\infty) \rightarrow 0, g'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \text{ at } \eta \rightarrow \infty \]  

(12)

Where the corresponding non-dimensional parameters \( M^2, \lambda, \alpha, \) Pr, B, \( Sc, \gamma, \) denote the Hartman number, Porosity parameter, Ratio parameter, Prandtl number, Heat generation/absorption coefficient, Schmidt number and Chemical reaction parameter respectively.

\[ M^2 = \frac{\sigma B^2}{a \rho}, \quad \gamma = \frac{K}{a}, \quad Sc = \frac{\nu}{D_{b}}, \quad \lambda = \frac{Ka}{\nu}, \quad \alpha = \frac{b}{a}, \quad Pr = \frac{\nu}{\sigma}, \quad B = \frac{Q}{a \rho c_{p}} \]

3. SKIN FRICTION, HEAT AND MASS TRANSFER COEFFICIENTS

The Physical quantities of interest are the skin-friction coefficients, the local Nusselt number, the local Sherwood number along x and y directions are defined as follows:

\[ C_{fx} = \frac{\tau_{w,x}}{\rho u_{w}^{2}}, \quad C_{fy} = \frac{\tau_{w,y}}{\rho u_{w}^{2}} \]  

(13)

\[ Re_{x}^{\frac{1}{2}} C_{fx} = \left(1 + \frac{1}{\beta}\right)f''(0) \]  

(14)

\[ Re_{y}^{\frac{1}{2}} C_{fy} = \left(1 + \frac{1}{\beta}\right)g''(0) \]  

(15)

\[ Nu_{x} = \frac{-x q_{w}}{k(T_{w} - T_{c})} \]  

(16)

\[ Re_{x}^{\frac{1}{2}} Nu_{x} = -\theta'(0) \]

\[ Sh_{x} = \frac{x j_{w}}{D(C_{w} - C_{c})} \]  

(17)

\[ Re_{x}^{\frac{1}{2}} Sh_{x} = -\phi'(0) \]
Where $Re_x = \frac{u_x X}{v}$ is the Reynold’s number.

4. METHODOLOGY
The framework of MHD Casson fluid flow over a stretching surface with existence of chemical reaction and variable thermal conductivity model are highly non-linearity and simultaneous boundary value problem. First, we reduce the higher order differential equations into first order differential equations by taking addition variables. Using MATLAB bvp4c numerical procedure, the above system of equations is solved.

5. RESULTS AND DISCUSSION
Equations (8) – (11) depending on the boundary conditions (12) are solved by employing the numerical technique. This part is shown to captivate the focus of researchers by investigating the impact of adopted parameters on transport equations. The influence of various parameters namely, Casson Parameter ($\beta$), Hartmann number ($M^2$), ratio parameter ($\alpha$), Porosity parameter ($\lambda$), Thermal conductivity parameter ($\epsilon$), chemical reaction parameter ($\gamma$) and Schmidt number ($Sc$). The obtained computational results are presented graphically in Fig.1-16. The numerical values for the parameters are taken to be fixed as: $\beta = 1.5, \alpha = 0.5, M = 0.6, B = 0.4, \gamma = 0.3, Sc = 0.6, Pr = 0.9, \epsilon = 0.7$. 

![Graph 1](image1.png)

![Graph 2](image2.png)

![Graph 3](image3.png)

![Graph 4](image4.png)
Fig. 5 Upsilon of β on θ(η)

Fig. 6 Upsilon of β on θ(η)

Fig. 7 Upsilon of γ on φ(η)

Fig. 8 Upsilon of Sc on φ(η)

Fig. 9 Upsilon of M Vs β on Skin-Friction

Fig. 10 Upsilon of M Vs β on Skin-Friction

Fig. 11 Upsilon of M Vs β on Nusselt Number

Fig. 12 Upsilon of M Vs β on Sherwood Number
Fig. 1 and Fig. 2 are made to contemplate the influence of Hartmann Number (M) and Casson parameter (β) on velocity profiles. Due to Lorentz force has a tendency to lessen the momentum boundary layer thickness. It provides resistance to the flow field so that the velocity profile reduced for higher values of M. In Fig. 2 it is seen that when the Casson parameter (β) is sufficiently huge, the non-newtonian behaviors disappear, and the fluid acts like a Newtonian fluid. In view of that an implement in Casson parameter (β) makes both velocity boundary layer thickness shorter.

Fig. 3–Fig. 6 are displayed to investigate the impacts of ratio parameter(α), Porosity parameter(λ), Heat generation/absorption parameter(B) and thermal conductivity parameter(ε) on Temperature profiles. We tracked down that the higher values of α and λ diminishes the both temperature and thermal boundary layer thickness. In Fig. 5 temperature is rising function for Heat generation/absorption parameter (B). The plots of temperature distribution in Fig. 6 depict the influence of variable thermal conductivity (ε). The presence of variable conductivity causes a rise in temperature across the fluid layers as expected since increase in thermal conductivity of the fluid leads to further heating of the fluid resulting in higher temperature.

The nature of chemical reaction parameter (γ) and Schmidt Number (Sc) on dimensionless concentration profiles have been displayed in the Figs. 7 & 8. It is noteworthy that a larger value of Sc and γ reduces the concentration flow fields. This causes the species buoyancy effects to diminish yielding a reduction in the fluid concentration and simultaneous reductions in the concentration boundary layer thickness.

Fig. 9–Fig. 12 are drawn to discuss the effect of M over the Skin friction coefficient, Nusselt and Sherwood number against Casson parameter (β). We found that bigger values of M leads to increases the non-dimensional skin friction and Sherwood number profiles and an opposite behavior made for Nusselt number. Fig. 13 is attracted to talk about the impact of chemical reaction parameter (γ) over the Sherwood number (Sh) against Schmidt number (Sc). We tracked down that an increment in γ reduces the Mass transfer rate profile.

6. CONCLUSION
The present study deals with numerical analysis of three dimensional, steady non-linear incompressible Casson fluid flow over a stretching surface in porous medium channel with chemical reaction and variable thermal conductivity. A parametric study on dimensionless velocity, temperature, concentration, skin-friction coefficient, Heat and Mass transfer rate are carried out. From the results of the present investigation some of the significant conclusions are drawn as below:

![Graph showing the influence of Hartmann Number (M) and Casson parameter (β) on velocity profiles.](image-url)
• Dimensionless velocity and Momentum boundary layer thickness gets decelerated due to Lorentz Force for higher values of M. An implement in Casson parameter (β) makes both velocity boundary layer thickness shorter.

• The higher values of α and λ diminishes the both temperature and thermal boundary layer thickness. An expansion in B and ε prompts an increment in the temperature flow field.

• A larger value of Sc and γ reduces the Concentration flow fields. This causes the Species buoyancy effects to diminish yielding a reduction in the fluid concentration and simultaneous reductions in the concentration boundary layer thickness.

• Larger values of M Vs β leads to increases the non- dimensional skin friction and Sherwood number profiles but an opposite behavior made for Nusselt number profile.

• The rate of mass transfer was found to be decreasing function of chemical reaction parameter (γ) Vs Schmidt number (Sc).

REFERENCES


