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A Closed Burst Error Correcting Code with Cantor Fractal Approach

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Abstract
The nature of the errors depends on the behaviour of channel, there is a need to study different types of error pattern. This paper explores the new type of error called closed burst and obtained the possibilities of the existence of two types of block wise burst error correcting codes of length 3^N and 3N respectively with cantor fractal approach over GF(2). The code envisaged are meant to correct all the closed bursts of length b_i in the i^{th}iteration in the first and last sub-blocks of equal length l_i.

Keywords: Burst; Block-wise error correcting codes; Error Pattern; Syndromes; Parity-check digits; Bounds; Sub-blocks; Parity-check matrix; Linear codes.

1. Introduction

Cantor (1845-1918), Known for axiomatic set theory and more interesting sets and published a description of a set known as Cantor set which is defines as the set of all $x\epsilon[0,1]$ with the ternary expansion $x = \sum_{n=1}^{\infty}\frac{a_n}{3^n}$, and $a_n = 0,2$ for all $n\epsilon N$.

Shaver, (2020) explored the cantor ternary set from the perspective of fractals and observed that the cantor set can be understood geometrically by imagining the continuous removal of a set portion of a shape in such a way that at every stage of removal, the chucks of the shape that remain each will have the same percentage removed from their centre. In fractal geometry this set is one of the classical examples of fractal and it is usually known as cantor fractal.

In this paper, with cantor fractal approach, we have studied block-wise burst error correcting code in two cases on the basis of length of the codes.

Burst is a well - known type of error in coding theory. It is found that instead of random error correcting codes burst error correcting codes are more efficient and economical. Fire, (1959), Chein, et al.,(1965) gave their own definition of burst. Dass, (1980) modified this definition which is defined as

Definition 1: A burst of length b(fixed) is an n- tuple whose non zero components are confined to any b-consecutive positions, the first of which is non zero and the number of its starting position is the first $(n-b+1)$ positions.

When in a burst of length b, in which the first and last components are non-zero with the number of its starting position is the first $(n-b+1)$ positions among b-consecutive positions, we call such type of error as closed burst. A closed burst may be defined as

Definition 2: A closed burst of length b is a vector whose first and last component are non- zero with the number of its starting position is the first $(n-b+1)$ positions. According to this definition closed burst of length 3 in a vector of length 4 are given by 1010,1110,0101,0111.

Block-wise burst error correcting code is a code in which a burst error can occur only within the selected part of the sub-block of a code. Dass, et al.,(1980) developed a class of codes in which the length of code n is divided in two subblocks of length n_1 and n_2 which could correct the bursts errors of length b_1 in first sub-block and bursts errors of length b_2 in the second sub-block. Tyagi, et al.,(2011, 2013) had generalised it into three sub-blocks and obtained interesting results. But the behaviour of the burst errors may vary from channel to channel and it is possible that the errors may occur only in the end parts of the code, so there is a need to study new types of error pattern.

In this paper we explore the possibilities of the existence of codes in which error do occur but only in first and last part sub-block of the code of length $n(= n_1 + n_2 + n_3)$ not in middle part of the code from the perspective of cantor fractal. We have discussed two cases on the basis of the length of a code with keeping in mind that the lesser the number of parity check digits, the better the rate of transmission.

The paper is organised as follows. Section 1 is introduction which gives brief view of the concepts related to the paper. Section 2, shows the construction of codes of length 3^N and 3N form in which all closed burst of length $b_i(i = 1\ to\ m)$, where m is the number of iterations, is to be corrected in the first and the third sub-block only, not in the second sub-block. In Section 3, the proof of the lower bound on the number of parity check digits for the codes so constructed, capable correcting all closed bursts is provided and some examples for the same comprise this section.

2. Construction of the code whose length is of the form of 3^N and 3N

Case1: When the code length is of the form of $n = 3^N$

Construction of an (n, k) linear codes over GF(2) that are capable of correcting closed burst error of length b_1 in the first and the last sub-block of equal length followed by successive iterations on the division of sub-blocks. The steps are as follows:

1st Iteration: Divide the code of length, $n = 3^N$ in to three sub-blocks of equal length, that is, $n_1 = n_2 = n_3 = l_1 = \frac{3^N}{3}$. Here the code is capable of correcting all closed bursts of length b_1 in the first and last sub-block only.

2nd Iteration: Divide the first and last sub-block of length l_1 again into three equal parts and the closed bursts errors b_2, is to be corrected, only in the end part of the subdivided sub-blocks each of length $l_2 = \frac{3^N}{3^2}$.

The process continues till the length of the subdivided sub-blocks in the t^{th} iteration is

$l_t = \frac{3^N}{3^t} = 3$ and it is capable of correcting only single closed burst error, $b_t = 1$.

For example, if $n = 27$ then the length of the sub-blocks in the successive iterations are $l_1 = 9$, $l_2 = 3$ which can correct all closed bursts of length b_1 and $b_2(= 1)$ respectively. Hence only two iterations are possible.

Case 2: When the code length is of the form of 3N

Consider n=3N, to construct such codes which are capable of correcting closed bursts we add some redundant vectors so that every subdivided sub-block will be the length of the form 3N. The steps are as follows:

1st Iteration: Divide the code of length, $n = 3^N$ in to three sub-blocks of equal length, that is, $n_1 = n_2 = n_3 = l_1 = \frac{3N}{3}$. The code is capable of correcting closed burst of length b_1 in the first and last sub-block only.

2nd Iteration: Add some redundant vectors say r_i in the middle of each sub-block obtained in 1st iteration so that its length is again of $t_1 = 3N$ type. Divide the first and last sub-block of length t_1 again in three equal parts and considered the closed burst errors, corrected only in the end part of the subdivided sub-blocks each of length $t_2 = \frac{l_1+2r_1}{3^2}$.

The process continues till the length of the subdivided sub-blocks in the m^{th} iteration is $m_t = \frac{N+2^{m-1}r_{i-1}}{3^t} = 1$ and is capable of correcting only single closed burst error, $b_t = 1$ in its end parts only.

For example, if $n = 15$ then the length of the sub-blocks in the first iterations is $l_1 = 5$, which can correct all closed bursts of length b_1. For the next iteration we add redundant vector r_1=1 in first and last sub-block so that the length of the sub-block $t_2 = 3$ (3N type) which can correct all closed bursts of length b_2. In the third iteration, by adding redundant variable r_2=1 and t_3=1 which is capable of correct all closed bursts of length $b_3 = 1$. Thus, for $n = 15$ three iterations are possible.

Lower Bound

Theorem 1. An (n, k) linear code over GF(q) that can correct all closed bursts of length, b_i In the i^{th} iteration in their respected sub-blocks of length l_i must have at least parity check digits **Proof**. The proof is based on total number of correctable error vectors and comparing with the total number of available cosets q^{n-k}.

The number of closed bursts of length b_i in the iterative sub-blocks each of length l_i is given by

$$2(l_i - b_i + 1)q^{b_i-2}(q-1)^2. \tag{1}$$

Thus the total number of correctable error vectors including zero vector is given by

$$1 + \sum_{i=1}^{m} 2^i(l_i - b_i + 1)q^{b_i-2}(q-1)^2 \tag{2}$$

Where summation is carried over the number of iterations.

For correction, all these vectors must belong to different cosets. Thus, we have

$$q^{n-k} \geq 1 + + \sum_{i=1}^{m} 2^i(l_i - b_i + 1)q^{b_i-2}(q-1)^2.$$

Example 1: Consider $(9 = 3^2, 3)$ linear code with the 6 x 9 Parity check matrix, which is capable of correcting all closed bursts error b_1=1 in the first and the last sub-block each of length 3.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It may be verified from the following error pattern -syndrome table1 that the syndromes of all the closed burst are non-zero and distinct.

Table 1

Error Patterns	**Syndromes**
First Iteration	
First Sub-block	
100 000 000	100000
010 000 000	010000

001 000 000	001000
Third Sub-block	
000 000 100	100100
000 000 010	010010
000 000 001	001001

Example 2: Consider $(27 = 3^3, 21)$ Linear code with the 6 x 27 Parity check matrix, which capable of correcting all closed bursts error b_1=2 in the first and the last sub- block each of length 9 in first iteration along with call closed bursts error b_1=1 in the second iteration where the length of each sub- block is 3.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

It may be verified from error pattern -syndrome table 2 that the syndromes of all the closed burst are non zero and distinct.

Table 2

Error Patterns	**Syndromes**
First Iteration	
First sub- block each of length 9	
110000000 000000000 000000000	101001
011000000 000000000 000000000	000010
001100000 000000000 000000000	110001
000110000 000000000 000000000	110110
000011000 000000000 000000000	001100
000001100 000000000 000000000	011000
000000110 000000000 000000000	110111
000000011 000000000 000000000	011111
Third sub- block each of length 9	
110000000 000000000 000000000	000110
011000000 000000000 000000000	011001
001100000 000000000 000000000	001000
000110000 000000000 000000000	000100
000011000 000000000 000000000	100100
000001100 000000000 000000000	110100
000000110 000000000 000000000	001111
000000011 000000000 000000000	011100
Second Iteration	
First Sub-block each of length 3	
100 000 000	101000
010 000 000	000001

001 000 000	000011
000 000 100	010000
000 000 010	100111
000 000 001	111000
Second Sub-block each of length 3	
100 000 000	100000
010 000 000	100110
001 000 000	111111
000 000 100	100011
000 000 010	101100
000 000 001	110000

Example 3: Consider $(12 = 3 * 4, 7)$ Linear code with the 5 x 12 parity check matrix, which capable of correcting all closed bursts error b_1=3 in the first and the last sub- block each of length 4 in first iteration along with all closed bursts error b_2=2 in the sub- blocks of length 2 in the second iteration and all closed bursts error b_2=1 in the sub-block of length 1 in the third iteration.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

It may be verified from error pattern -syndrome table 3 that the syndromes of all the closed burst are non zero and distinct.

Table 3

Error Patterns	**Syndromes**
First Iteration	
First Sub-block each of length 4	
1010 0000 0000	10100
1110 0000 0000	11100
0101 0000 0000	01010
0111 0000 0000	01110
Third sub- block each of length 4	
1010 0000 0000	10010
1110 0000 0000	011001
0101 0000 0000	001000
0111 0000 0000	000100
Second Iteration	
First sub-bock each of length 2	
11 00 00	11000
00 00 11	00110
Third Sub-block each of length 2	

11 00 00	10110
00 00 11	01001
Third Iteration	
First Sub-block each of length 1	
1 0 0	11000
0 0 1	00110
1 0 0	10110
0 0 1	01001
Third Sub-block each of length 1	
1 0 0	00001
0 0 1	10111
1 0 0	10011
0 0 1	11010

Discussion and Conclusion

This paper presents the codes on the concept of cantor fractal, which can correct all closed bursts error in their respective iterative sub-blocks occur only at the end parts of the code. Such codes may be possible for large value of code length which can correct multiple errors in the same sub-blocks of different lengths in the successive iterations but at different positions.

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