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Mathematical Modelling Approach for Administration of Palatable Diet to Quantify Blood Glucose Level: A Reference to Systolic Blood Pressure

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Abstract:
The study concerns the Mathematical analysis of some variations that arise in systolic blood pressure (SBP) for different human age groups due to endocrine disorders (diabetes). An attempt has been made to improve the blood glucose levels (both pre-prandial and post prandial) associated with SBP by the suitable administration of palatable diet. Red portions (Protein and Fat) and Black Portions (Carbohydrates containing sugar and starch) have been considered with the assumption of "weight" of an individual as constant. A Mathematical model has been developed by using coupled nonlinear ordinary differential equations to study the behaviour of blood glucose levels. These equations involve the terms as, the change in glucose and the change in insulin with respect to time, the food intake sensitivity constants and the step function which can provide the quantification of normal blood glucose levels. The effectiveness in the variation of systolic blood pressure is analysed for various age groups. The response of men and women aged 25 years and above and the case of juvenile is studied using multiple linear regression model. The assumption that the weight of an individual as constant is confirmed by ANOVA for the determination of blood-Glucose levels. The palatable diet composed of Protein (P), Fat (F) and Carbohydrate (C) as PFC model is simulated with Joselin's Principle.

Keywords: Diabetes, Pancreas, Palatable Diet, Regression, Glucose

Introduction:

Diabetes is a disorder of nutrition in which the body looses the ability to utilize carbohydrates i.e sugars and starches contained in the daily food intake. As a result, the blood and tissues become surcharged with undigested sugar. This abnormal presence of unused and unusable sugar produces a state of depleted alkanity of the tissues and blood. This gradually borders on acidity ushering in the grave and fatal stages of diabetes. Nature endeavors to relieve this abnormality by passing quantities of sugar

through kidney in the urine. It is this sugar in the urine which constitutes the most familiar symptom of diabetes. Recent investigations/studies have shown that normally there is present in the body tissues a complex chemical substance making possible, by its chemical action, the conversion of sugar and starch into body heat, muscle and nerve energy. This chemical sugar digesting substance is largely produced and doled out to the blood, by that long glandular organ lying behind the stomach, called the "Pancreas". This is the reason for which diabetes is therefore regarded as an aggravated and specialized form of indigestion, due to failure of pancreas and of its chemical contribution to the bodily functions.

In 1922 Doctors Banting and Best of the Toronto university announced that they had succeeded in isolating from the pancreas this sugar digesting chemical which they names "Insulin". They also announced that this insulin substance could be injected hypodermically into the blood stream of a diabetic patient with the result that both blood and urine becomes free from sugar. It was necessary to make from one to three such injections daily to keep the patient free of sugar and the consequences of its presence. This effect is maintained as long as the injections are continued. When the injections are discontinued, the evidence of excess sugar reappear. Desiccated pancreatic preparations are now used with success.

As diabetes is a nutritional disorder it is evident that dietary regulations is an important factor in reestablishing a normal nutritional balance. In selecting a plan of diet it must be remembered that the diabetic individual has lost part of his digestive power for starches and sugars and cannot utilize the normal amount of these foods. If this person now be fed the same quantity of food as when in health, not only would part of the food be lost through the kidneys, but it no longer acts or serves as food. On the contrary it actually becomes a poison to him setting up the train of symptoms peculiar to diabetes.

The two types of Diabetes are Diabetes insipidus and Diabetes mellitus. Diabetes insipidus the rare metabolic disorder in which the body passes large quantity of colourless urine that contains more water causing thirsty, dry hands, constipation. This is due to the failure of kidney's function where in the water is to be reabsorbed. Diabetes mellitus is the disorder of carbohydrate metabolism in which sugars in the body are not oxidized to produce energy due to lack of pancreatic hormone insulin.

Gary et. al[12] presented the effect of body weight of a free 76 kilojoules (320 calories) daily supplement of almonds for six months. Katiyar and Basavarajappa et. al[20] analysed the diabetes mellitus under palatable composition of quantitative diet with varying body frames. Allick Gideon et. al[1] compared the effects of a caloric high carbohydrate and high fat improves glucoregulation in type-II diabetes mellitus by reducing post absorptive glycogenolysis. Venkatapuram et. al[30] explained metabolic syndrome is considered to be a metabolic precursor of type – II diabetes mellitus and is an independent risk factor in the pathogenesis of atherosclerosis. Meena Verma et. al[22] presented the effect of increasing duration of diabetes mellitus type –II on glycated haemoglobin and insulin sensitivity. Villegas et. al[31] analysed that high intake of foods with a high glycemic index and load, especially rice, the main carbohydrate contributing food may increase the risk of type- II diabetes mellitus in Chinese women. Gatewood et. al[13] explained the effect of hormone glucose and insulin secreted by pituitary and thyroxin produced by thyroid as simplest mathematical form. Brownlee et. al[9] described glycosylation end products in tissue and the biochemical basis of diabetic complications. They also analysed the fluctuations of chemical compounds in diabetic patients. Kapur[19] presented a compartment model for diabetes mellitus taking the interaction of blood glucose with insulin at different time intervals. Bankroft et. al[5] made a comparative study of dysfunction in men with and without diabetes mellitus. Bartholovistsch et. al[8] studied the behaviour of viscosity of the blood in diabetic

cases and non-diabetic cases

In view of the nutritional disorder and the presence of undigested sugar A palatable dietary plan is modelled mathematically to regulate the blood sugar level with normal values. There appears the significant improvement with PFC model which require modifications according to individual conditions found in each patient. The objects aimed at in treatment are first to reduce and prevent acid accumulation and second to find the patient's carbohydrate digestive powers, and then to keep the food intake safety within this deficient digestive function. For three days, the patient is given exclusively green leafy vegetables, boiled, cooked and raw. Only water is allowed in addition. If at the end of three days, the urine is not yet free of sugar, then the green vegetable days are continued for further period, until urine is free of all sugar. Now add to this green vegetable diet, one form of carbohydrate food such as either rice, potatoes, or oatmeal. Begin with only one Teaspoon full on the first carbohydrate day. This food is then gradually increased day by day and accurately measured each time. The urine must be tested for sugar. When this begins to appear as the food is being increased, it indicates that patients limit for carbohydrate food has been reached. The quantity of rice or potatoes should then reached by one third of what was given when sugar appeared and the patient must content himself within this limit for a week or longer before the carbohydrates are increased. Green leafy vegetables are used to liberty, this constitutes the bulk and basis of the diet plan.

After remaining sugar free for a time a gradual increase in starchy foods may again be attempted till sugar reappears, when the same plan of reduction by one third is followed.

In addition to the above mentioned carbohydrate foods, the patient may be given eggs and cheese, skimmed milk, butter sparingly, olive oil, gluten meal or gluten bread and legumes and nuts. The last two should be added with caution. No sweet and non- starchy fruits are permissible. Meat foods are none favored because of their acid producing tendency.

The study is aimed at computation of normal blood sugar level x(t) in the form of palatable diet using sensitivity values. The case of variation in the systolic blood pressure(SBP) against age is employed in the model. This could explore the details of attaining the normal values of x(t) referred to change in viscosity of blood which may cause changes in flow rate of the blood. System of differential equations is considered by introducing the sensitivity value in terms of systolic blood pressure. Multiple linear regression models are studied for obtaining various SBP values in terms of age. Numerical results give the satisfactory comparison for attaining normal values of x(t).

Formulation

Model refers to the quantities as 'x(t)' for blood glucose level, 'y(t)' for blood insulin level, 'z' for food input, 't' for time and 'I' for insulin input. Under the assumption of the normal fasting level of blood glucose with 70-100 mg/100ml before breakfast and 120-140 mg/100ml following a meal, the gradients of blood sugar (x) and the insulin (y) are modelled as,

$$\frac{dx}{dt} = -\alpha xy + \beta x_0 H(x_0 - x) - \beta x H(x_0 - x) + \gamma Z(t) \quad (1)$$

$$\frac{dy}{dt} = (x_{S,A}\, x) \times H(x - x_0) - (x_{S,A}\, x_0) H(x - x_0) - \psi y_0 + \xi I(t) \quad (2)$$

Here α, β and γ are the positive constants called sensitivity values for insulin, the low blood sugar level, high blood sugar level and the input level respectively for the sugar level (gradient). We consider $x_{S,A}$, ψ and ξ as the positive constants and are taken as the sensitivity values for high blood sugar level and increase in blood pressure level, insulin level and the input level respectively, 'H' is the unit step

function which controls the quantity of food intake sensitivity of the diet plan of four different meals. The input to the blood sugar level is via food source. The food store can be assumed to be filled periodically and the contents at any stage are reduced in a simple exponential manner.

The source term Z(t) in terms of quantity of food can be expressed as,

$$\xi = Z(t) = \begin{cases} 0 & , \ t < t_0 \\ Qe^{-\delta(t-t_0)} & , t \geq t_0 \end{cases} \quad (3)$$

where Q - quantity of meal, δ - delay parameter and t_0- time of the meal. The subcutaneous injection at periodic intervals leaks its contents into the system over a period of time. Then we take the maximum effect to mean leakage rate as I(t). The values of $x_{S,A}$ in terms of age for men and women are calculated for the average values taken for the coefficients of regression equation.

The food intake for the first meal is modelled as,

$$I(t) = \frac{\rho t_1}{t_1 - t_0} + bt + k \quad (4)$$

$$z(t) = Qe^{-\delta(t_1 - t_0)} \quad (5)$$

Multiple linear regression analysis between the input as variation of age associated with varying body frames give rise to total calorie. Regression coefficients control the change in blood glucose level for the chosen palatable diet for four different meals. The quantities of four different meals in the form of variations of SBP v/s age for various body frames has been taken into consideration in the following regression equation. The sensitivity value in the equation (2) is modelled by multiple linear regression equation as

$$x_{S,A} = 132.3 - 0.1414x_A \text{ (For men)}$$
$$x_{S,A} = 100.82 + 0.3387x_A \text{ (For women)} \quad (6)$$

Analysis

The blood sugar and insulin levels are given by equations (1) and (2). Changing the non-linear term αxy as $\alpha\bar{x}y$ (linear term of x and linear term of y are to be taken as independent Solutions) then, equations (1) and (2) become,

$$\frac{dx}{dt} = -\alpha\bar{x}y \quad (7)$$

$$\frac{dy}{dt} = x_{S,A}\, x - x_{S,A}\, x_0 - y \quad (8)$$

On solving the above differential equations, we have,

$$x(t) = C_5 e^{m_1 t} + C_6 e^{m_2 t} + x_0 + \frac{\gamma Q(\Psi - \delta)e^{-\delta(t-t_0)}}{\delta^2 - \Psi\delta + \alpha x_{S,A}\,\bar{x}} \quad (9)$$

$$y(t) = C_7 e^{m_1 t} + C_8 e^{m_2 t} + \frac{\gamma Q e^{-\delta(t-t_0)}}{\delta^2 - \Psi\delta + \alpha x_{S,A}\,\bar{x}} \quad (10)$$

We redefine as,

$$x = x^{PD}, y = y^{PD}, C_1 = C_1^{PD}, C_2 = C_2^{PD}, Q = Q^{PD} = \gamma Q^{PD}$$

with 'PD' = palatable diet. For PFC by introducing regression model.

By introducing food source Z(t),

$$z(t) = Qe^{-\delta\left\{\frac{\rho t_1}{I(t)-b\,t-k}\right\}} \quad (11)$$

We consider the quantity of the meal Q^{PD} in calories with palatable choice with red portions and black portions ofavailable foods. Analysing the blood sugar and insulin levels by providing the palatable diet at four different meals divided to four different values of t, i.e. at t:t_0=3.00 hours (breakfast), t_1= 6.00 hours (noon), t_2= 9.00 hours (evening) and t_3=12.00 hours (late evening), the new types of palatable diets are computed. Then x (t) and y (t) areobtained from equations (14) and (15) as,

$$x^{PD} = {C_1}^{PD}e^{-t}\sin t + {C_2}^{PD}e^{-t}\cos t + {x_0}^{PD} + \frac{\gamma Q^{PD}(\psi-\delta)e^{-4.16(t-t_0)}}{\delta^2-\psi\delta+\alpha x_{S,A}\,\bar{x}} \quad (12)$$

$$y^{PD} = \left({C_1}^{PD} + {C_2}^{PD}\right)(0.25)e^{-t}\sin t + \left(-{C_1}^{PD+}{C_2}^{PD}\right)(0.25)e^{-t}\cos t + \frac{x_{S,A}\,\gamma Q^{PD}e^{-4.16(t-t_0)}}{\delta^2-\psi\delta+\alpha x_{S,A}\,\bar{x}} \quad (13)$$

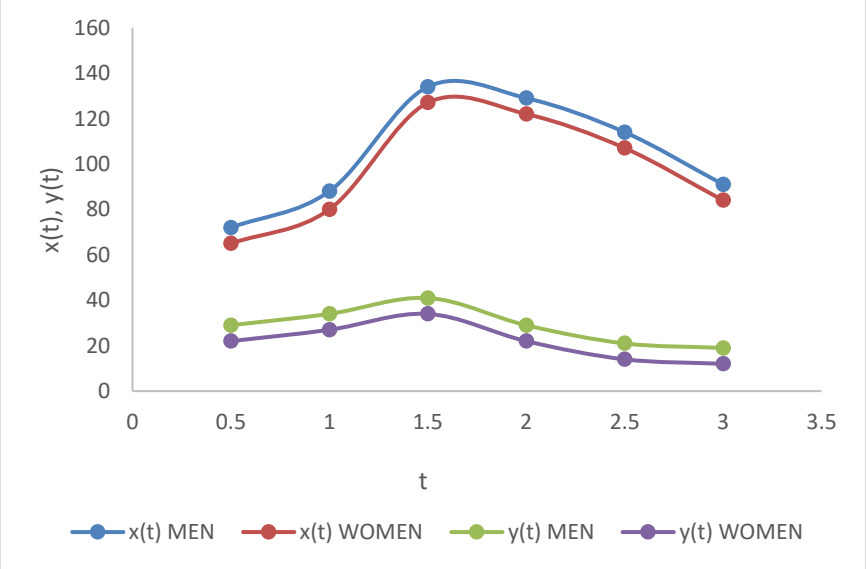
Equations (9) to (13) give the values of x(t) and y(t) for non – palatable and palatable diet during and after the breakfast. Then

$$x^{PD} = {C_1}^{PD}e^{-t}\sin t + {C_2}^{PD}e^{-t}\cos t + \frac{\gamma Q^{PD}(\psi-\delta)e^{-4.16(t-t_0)}}{\delta^2-\psi\delta+\alpha x_{S,A}\,\bar{x}} - \frac{\mu t}{\varphi} + \frac{\psi\mu-\alpha\bar{x}x_{S,A}}{\alpha\bar{x}{x_{S,A}}^2} \quad (14)$$

$$y^{PD} = \left({C_1}^{PD} + {C_2}^{PD}\right)(0.25)e^{-t}\sin t + \left(-{C_1}^{PD+}{C_2}^{PD}\right)(0.25)e^{-t}\cos t + \frac{x_{S,A}\,\gamma Q^{PD}e^{-\delta(t-t_0)}}{\delta^2-\psi\delta+\alpha x_{S,A}\,\bar{x}} + \frac{\mu}{\alpha\bar{x}x_{S,A}} \quad (15)$$

Results and Discussion

Model predicts the introduction of MLR model for achieving the normal values of x(t) in reference to palatable composition as Protein: Fat: Carbohydrates (P: F: C).The raise in the SBP for various age inputs as computed analytically give the values of x(t) for acceptable range. The graphs referred to solution of system of differential equations using multiple linear regression model explain the closeness of Numerical values for the normal values of x(t) in response to corresponding values of y(t).



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