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Fuzzy MCDM for Analyzing Employees Efficiency: A Comprehensive Framework

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Abstract:

The Fuzzy Choquet Integration is an advanced fuzzy decision-making method that can effectively handle multi-criteria decision-making (MCDM) problems where interactions or interdependencies between criteria need to be taken into account. When applied to employee efficiency evaluation, the Choquet Integral allows decision makers to assess not only individual criteria but also the synergistic effects and interactions between those criteria, which is often crucial in efficiency appraisals. Although fuzzy MCDM methods have been widely applied to employee efficiency appraisal, Choquet integration has been relatively under explored in this domain. Fuzzy Choquet Integration is designed to capture interactions among efficiency criteria, which can provide a more accurate and holistic evaluation of employees. This research aims to demonstrate how Choquet Integration can be applied to employee efficiency evaluation, considering both the individual efficiency of criteria and their interdependencies by considering a case study and results are compared with the weighted averages.

Keyword: λ-measure, Choquet integration, MCDM, efficiency score.

1. Introduction:

In today's rapidly changing and innovative contexts, choosing the right Efficiency Appraisal (EA) methodologies for organizations and determining their funding tiers is a challenging task. In contemporary organizations, performance reviews are now a strategic component of the integration of human resource operations and business policies. It is more difficult to identify the best EA approach when there are several criteria in the decision-making process. Employee efficiency analysis is a critical activity in organizations, as it directly impacts productivity, promotions, rewards, and development plans. Traditional methods like rating scales are often subjective and may not fully capture the complexities of performance. Efficiency appraisal is a critical process in organizations for evaluating employee contributions, guiding decisions regarding promotions, rewards, and training. Traditional evaluation methods often struggle to account for the complex relationships between multiple efficiency criteria, such as technical skills, teamwork, leadership, problem-solving abilities, etc. These qualitative relationships can be non-linear and interdependent, which makes conventional evaluation methods, like simple weighted averages, less effective. Fuzzy MCDM introduces the concept of combining fuzzy logic with MCDM to evaluate complex decision-making problems with uncertain or vague criteria. Fuzzy MCDM provides a review of existing studies applying to decision-making problems. Fuzzy Choquet Integration offers a more sophisticated approach to determine efficiency, as it can account for interactions and the relative importance of different criteria. By using fuzzy logic, this method can handle subjective assessments, uncertainty, and the nuances of human judgment, making it particularly suitable for employee efficiency analysis. In Literature there are many Fuzzy MCDM tools like:

1. Fuzzy AHP: For determining the relative importance of performance criteria.
2. Fuzzy TOPSIS: For ranking employees based on their performance in each criterion.
3. Fuzzy DEMATEL: Used for establishing relationships between different performance criteria and understanding cause-effect relationships.
4. Choquet Integration: For determining relative importance, interdependencies between various criteria and interaction index.

In present paper we consider Choquet integration as an aggregation tool for information fusion. Now, we consider the next section which gives the literature review.

2. Literature Review:

The concept of fuzzy sets was established by L.A. Zadeh[16]. To combine the interrelated information, Choquet integral is used as aggregation operator. Various evaluation problems are solved by Choquet integration. Shapely Value standard and interaction index is used by Choquet integral fuzzy measures. Choquet integration is used to aggregate customers' satisfaction [1]. Women vulnerability index using different MCDM approach and weighted average mechanism was observed along with its effect [2]. Employee performance evaluation model was established using fuzzy logic, fuzzy type -2 ranking method and fuzzy AHP model [3,4,6]. Fuzzy reasoning method is used for fuzzy rule based classification systems along with Choquet integration [5]. By using a multiple criteria decision analysis method, specifically MULTIMOORA integrated Shannon's entropy significant coefficient, [7] addresses an assessment of the PA method. Using two approaches—MULTIMOORA and Entropy MULTIMOORA—a case study on the best PA technique selection is examined by determining the criteria and alternatives based on the literature and case-study expert opinions [7]. Applications of fuzzified Choquet integral was studied [8]. Using interval valued fuzzy numbers (VIKOR) evaluated the performance of three intercity bus companies along with soft computing technique [9]. Using fuzzy MCDM approach safety of construction labours were analyzed [11]. Various fuzzy MCDM technique and their uses from 1994 to 2014 reviewed [10]. Two dimensional model was designed to evaluate the performance of employee using fuzzy logic and fuzzy TOPSIS method [12]. Assessing lecturers' research productivity can be considered as a MCDM problem in an uncertain context, so using Fuzzy MCDM approach lecturers research output was examined [13]. Fuzzy extensions of different TOPSIS method was introduced and its application was considered as case study[15].

3. Methodology:

This section gives step-by-step process for applying fuzzy MCDM to employee efficiency analysis:

1. Criteria Selection: Identifying the key criteria for analyzing employee efficiency, such as technical ability, problem-solving skills, interpersonal communication, punctuality, and leadership.
2. Fuzzy Set Representation: Employees' performance in each criterion is rated on fuzzy scales (e.g., "low", "medium", "high", or linguistic terms like "very good", "satisfactory", "poor").
3. Assigning fuzzy weights to each criterion to reflect its importance. These weights could be determined through expert opinions, surveys, or past efficiency data.

4. The fuzzy ratings and weights are combined using aggregation operators like the weighted average, fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), fuzzy AHP (Analytic Hierarchy Process), fuzzy integrals namely Choquet integration or DEMATEL.
5. Converting the fuzzy results into a crisp value for each employee, which provides a final performance score (Defuzzification).
6. Ranking of Employees is based on the defuzzified scores, employees can be ranked in terms of their overall performance.

The present section gives some preliminaries regarding λ-fuzzy measure and Choquet Integration. The fuzzy measures are called as non-additive measure because of its characteristic non-additivity. This characteristics helps vital role in MCDM.

Definition 2.1 [14] "Let $\lambda \in (-1, \infty)$. A normalized set function g_λ defined on $\mathcal{G} = 2^\Theta$ is called as λ-fuzzy measure on Θ if for every pair of disjoint subsets θ_1 and θ_2 of any nonempty set Θ , we have,

$$g_\lambda(\theta_1 \cup \theta_2) = g_\lambda(\theta_1) + g_\lambda(\theta_2) + \lambda \cdot g_\lambda(\theta_1) \cdot g_\lambda(\theta_2)."$$

Obviously, "if $\lambda = 0$, then a λ-fuzzy measure is a normalized additive measure i.e. probability measure. Following theorem is used to determine the parameter λ is calculated.

Theorem 2.1 [14] "Let 2^Θ be the class of all subsets of $\Theta = \{\beta_1, \beta_2, \dots, \beta_n\}$ be the finite set, the fuzzy measure $g_\lambda(\Theta) = g_\lambda(\{\beta_1, \beta_2, \dots, \beta_n\})$ can be formulated as

$$g_\lambda(\{\beta_1, \beta_2, \dots, \beta_n\}) = \frac{1}{\lambda}\left[\prod_{i=1}^{n} [1 + \lambda g_\lambda(\beta_i)] - 1\right] \qquad (1)$$

where, $\lambda \in (-1, \infty) \cup \{0\}$.

As $g_\lambda(\{\beta_1, \beta_2, \dots, \beta_n\}) = 1$ the formula becomes

$$\lambda + 1 = \prod_{i=1}^{n} [1 + \lambda g_\lambda(\beta_i)]." \qquad (2)$$

Definition 2.2 [14] "Let f be a nonnegative measurable function on $(\Theta, \mathcal{G})$ and $\theta \in \mathcal{G}$. Let μ be a monotone measure defined on Θ. Then Choquet integration with the measure μ is given as

$$\tilde{C}_\mu(f) = \int_0^\infty \mu({}^\alpha F \cap \theta) d\alpha$$

where ${}^\alpha F$ is the α-level set of f, for $\alpha \in [\alpha, \infty)$. When $\theta = \Theta$, the Choquet integral may also be defined as $\tilde{C}_\mu(f)$."

Definition 2.3 [14] "Let f be a nonnegative measurable function on $(\Theta, \mathcal{G})$. The Choquet integral of f with respect to g_λ is defined by

$$\tilde{C}_\mu(f) = \sum_{i=1}^{n} \left(f(\beta_i) - f(\beta_{i-1})\right) g_\lambda(\theta_i) \qquad (3)$$

where $\theta_i = \{\beta_i, \beta_{i+1}, \cdots, \beta_n\}, f(\beta_0) = 0$ and $(\beta_1, \beta_2, \cdots, \beta_n)$ is a numbering of the elements of Θ satisfying the condition that $f(\beta_1) \le f(\beta_2) \cdots \le f(\beta_n)$.

4. Case Study:

A sample of 50 employees working in MIDC Sangli is chosen, and efficiency data is collected through surveys, feedback, and manager evaluations. Out of 50 employees, we have considered here 20 employees and their efficiency data. Efficiency is typically evaluated using multiple criteria such as: technical skills, problem solving ability, communication skills, teamwork, leadership, punctuality, work ethics, creativity, etc. Here we have considered eight characteristics. Each criterion is given a weight based on its importance, such as:

Criteria	Technical Skills	Problem Solving Ability	Comm. Skills	Team Work	Leadership	Punctuality	Work Ethics	Creativity
	(x_1)	**(x_2)**	**(x_3)**	**(x_4)**	**(x_5)**	**(x_6)**	**(x_7)**	**(x_8)**
λ-measure	**0.8**	**0.9**	**0.7**	**0.8**	**0.6**	**0.7**	**0.5**	**0.6**

The weight based on importance is given by λ-measure as below:

$$g_\lambda(x_1) = 0.8,\quad g_\lambda(x_2) = 0.9,\quad g_\lambda(x_3) = 0.7,\quad g_\lambda(x_4) = 0.8,$$
$$g_\lambda(x_5) = 0.6,\quad g_\lambda(x_6) = 0.7,\quad g_\lambda(x_7) = 0.5,\quad g_\lambda(x_8) = 0.6$$

The efficiency score for 20 employees is given in the Table 1 and Figure 1 gives its graphical representation. For the evaluation of interdependency measures MATLAB Software is used and its pseudocode is given after Table 1. Here we have considered eight criteria. We denote criterion set as:

$\Theta = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$. Therefore, we have to consider the interdependency measure of $2^\Theta = 2^8 = 256$ sets i.e. interdependencies between single criteria, two criteria, three criteria, four criteria, five criteria, six criteria, seven criteria and all the eight criteria. To calculate interdependency measure, firstly calculate λ using equation (2). By MATLAB programming we have the polynomial equation in λ as :

$$4.6\,\lambda + 13.66\lambda^2 + 18.956\lambda^3 + 16.3669\lambda^4 + 9.00284\lambda^5 + 3.0807\lambda^6 + 0.5996\lambda^7 + 0.0508x^8 = 0.$$

The roots of this equation are given as $\{-2.9375069 + 0.6555001i, -2.9375069 - 0.6555001i, -1.8849668 + 1.4761798i, -1.8849668 - 1.4761798i, -0.5783344 + 1.187149i, -0.5783344 - 1.187149i, -0.9999712, 0\}$. There are six complex roots, we discard these roots [14]. If we consider $\lambda = 0$ then we get additive measure (see [14]). So only we have to consider $\lambda = -0.9999712$. As this $\lambda \in (-1, \infty)$, we find interdependency measures using $\lambda = -0.9999712$.

Figure 1 Efficiency Score of each employee with different criterion

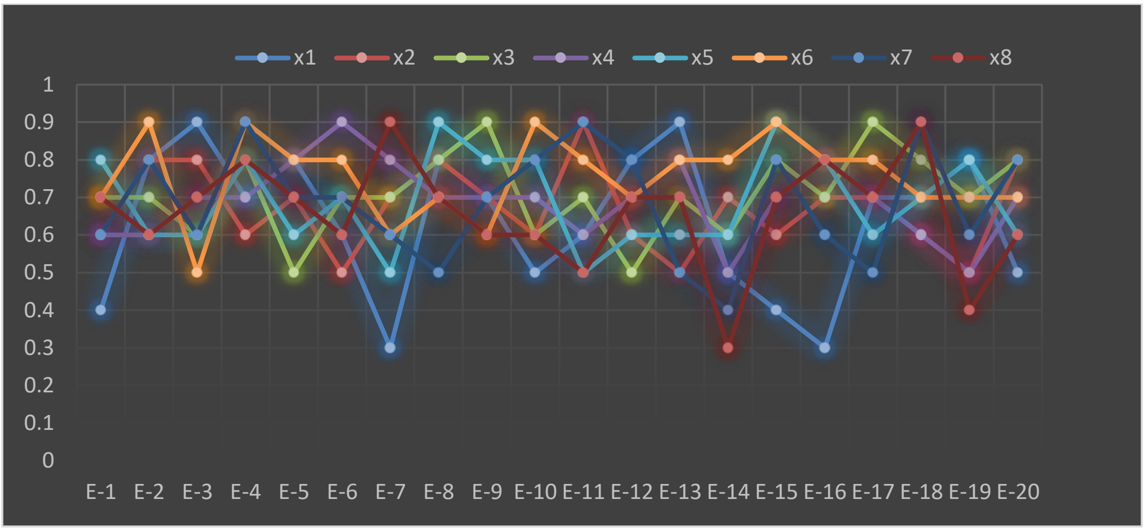

Table 1: Efficiency Score Values

Criteria → Employees ↓	Technical Skills	Problem Solving ability	Communication Skills	Team Work	Leadership	Punctuality	Work Ethics	Creativity
	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(x_6)	(x_7)	(x_8)
E-1	0.4	0.6	0.7	0.6	0.8	0.7	0.6	0.7
E-2	0.8	0.8	0.7	0.6	0.6	0.9	0.8	0.6
E-3	0.9	0.8	0.6	0.7	0.6	0.5	0.6	0.7
E-4	0.7	0.6	0.8	0.7	0.8	0.9	0.9	0.8
E-5	0.8	0.7	0.5	0.8	0.6	0.8	0.7	0.7
E-6	0.6	0.5	0.7	0.9	0.7	0.8	0.7	0.6
E-7	0.3	0.7	0.7	0.8	0.5	0.6	0.6	0.9
E-8	0.8	0.8	0.8	0.7	0.9	0.7	0.5	0.7
E-9	0.7	0.7	0.9	0.7	0.8	0.6	0.7	0.6
E-10	0.5	0.6	0.6	0.7	0.8	0.9	0.8	0.6
E-11	0.6	0.9	0.7	0.6	0.5	0.8	0.9	0.5
E-12	0.8	0.6	0.5	0.7	0.6	0.7	0.8	0.7
E-13	0.9	0.5	0.7	0.8	0.6	0.8	0.5	0.7
E-14	0.5	0.7	0.6	0.5	0.6	0.8	0.4	0.3
E-15	0.4	0.6	0.8	0.7	0.9	0.9	0.8	0.7
E-16	0.3	0.7	0.7	0.8	0.8	0.8	0.6	0.8
E-17	0.7	0.7	0.9	0.7	0.6	0.8	0.5	0.7
E-18	0.7	0.6	0.8	0.6	0.7	0.7	0.9	0.9
E-19	0.8	0.5	0.7	0.5	0.8	0.7	0.6	0.4
E-20	0.5	0.8	0.8	0.7	0.6	0.7	0.8	0.6

Now, we consider Pseudocode Algorithm to find Choquet indices and interdependency measure between two to eight criteria. All evaluations are done using MATLAB software. The pseudocode is given below:

Pseudocode Algorithm : To obtain Choquet Integration with Interdependency Measure

Description: To calculate Choquet integration values of the employees.
Input: integer n (number of criteria), efficiency score of the employee for each criteria, λ-measure of each criteria.
Output: Choquet integrated values for the employee.

1. Input the number of criteria n.
2. Input the interdependency measure for each criteria.
3. Input the efficiency score of each employee.
4. Determine the value of λ.
5. Determine the interdependency measure between Two criteria.

```
for i=1to n-1 do
    for j=i+1:1:n do
        s=1;
        s=s*(1+ λ * g_λ (i))*(1+ λ*g_λ (j));
        g_λ (i,j)=(1/ λ)*(s-1);
    end
end
```

6. Determine the interdependency measure between Three criteria like step 5.
7. Determine the interdependency measure between Four criteria like step 5.
8. Determine the interdependency measure between Five criteria like step 5.
9. Determine the interdependency measure between Six criteria like step 5.
10. Determine the interdependency measure between Seven criteria like step 5.
11. Determine the interdependency measure between Eight criteria like step 5.
12. Calculate Choquet integration index using formula $\tilde{C}_\mu(f) = \sum_{i=1}^{n} \left(f(\beta_i) f(\beta_{i-1})\right) g_\lambda(\theta_i)$.

Choquet Integration indices and weighted average are calculated and given in the Table 2.

Table 2: Comparison between Choquet Integration indices and weighted average

Employees	Choquet	Weighted	Employees	Choquet	Weighted
E-1	0.7479781	3.52	E-11	0.8670055	3.87
E-2	0.8450021	4.08	E-12	0.7779956	3.76
E-3	0.8699967	3.85	E-13	0.8460038	3.89
E-4	0.8689761	4.26	E-14	0.7510005	3.15
E-5	0.7739957	3.95	E-15	0.8649805	3.97
E-6	0.8539765	3.83	E-16	0.7809683	3.82
E-7	0.8339811	3.56	E-17	0.8529999	3.97
E-8	0.8539958	4.18	E-18	0.8530031	4.04
E-9	0.8480023	3.99	E-19	0.7679995	3.49
E-10	0.8399905	3.79	E-20	0.7789911	3.85

5. Result and Discussions:

Table 2 gives Choquet indices and weighted averages. This shows that Choquet integration aggregates qualitative data instead of quantitative data. In weighted averages as it only depends upon weight, it may result in false decision. Ranking using Choquet integration and weighted average is considered as below:

Ranking According to Choquet Indices		Ranking According to Weighted Average	
E-3	0.8699967	E-4	4.26
E-4	0.8689761	E-8	4.18
E-11	0.8670055	E-2	4.08
E-15	0.8649805	E-18	4.04
E-8	0.8539958	E-9	3.99
E-6	0.8539765	E-15	3.97
E-18	0.8530031	E-17	3.97
E-17	0.8529999	E-5	3.95
E-9	0.8480023	E-13	3.89
E-13	0.8460038	E-11	3.87
E-2	0.8450021	E-3	3.85
E-10	0.8399905	E-20	3.85
E-7	0.8339811	E-6	3.83
E-16	0.7809683	E-16	3.82
E-20	0.7789911	E-10	3.79
E-12	0.7779956	E-12	3.76
E-5	0.7739957	E-7	3.56
E-19	0.7679995	E-19	3.49
E-14	0.7510005	E-14	3.15
E-1	0.7479781	E-1	3.52

Fuzzy Choquet integration provides more accurate rankings by considering both individual criteria and their interdependencies. The method successfully captures the interaction between performance criteria (e.g., how leadership might depend on communication and teamwork), leading to more realistic and fair evaluations. Results from fuzzy Choquet integration often align more closely with qualitative insights from managers, as the method accounts for nuances that traditional approaches.

The fuzzy Choquet integral is more complex to implement than traditional methods, especially when defining fuzzy measures and determining interactions between criteria. The accuracy of the model depends on the expert's ability to define appropriate fuzzy measures and weights, which can introduce subjectivity.

6. Conclusion:

The paper demonstrates that Choquet Integration is a powerful tool of fuzzy MCDM for analyzing employee efficiency, especially when dealing with complex, interacting criteria. This method accounts for interdependencies between performance criteria and provides more accurate, consistent, and fair rankings compared to traditional methods.

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