

Studies on Power Radiated in Central Cone by Planar Undulator in THz Domain

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Abstract

In this study, electron trajectories are derived and analytically & numerically investigated for planar undulator in THz domain. Power of THz radiation in central cone is derived and calculated in THz region. The spectral brightness and average power produced by planar undulator is derived and calculated using Generalized Bessel Functions (GBFs). The applications of THz radiation produced with few μW power level by the planar undulator for various applications is discussed.

Keywords: THz Radiations, Planar Undulator, Lorentz Force Equation, Power radiated in central cone.

1. Introduction

Terahertz (THz) radiations designated as THz gap from 1 THz to 10 THz and corresponding wavelength from 30 μm to 1000 μm in electromagnetic spectrum having higher spatial coherence, spectral brightness and wide tunability are greatly demanded for advanced scientific research and commercial applications [1-4]. Parametric oscillators and time domain Lasers techniques are conventional methods produce significantly lower average power THz radiations [5-7]. However, Backward Wave Oscillators (BWOs), Synchrotron Radiations (SRs) and Free Electron Laser (FELs) produce widely tunable radiations with higher spectral brightness and average power. When a low electron beam emittance together with minimal energy spread produced by linear accelerator is passed through a spatially varying periodically magnetic field of the planar undulator, its interaction produces coherent THz radiation via stimulated emission in a resonator. Electron beam in undulator travels in vacuum i.e. no laser medium, therefore no upper limit on average power produced by FEL, compared to quantum/conventional lasers [8-13]. In addition, a new technique based on slice energy spread modulation using the periodic intensity envelope of a pulsed laser enables strong sub-picosecond density bunching in relativistic electron beams, leading to the generation of high-power, narrowband THz radiation in the 1–10 THz range [14]. THz radiation is useful in treating skin cancer, angina, lung and oral diseases [15]. However, in comparison to X-rays and gamma rays, THz radiations are non-ionizing to living tissues and safe for humans [16]. The demand and faster growth in THz technology, it has enabled diverse applications including advanced cellular communication, with 5G systems already deployed and 6G development underway, supported by THz's wide bandwidth and low power consumption [17]. Moreover, time-domain THz spectroscopy serves as a non-contact, non-destructive tool for analysing pure and degraded oils, hydrocarbons, and other chemical products [18].

Being smaller energy of about 4 meV at 1 THz, it is non-ionizing and safe for biological samples and biomolecules, unlike X-rays and microwaves, which can be harmful for tissues due to their ionizing nature.

It can uniquely identify the rotational and vibrational spectra of many molecules in the THz range. Well established studies and advanced THz technology have shown that the spectral signatures of food commodities in this range enable the development of THz-based detection systems for real-world applications. THz radiation can recognise food contamination and determine quality by detecting harmful substances such as pesticide residues, additives, antibiotics, and pathogens [19]. Moreover, its ability to penetrate nonpolar materials like plastics, wood, metals, and paper allows non-destructive inspection of packaged goods without unpacking, particularly when high-brightness sources such as FEL-based THz radiation are used [20]. Furthermore, THz waves undergo much lower losses from Rayleigh scattering caused by atmospheric dust, smoke, or fog [21].

The development of advanced Free Electron Lasers (FELs) has empowered systematic analysis of intense laser interactions with biological samples. The optimal wavelength for tissue ablation depends on the differential absorption of tissue components such as water, minerals, and proteins, with absorption spectra identifying wavelengths that maximize efficiency while minimizing collateral damage [22]. On account to their low photon energy and non-ionizing nature, THz radiations are well-suited for non-destructive, non-invasive material inspection in security applications. THz detection systems can accurately locate standoff objects and improvised explosive devices (IEDs). Their ability to penetrate plastics, polymers, and fabrics concealing weapons, IEDs, or explosives like RDX reduces potential threats. Furthermore, THz spectral data obtained through imaging systems serve as input for signal-processing algorithms to identify explosive signatures [23–25].

This research work presents analytical studies of electron trajectories using Generalized Bessel Functions (GBFs), supported by simulations with the fourth-order Runge–Kutta method [26, 27]. The longitudinal ponderomotive force driving microbunching is derived and analysed using the relativistic Lorentz force equation [28–30]. Spectral brightness of THz radiation is examined analytically as a function of electron beam parameters with minimal energy spread to enable laser action in a Free Electron Laser (FEL). Expressions for on-axis and off-axis spectral brightness in a linearly polarized undulator are derived and evaluated. Numerical results confirm the generation of THz radiation with high spectral brightness in the undulator [31, 32]. Power radiated in the central radiation cone produced by N_e electrons in planar undulator is investigated and presented analytically [33].

Planar Undulator

The magnetic field of linearly polarized undulator is given by, $\vec{B} = 0.\hat{x} + B_0 \sin(\omega_u t)\hat{y} + 0.\hat{z}$, where B_0 is the amplitude of the undulator field. The formula required for the amplitude of undulator field B_0 produced by Halbach configuration with Pure Permanent Magnet (PPM) is given by

$$B_0 = 2\epsilon B_{\text{rem}} (1 - e^{-k_u L'}) (e^{-k_u g_u / 2}) \text{sinc}(\epsilon\pi/M) \quad (1)$$

where B_{rem} is the remnant field of pure permanent magnet and $B_{\text{rem}}=1.1$ Tesla in case of NdFeB [34]. In our study, we choose $\epsilon = 1$ and $M=4$ which represents number of magnets required to complete one period, $L' = \lambda_u / 4$. Using equation (1), we obtain the peak value of undulator field which is given by

$$B_0 = 1.57(\text{Tesla}) \exp(-\pi g_u / \lambda_u) \quad (2)$$

The expression of peak value of undulator parameter K is given by

$$K = 0.148\lambda_u (\text{mm}) \exp(-\pi g_u / \lambda_u) \quad (3)$$

where g_u is undulator gap in mm. We carry out the analysis of practically designed parameters of undulator to produce THz radiations in the next section.

Electron Trajectories

By applying the Relativistic Lorentz Force (RLF) equation $d\vec{p}/dt = -ec(\vec{\beta} \times \vec{B})$ for the calculations of velocity and electron trajectories using Generalised Bessel Function (GBF) and Relativistic Lorentz Force (RLF) equation, the 3D trajectory in undulator is written as

$$\vec{r}(t) = -\frac{Kc}{\gamma\omega_u} \sin(\omega_u t) \cdot \hat{x} + 0 \cdot \hat{y} + \left[\beta^* ct - \frac{K^2 c}{8\gamma^2 \omega_u} \sin(2\omega_u t) \right] \cdot \hat{z} \quad (4)$$

$$\text{where } \beta^* = 1 - \frac{1}{2\gamma^2} (1 + K^2/2 + (\gamma\psi)^2) \text{ is average velocity of electron beam.} \quad (5)$$

Equation (4) describes the electron trajectory with axial and longitudinal oscillations. Longitudinal oscillations take place at double of the fundamental frequency whose contribution to the oscillation process leads zero contribution.

Power Radiated in the Central Radiation Cone

The normalized electron velocity of electron through undulator structure is derived by equation (4)

$$\beta_x = -\frac{K}{\gamma} \cos(\omega_u t) \quad (6)$$

The maximum amplitude of electron transverse normalized velocity oscillation is described by equation (6) which is responsible for emission of radiation. The off axis average power radiated due to the electron beam of current I_b is derived (33) and follows as

$$\tilde{P}_{ave} = \frac{\pi e \gamma^2 I_b (A) [T_{xm}^2 + T_{ym}^2] \sin^2 c^2 (\gamma T/2)}{\epsilon_0 \lambda_u} \left[\frac{mK}{(1 + K^2/2 + (\gamma\psi)^2)} \right]^2 \quad (7)$$

On using practical units, on axis, we get the expression of power emitted from undulator

$$\tilde{P}_{ave} = \frac{(5.67 \times 10^{-6} \text{ W}) \gamma^2 I_b (A) [J_{\frac{m+n}{2}}(Q_m) + J_{\frac{m-n}{2}}(Q_m)]^2 \sin^2 c^2 (\gamma T/2)}{\lambda_u (\text{cm})} \left[\frac{mK}{(1 + K^2/2)} \right]^2 \quad (8)$$

However, on axis average power of central cone ($n=1$) of planar undulator is simplified from equation (8) as

$$\tilde{P}_{cen} = \frac{(5.67 \times 10^{-6} \text{ W}) \gamma^2 I_b (A)}{\lambda_u (\text{cm})} \left[\frac{mK}{(1 + K^2/2)} \right]^2 [J_1(Q_m) + J_0(Q_m)]^2 \quad (9)$$

The on axis spectral brightness of electron beam having current I_b and beam energy from planar undulator is specified as (35)

$$\tilde{B}(dN/d\Omega, d\omega/\omega, 0.1\%BW) = \frac{7.53 \times 10^6 \gamma^2 N_u^2 I_b (A) [J_{\frac{m+n}{2}}(Q_{ms}) + J_{\frac{m-n}{2}}(Q_{ms})]^2 \sin^2(\nu_m T/2)}{\sigma_x (mm) \sigma_y (mm) \sqrt{1 + \frac{\sigma_x^2}{\theta_{cen}^2}} \sqrt{1 + \frac{\sigma_y^2}{\theta_{cen}^2}}} \left[\frac{mK}{(1 + K^2/2)} \right]^2 \quad (10)$$

The SI unit of spectral brightness is $\left[\frac{\text{Photons/sec}}{\text{mm}^2 \text{mrad}^2 (0.1\%BW)} \right]$. Spectral brightness expression reveals explicitly various parameters of electron beam. It explains how beam current, beam emittance and beam divergence affect the spectral brightness. It is also interpreted that beam current and electron beam emittance should be balanced well, otherwise, numerical value of spectral brightness is degraded drastically.

Results and Discussion

In the current investigation, the electron beam parameters of ILU-14 accelerator are used for analytical calculations for planar undulator. The accelerator produces an electron pulse with an energy-spread ranging from 8 % at 7.5 MeV to 7.7 % at 10 MeV, operating at a repetition rate of 50 Hz. Its operating frequency is 176 MHz, with a total efficiency of 26 %. The accelerator achieves maximum energies of 7.5 MeV and 10 MeV, with an average power of approximately 100 kW. The characteristic impedance is 1240 Ω , and the shunt impedance is 27.3 M Ω [36]. The relative spectral bandwidth of the radiation is defined as $\Delta\lambda/\lambda = 1/N_u$ which has numerical value for planar undulator parameter is 6.7%.

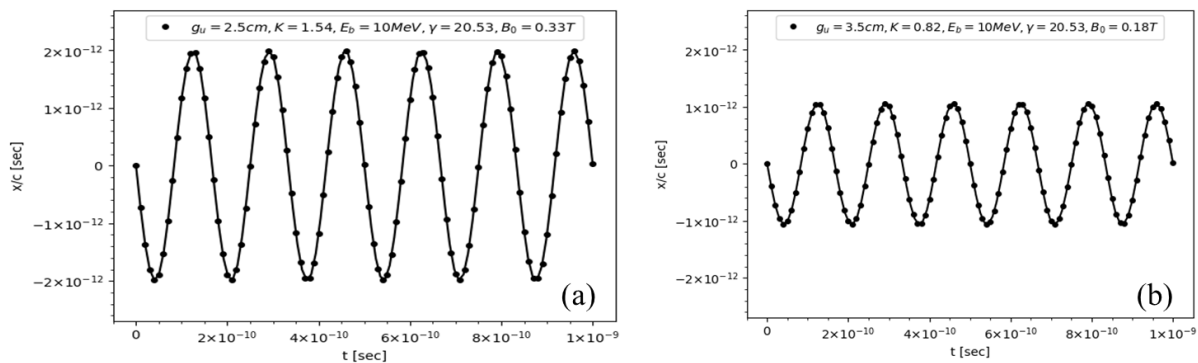


Fig.1(a) & 1(b) shows the transverse oscillations confined in x-axis through undulator due to decreasing value of K and B_0 . Both reduces on increasing undulator gap from (a) $g_u = 3.0 \text{ cm}$ to (b) $g_u = 3.5 \text{ cm}$.

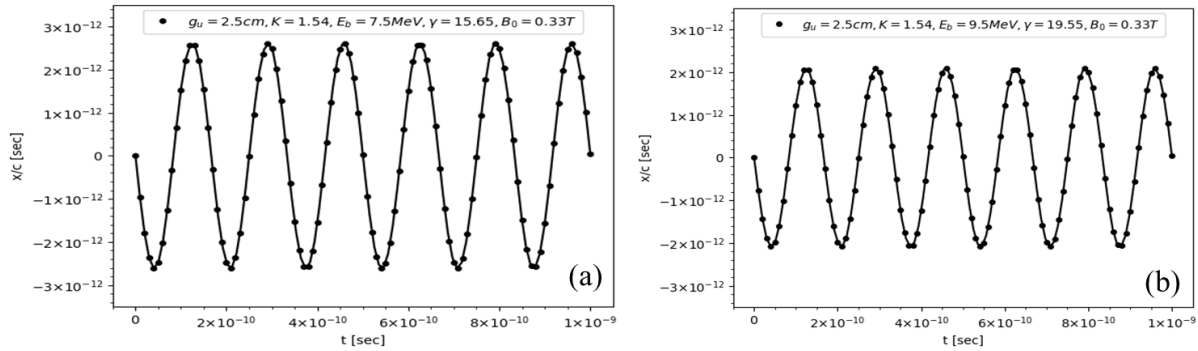


Fig. 2(a) & (b) shows amplitude of the transverse oscillations confined in x-axis due to increasing electron beam energy which directly affects the motion of electrons.

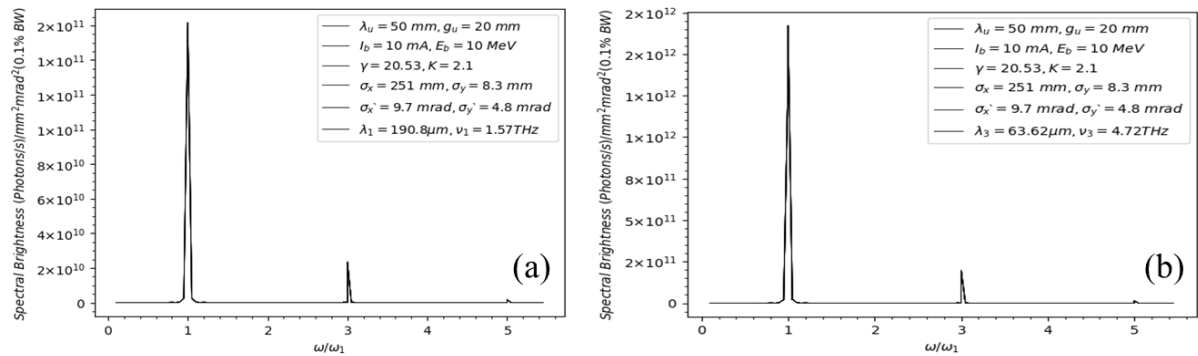


Fig.2 (a) & 2 (b) shows the on axis spectral brightness of fundamental frequency (n=1) is 2×10^{11} (photons/sec/mm²mrad² 0.1%BW) at 190.8 μm wavelength and 1.57 THz frequency, (b) of third harmonic frequency (n=3) is approximately 2×10^{11} (photons/sec/mm²mrad² 0.1%BW) at 63.62 μm wavelength and 4.72 THz frequency with electron beam energy $E_b = 10$ MeV.

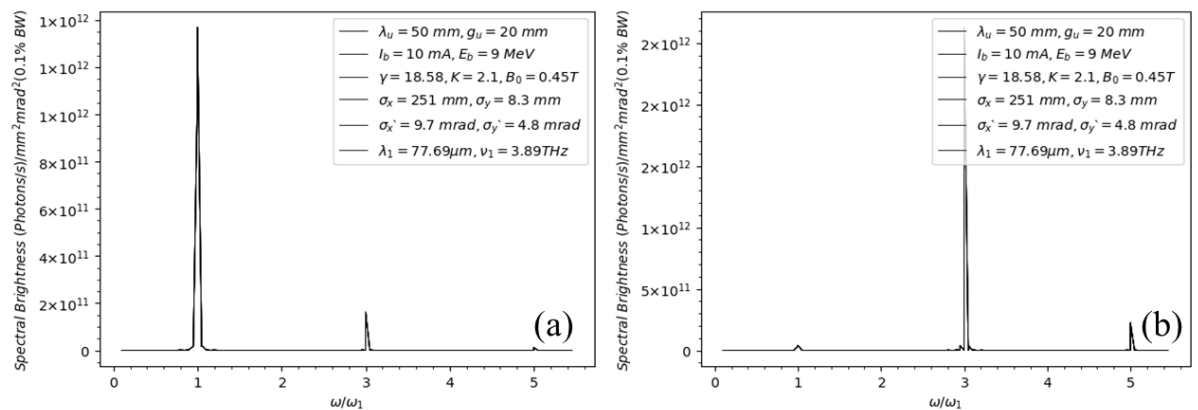


Fig.3(a) & 3(b) shows the on axis spectral brightness of first harmonic frequency (n=1) is approximately 13.5×10^{10} (photons/sec/mm²mrad² 0.1%BW) at 233.1 μm wavelength and 1.29 THz frequency, (b) of third harmonic frequency (n=3) is approximately 2.5×10^{12} (photons/sec/mm²mrad² 0.1%BW) at 77.69 μm wavelength and 3.86 THz frequency with electron beam energy $E_b = 9$ MeV.

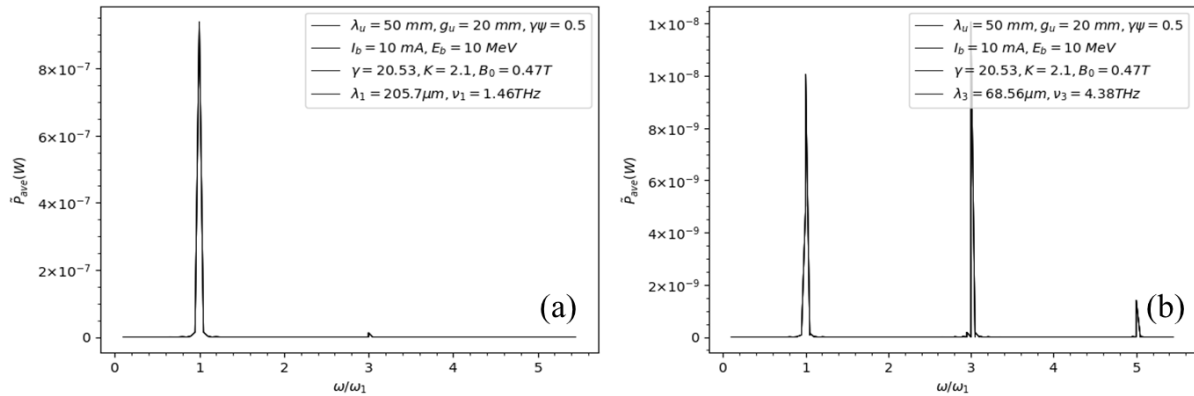


Fig. 4(a) & 4(b) shows the off axis $\hat{P}_{ave}(W)$ of first harmonic frequency ($n=1$) is approximately 9.5×10^{-7} (W) at $205.7 \mu m$ wavelength and 1.46 THz frequency, (b) shows the off axis $\hat{P}_{ave}(W)$ of third harmonic frequency ($n=3$) is approximately 1.2×10^{-8} (W) at $68.56 \mu m$ wavelength and 4.38 THz frequency with electron beam current $I_b=10$ mA.

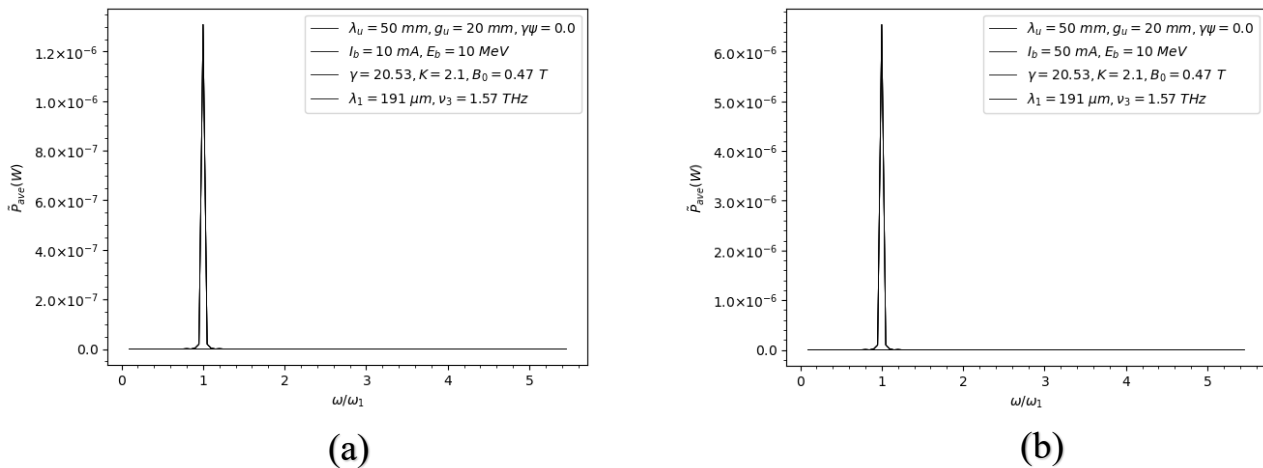


Figure 5 (a) and 5 (b) demonstrate the power radiated in central cone for various parameters of electron beam. It is revealed that on increasing the current of electron beam, the radiated power in central cone is enhanced which is consistent with equation (9). The value of radiated power in central cone is displayed from $1.4 \mu W$ to $6.3 \mu W$ for corresponding current from 10 mA to 50 mA.

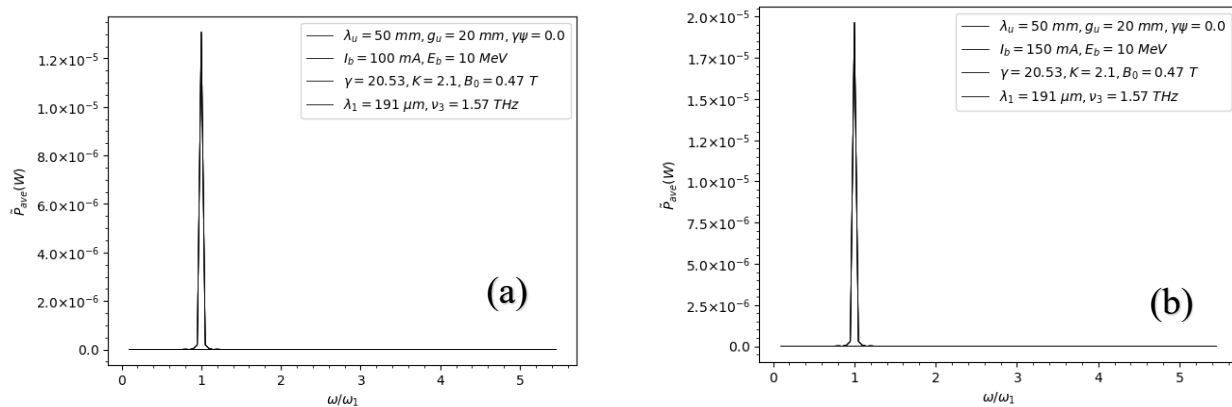


Figure 6 (a) and 6 (b) demonstrate the power radiated in central cone for various parameters of electron beam. It is revealed that on increasing the current of electron beam, the radiated power in central cone is enhanced which is consistent with equation (9). The value of radiated power in central cone is displayed from 12.2 μW to 20.0 μW for corresponding current from 100 mA to 150 mA.

Thus, current calculations demonstrate the spectral brightness and average power radiated in central cone for various undulator parameter and linear accelerator parameters shown in various figures are consistent with equations mentioned earlier. Analytical results obtained are useful for the development of THz-FEL for various applications.

Conclusion

Finally, the results achieved confirm the production of 1 THz to 5 THz radiations at wavelength from 66 μm to 250 μm using planar undulator. The calculations showed the spectral brightness of 2×10^{11} (photons/sec/mm²srad² (0.1%BW)) can be applied to non-invasive identification of biomolecular vibrational modes, particularly for protein hydration and conformational dynamics in living cells, at 190.8 μm wavelength with 1.57 THz frequency. A THz-FEL delivering 9.5×10^{-7} Watt at 205.7 μm wavelength and 1.46 THz frequency enables label-free identification and differentiation of healthy versus cancerous tissues in biomedical imaging, leveraging the strong THz absorption contrast of water and biomolecules [37].

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