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Students' Common Errors in Geometric Proving: Basis for an Enhancement Program

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ABSTRACT

This Qualitative Research was conducted in November 2024 among three Geometry teachers in one of the public secondary schools in Dingle. It addressed in identifying common errors in geometric proof-writing among students and the underlying causes of these errors as basis for an enhancement program. The respondents were interviewed using guide questions which underwent content validation by three experts in the field of Mathematics. Thematic Data Analysis was used to analyze the responses of the teachers. The study revealed that common errors made by students during geometric proof-writing were incorrect or missing statements, organizational issues, and algebraic errors. These errors were caused by insufficient grasp of definitions and postulates, weak algebraic skills, difficulty in logical reasoning, and lack of confidence in mathematical abilities. The enhancement program, Remedial Class, on geometric proof-writing is recommended to the Geometry teachers in improving students' ability to write accurately and effectively. These have helped the students develop a deep understanding of the concepts, strong problem-solving skills, and systematic approach to proof-writing. This study concluded that identifying common errors among students can significantly help address their difficulties in tackling problems involving geometric proof-writing. This process also promotes critical-thinking by encouraging precision in constructing valid proofs.

Keywords: common errors, geometric proof-writing

INTRODUCTION

Geometry and reasoning have been connected in the writing of Euclid's *Elements*, which provided a paradigm for deductive reasoning in the context of geometric proof. This connection has motivated, in part, the inclusion of geometric proof in high school mathematics curricula (González & Herbst, 2006; Sinclair, 2008).

Decades of research suggests that students have struggled with learning proof in Geometry (Cirillo and Hummer, 2019). Many students struggle with constructing rigorous and logically sound proofs, leading to misconceptions and hindering the overall mathematical understanding.

Research suggests that teachers struggle to find effective ways to introduce proof. In 1940, Smith argued that being aware of student misconceptions in Geometry is the first step in preparing to address the fundamental challenges of learning to prove. Through careful study, he identified and analyzed "three serious learning difficulties" that students have in connection with; lack of familiarity with geometric figures, not sensing the meaning of if-then relationship, and inadequate understanding of the meaning of

proof (1940, p.100). Smith found that when these things were attended to explicitly improved students' results.

The study of Dreyfus, et. al (2022) revealed the findings point clearly to effects of elements within the students' understanding of definitions on their understanding of proofs and on their ability to prove. These elements are as follows; the difficulty to internalize that an incomplete definition is an incorrect definition and may lead to an incomplete proof, followed by the difficulty to deal with non-economical definitions forms the basis to non-economical proofs, then students have difficulties to accept equivalent definitions to the same geometrical concept, and the students' lack of understanding the origin of a constructive definition of geometrical concept.

The study of Mwadzaangati (2020) concluded that the textbook presented the geometric proof development tasks at a high level as they included both empirical exploration tasks and formal proof tasks. Despite this task setup in the textbook, only one teacher involved the learners in empirical exploration tasks and maintained the high cognitive level of the tasks during instruction. The other two teachers only presented the formal proof tasks. Although the formal proof tasks that were set up by the two teachers were of a high cognitive level, the procedures that were used during task implementation resulted in reduction of the cognitive level of the proof tasks.

This research addressed to identify these prevalent errors, analyze the underlying causes and propose targeted enhancement program to improve proof-writing skills in order to develop effective enhancement programs in school. By understanding the nature of these errors, teachers can design a more effective instructional strategies and assessment tools to support students' learning and mathematical proficiency.

This agreed with the study of Bautista (2022) whose mean scores of the student taught with or without error analysis before and after the intervention was "did not meet expectation" and "fairly satisfactory", respectively. There was no significant difference found in performance of the students in the control and experimental groups after controlling the pre-test scores. Thus, the use of error analysis as a formative assessment can be an innovative teaching technique in improving the performance of the students.

This study underscores the crucial need to enhance geometric proof-writing among students. By addressing a gap in mathematics education, it aims to equip students with mastery of proof techniques. Furthermore, the study will inform the development of an evidence-based enhancement program that directly targets students' weaknesses in proving. Ultimately, it contributes to a deeper understanding of the cognitive processes involved in geometric proofs, thereby informing the on-going development of effective teaching and learning strategies in this area.

Specifically, it sought answers to the following questions:

1. What are the most common errors made by students in geometric proof-writing?
2. What are the underlying causes of these common errors?
3. What enhancement program can be done to effectively mitigate these difficulties and improved students' geometric proof-writing performance?

METHODOLOGY

This study used Qualitative Research. It is defined as "the study of the nature of phenomena", including "their quality, different manifestations, the context in which they appear or the perspectives from which they can be perceived", but excluding "their range, frequency and place in an objectively determined chain of cause and effect" (Busetto, et.al, 2020).

Brannan (2022) added that this type of research gathers participants' experiences, perceptions and behaviour. It answers the hows and whys instead of how many or how much. It could be structured as a standalone study, purely relying on qualitative data, or part of mixed-methods research which combines qualitative and quantitative data. It introduces the readers to some basic concepts, definitions, terminology and applications of qualitative research.

The respondents of this study were the three Geometry Teachers of Rufino G. Palabrica Sr. National High School teaching Geometry for more than five years. They were interviewed based on three guide questions related to geometric proof-writing. Questions underwent content validation by three experts in the field of mathematics. The responses of the teachers were recorded and analyzed to identify recurring themes and patterns related to the causes of errors. Thematic Data Analysis was used to emphasize identifying, analysing and interpreting patterns of meaning or themes within qualitative responses.

RESULTS

This study has identified common errors in geometric proof-writing among Grade 9 students of Rufino G. Palabrica Sr. National High School. The responses derived from interview, provided rich insights in identifying common errors in proving, the underlying reasons of these errors, and possible enhancement program to be implemented to enhance students' geometric proof-writing performance.

The three teachers unanimously agreed that students' common errors were incorrect or missing statements, organizational issues, and algebraic errors. Themes on errors in incorrect or missing statements were

omission of mathematical definitions, theorems or logical steps. Errors in organizational issues can be categorized to missteps in proof structuring. While algebraic errors identified were mistakes in algebraic manipulation, simplification or arithmetic.

In terms of the underlying causes of these errors, these were caused by insufficient grasp of definitions and postulates, weak algebraic skills, difficulty in logical reasoning, and lack of confidence in mathematical abilities. Insufficient grasp of definitions and postulates in code can lead to various types of errors, including conceptual misunderstanding, limited practice, and language and interpretation barriers. On the other hand, underlying causes of errors in weak algebraic skills were arithmetic and simplification errors, and misapplication of algebraic properties. As to difficulty in logical reasoning, lack of sequential thinking and inability to justify steps were the causes of errors. Avoidance of complex problem and relevance in memorization over understanding were identified errors under lack of confidence and mathematical abilities.

As to the enhancement program, remediation class is recommended to address these identified errors and improve students' performance in geometric proof-writing skills.

DISCUSSION

This Qualitative Research has identified the following geometric proof-writing errors made by Grade 9 students.

The figure below provides a visual overview of the study's results.

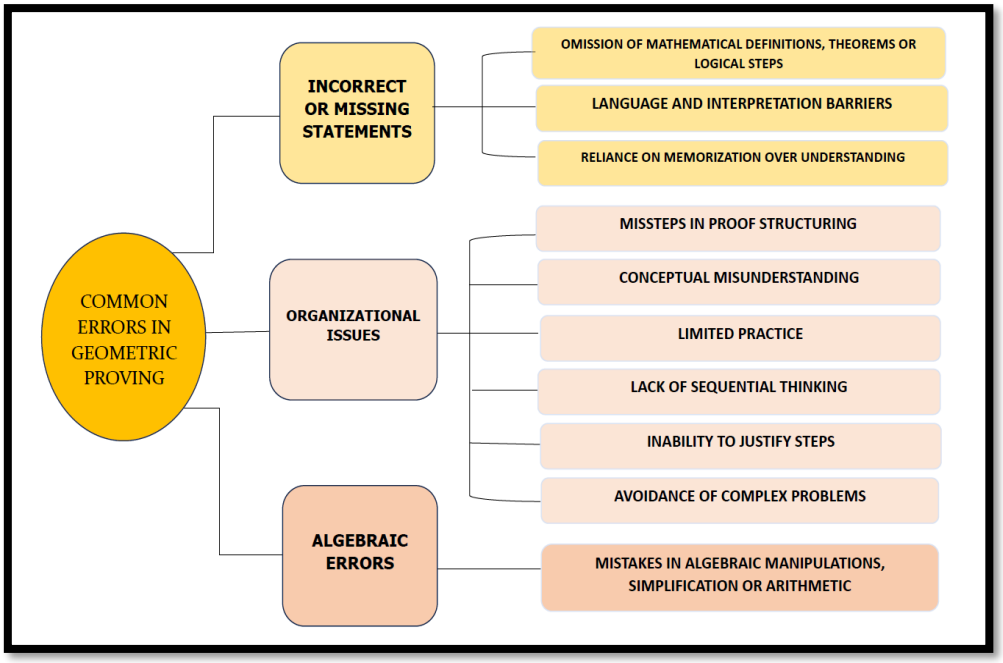

Figure 1 illustrates common errors made by students in geometric proofs, along with the identified themes associated with these errors.

Common Errors

Three teachers unanimously said that the common errors made by students during geometric proof-writing were incorrect or missing statements, organizational issues, and algebraic errors.

The following themes emerged as recurring patterns in the errors students made:

1. Incorrect or Missing Statements

Omitting mathematical definitions, theorems, or logical steps in geometric proof-writing can lead to incomplete or invalid arguments, as each element provides the necessary foundation for logical reasoning. Such omissions hinder the clarity and rigor of the proof, making it difficult for students to verify or follow the reasoning process.

Teacher A said, "*Ang mga estudyante nabudlayan mag-intindi ka definitions. Napang-omit da iban nga important words. Gasala sanda tapos nalipatan da ya mga fundamental concepts sa proving* (Students have misunderstanding of definitions. Some important words are omitted. They have misconceptions and lack understanding of fundamental concepts in proving)".

On the other hand, Teacher B added "*Napanglipatan ka iban ang mga theorems. Man an da gid importante daang theorems para maka-prove sanda ensakto pay* (Some students also missed some theorems that are important in proving statements accurately)".

As interviewed to Teacher C, she confirmed that *"Kalabanan ka mga estudyante nabudlayan magpalibog. Budlayan sanda magpasunod ka ideas. Muda gaka-confuse sanda, sala ya sabat sa punta, kag budlayan mag-prove* (Most of the students find difficulty in understanding reasoning. They struggled to connect ideas effectively, leading to confusion, incorrect conclusions, and difficulty in solving proofs)''.

With the given responses of the teachers, the students find difficulties in understanding geometric terms, formulating theorems, and to follow correct order of ideas in proving.

Based on the above statements given by the teachers, incorrect or missing statements where omission of mathematical definitions, theorems, or logical steps were the frequent errors committed by the students. The study of Mathunya and Moru (2022) identified most errors committed by the learners as persistent. Overgeneralizing the rules of prior knowledge to new knowledge, particularly in different contexts, was the most frequent cause of the errors. In addition to this was the misunderstanding and misinterpretation of correct meanings in the given context. Some of the identified errors overlapped with those in the reviewed literature while others did not.

2. Organizational Issues

In geometric proof-writing, students often struggle with maintaining logical flow, leading to disconnected statements that fail to link given information to the desired conclusion. Missteps also arise from a lack of clarity in their reasoning, such as skipping crucial steps or presenting incomplete justifications, which weakens the coherence and validity of the proof.

Teacher A mentioned that "*Ang iban nga mga estudyante kulang sa structure ang proof. Waay klaro anda pagparasunod. Nabudlayan sanda magpasunod ka proof. Amuda duru anda sala kag napang-omit* (Some students have lacked proof structure without clear and logical flow which was confusing and difficult to follow, leading to errors and omissions)''.

As to teacher B, "*May dayan nga mga estudyante nga waay gakaraamo anda reasoning ka ideas. Muda budlayan sanda mag-organize ka sakto nga pagparasunod ka proof.* (There are students that have disorganized reasoning of ideas made difficult to organize the correct flow of proof)".

Teacher C commented, *''May dayan gani nga sala ana nga steps sa pag –prove. May napanglaktawan nga steps* (There are even students with wrong steps in proving. Some steps were performed intermittently)".

Teacher responses indicate that students struggle with structuring of proofs and often lack clarity in the steps they take to prove a geometric statement.

The statements of the teachers on organizational issues regarding missteps in structuring proof as between ideas without logical flow or lack of clarity in steps was the errors made by students in geometric proving.

The findings in the study of Asvat and Stemele (2024) confirmed the error types identified in the literature and demonstrated a notable improvement in performance on the post-test following the intervention using algebra tiles. The results indicated that the intervention successfully rectified pre-existing errors and misconceptions, resulting in an 18% enhancement in overall performance among participants. The study aligned with a Vygotskian sociocultural perspective, emphasizing the pivotal role of manipulatives in facilitating learning within the zone of proximal development. The use of manipulatives aids learners in constructing conceptual understanding by reinforcing abstract ideas.

3. Algebraic Errors

Students often make errors in algebraic manipulation, such as incorrectly distributing signs or simplifying fractions, leading to flawed geometric proofs. Another common mistake involves inaccurate arithmetic calculations, resulting in incorrect values for lengths, angles, areas within the proof's arguments.

According to Teacher A, *''May mga estudyante nga naga-sala mag-solve ka equation. Sala pag-gamit da ka algebraic properties, labi na gid sa errors da sa sign ka numbers* (Some students have mistaken in equation solving, misapplications of algebraic properties, especially on sign errors of numbers)".

While Teacher B answered that, *"Indi iban kamaan paano mag simplify. Bisan mag-plus dulang gani* (Some students does not know how to simplify. Even addition poses a challenge for some students)".
On the other hand, Teacher C added that "*Basta may algebraic manipulations gani, iban indi gid kamaan a* (Some students are unsure of how to perform algebraic manipulations)".
Given the responses of the teachers, students have errors in algebraic manipulations, simplifications of equations, and operations of arithmetic.
Mistakes in algebraic manipulations, simplifications, or arithmetic were errors made by students in algebraic errors.
Bayaga and Bosse (2024) uncovered several prevalent error categories, including conjoining, cancellation, and problem-solving errors, indicating deficiencies in conceptual comprehension and procedural execution. Moreover, applying the SOLO taxonomy elucidates learners' diverse levels of understanding, with a majority position within the uni-structural or multi-structural stages. Theoretical implications underscore the necessity for tailored instructional approaches to mitigate learners' obstacles and foster a deeper grasp of algebraic principles.

Underlying Causes of Errors
As to the underlying causes of these common errors, the following themes emerged:
1. Insufficient Grasp of Definitions and Postulates
Students' unstable grasp of geometric definitions and postulates often leads to conceptual misunderstandings, hindering the ability to construct rigorous proofs. Limited practice, coupled with language and interpretation barriers, further exacerbates these difficulties, resulting in flawed reasoning and incomplete arguments.
Teacher A stated that *"Kalabanan sa anda nabudlayan kay indi gid nanda maintindihan ang buut hambalan ka gina-defined. Saulo da ang words, pay indi sanda ka internalize ka concepts* (Many students struggle because they don't truly understand the meaning behind the definitions. They memorize the words, but they don't internalize the concepts)".
As to Teacher B, "*Indi bastante ang basahon lang dayang mga definitions kag postulates. Uyon tana kadayang proving duru nga practice dapat. Kun waay it practice, budlayan gid sanda* (It's not enough to just read the definitions and postulates. Geometric proof requires a lot of practice. Without that practice, they're left floundering)".
"Ang language abi ka Geometry makasarala. Pirmi, nabudlayan sanda mag-interpret ka word problems kag mag-translate ka given information (The language of Geometry can be tricky. Students often struggle to interpret the wording of problems and translate the given information. This is especially true for students who are poor in English)", Teacher C quoted.
With the given responses of teachers, students' success in geometric proving hinges on a deep understanding of definitions and postulates, ample practice, and overcoming language barriers to ensure accurate problem interpretation.
Teachers' statement on the underlying causes of common errors in insufficient grasp of definitions and postulates were conceptual misunderstandings, limited practice, and language and interpretation barriers.
The findings of Chiphambo and Feza (2020) showed that daily reflective tests can serve as essential tools in helping mathematics this paper recommends mathematics teachers to make optimal use of diagnostic and reflective tests to guide them in identifying teacher discover more and more in-depth students' alternative geometric vocabulary, conceptual and terminology misunderstandings. In

conclusion students' alternative geometric vocabulary, conceptual and terminology misunderstandings in learning geometry.

2. Weak Algebraic Skills

Even with a strong grasp of geometric concepts, students' weak algebraic skills can derail the proof-writing process, leading to arithmetic and simplification errors within the algebraic manipulations necessary for a complete argument. Misapplication of algebraic properties within geometric proofs results in invalid conclusions, even if the geometric reasoning is sound.

According to Teacher A, *"May makita ako nga mga kabataan nga kaintindi ka Geomety, pay dayan gid sanda gasala sa solvings da sa simple nga arithmetic. Kis-a, daw na-frustrate takon mag-lantaw ka ubra da hay careless sa calculations da* (I see so many students who understand Geometry, but they lose points on simple arithmetic mistakes within their algebraic steps. It's frustrating to watch a students' work undone by a careless calculation)".

For Teacher B, "*Ang problema abi indi nga indi sanda makaintindi ka mga geometric principles. Pirmi gid, nabudlayan sanda mag-simplify ensakto. Kinahanglan nanda mag-practice pagid kadaya para mag-araram* (The problem isn't always a lack of understanding of the geometric principles. Often, students struggle to simplify their algebraic expressions correctly. They need more practice simplifying expressions to become more efficient)".

As for Teacher C, *"Amo gid daya pirmi ang sala nanda sa application ka algebraic properties. Ka-careless kananda sa signs* (A common issue is the misapplication of algebraic properties. Students are careless about signs of integers)".

With these statements of teachers, students' weak algebraic skills, manifesting as arithmetic errors, simplification difficulties, and misapplication of algebraic properties, significantly hinder students' ability to successfully complete geometric proofs, even when geometric understanding is strong.

Teachers' statements on the underlying causes of common errors in weak algebraic skills were arithmetic and simplification errors and misapplication of algebraic properties.

Results of the study of Marpa (2019) disclosed that pre-service teachers have difficulties in classifying polynomials according to the degree and polynomials in parenthesis. They also have difficulties in translating mathematical phrases into symbols. Thus, mathematics teachers should strengthen instruction on these topics. Mathematics teachers handling the subject are also encouraged to develop learning exercises that will develop the mastery level of the students.

3. Difficulty in Logical Reasoning

Students' common challenge in geometric proof-writing is difficulty with logical reasoning, often characterized by a lack of sequential thinking, where students struggle to connect steps in a coherent argument. This difficulty is further compounded by an inability to justify each step with relevant definitions, postulates, or previously proven theorems, resulting in incomplete or unconvincing proofs.

Teacher A talk about *"Nagalumpat anda reasonings, tapos ya mga sugpon na waay gaarangot sa isa kag isa. Kinahanglan madevelop nanda ang pagpanumdumdum ka pagparasunod ka mga ideya para nami resulta aka ubra da* (Students often jump around in their reasoning, making connections that don't logically follow. They need to develop a stronger sense of sequential thinking to build a solid argument)".

Teacher B bring up *"May mga statements sanda nga nahimo, pay indi nanda ma-explain ang reason. Waay gid nanda na-master ya parte sa geometric concepts kag kun paano magamit sa anda proof*

(They'll make a statement, but they can't explain the reason. They haven't mastered the connection between the geometric concepts and how to articulate them in a proof)".

Teacher C point out *"May manami sanda nga perception sa Geometry, pay magsulat sanda ka proof indi organisado kag bukon it klaro* (They often have a good perception about Geometry, but their proof writing is messy and unclear)".

Based on the statements of teachers, students' difficulties in geometric proof-writing often stem from a lack of sequential thinking and the inability to justify each step with appropriate geometric principles, resulting in flawed or incomplete logical arguments.

Teachers' statements on the underlying causes of common errors in difficulty in logical reasoning were lack of sequential thinking and inability to justify steps.

Braithwaite (2022) mentioned in his study that university students completed a scaffolded geometric proof construction task in which they selected justifications for proof steps, and assessments of probabilistic reasoning, conditional inference, and algebra. Accuracy on the geometric proof justification task predicted probabilistic reasoning when controlling for conditional inference or algebra, though not both. Variance in probabilistic reasoning that was uniquely explained by geometric proof justification was related to effective use of given information on the geometric proof justification task.

4. Lack of Confidence in Mathematical Abilities

Students' lack of confidence in their mathematical abilities often leads to avoid tackling more complex geometric proof problems, hindering their progress and development of problem-solving skills. This lack of confidence frequently manifest as a reliance on rote memorization of proof structures rather than a deeper understanding of the underlying geometric principles and logical reasoning involved.

Teacher A said that *"May dayan tana nga pagkita na nga mabudlay, tamadan dun magsabat. Uyon da tanda mahapos lang ya itao mo nga sabtan da* (Some students avoid challenging proofs. They stick the easy ones to answer)".

While Teacher B added that *"Duru nga mga bata ginasaulo da ya pattern mag-prove,nga imbis daad intindihon da danay ang logic ka problem* (Many students try to memorize proof templates instead of truly understanding the logic)".

Also, Teacher C commented that *"Ay pirmi lang amu daya. Waay tanda sarig sa kaugalingon da a. Amuda nga nalaktawan da kag mag-sarig dulang sa pag-memorize. Te kundi ano pag-intindi da kag gadura self-confidence da sa proving* (It's a vicious cycle. Lack of confidence leads to avoidance and reliance on memorization, which further hinders their understanding and reinforces their lack of confidence)".

Referring to the statements of the teachers, students' lack of confidence in geometric proof-writing often manifests as avoidance of challenging problems and over-reliance on memorization instead of developing a strong understanding of underlying geometric principles and logical reasoning.

Teachers' statements on the underlying causes of common errors in lack of confidence in mathematical abilities were avoidance of complex problems and reliance on memorization over understanding.

The study of Aisyah (2023) showed that students' prospective teachers did not prioritize proof in solving geometry problems, even though they worked hard to solve the problems independently until they were finished. The students' perseverance also impacts their mathematical reasoning in proving geometric theorems. Students with more hard work values tend to have more reasoning values. The results of this study have implications that there needs to be an effort from the teacher to get used to giving proof questions to support students' perseverance and mathematical reasoning abilities.

Proposed Enhancement Program

As to the enhancement program to be done, teachers could create remediation program which will help students improve geometric proof-writing abilities. It could help students who were struggling with proofs, focusing on areas where students require additional assistance. Through this enhancement program on proving, students will be empowered to achieve academic achievement and reach full potentials.

Somani (2023) mentioned that remedial classes and remedial education are important because they provide students with the support they need to succeed academically. Students who are struggling with a particular subject or skill may feel discouraged and overwhelmed, which can lead to a lack of motivation and a decrease in student performance. Remedial classes provide these students with the resources they need to overcome these obstacles and achieve academic success. By conducting remediation classes, teachers can identify further the gaps in a students' knowledge to provide targeted lessons that can help students to catch up with their peers (Thakuria, 2024).

Limitation of the Study

The limitation of this study is the challenge of clearly defining distinct categories of geometric proof errors, as underlying causes may overlap. This limitation may affect the accuracy of automated error detection. Future researchers could address this by incorporating cognitive factors that contribute to geometric errors, such as working memory capacity, spatial reasoning abilities, and understanding of abstract geometric concepts.

CONCLUSION

This study identified common errors made by students in geometric proof-writing, revealing significant implications for both teaching and learning Mathematics. Understanding these errors and their underlying causes can empower teachers to develop targeted interventions and programs to address these challenges

This study recommends that teachers implement remediation classes to help students' master geometric proof-writing.

Furthermore, identifying common errors among students can significantly help address their difficulties in tackling problems involving geometric proof-writing. This process also promotes critical-thinking by encouraging precision in constructing valid proofs.

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