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Construction of Balanced N-Ary T-Designs by Block Sum and Product (BSP) Methodology on 3-Designs

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Abstract:
In this paper we use 3-designs whose incidence matrix will take only binary values and constructed a series of balanced n-ary t-designs by using a tool Block Sum and Product (BSP) Methodology. The simple 3-(4, 3, 1) and 3-(5, 3, 1) designs are used as a parent 3-designs for the procedure. The parent 3-(4, 3, 1) design gives three new balanced n-ary t-designs and 3-(5, 3, 1) design gives twenty new balanced n-ary t-designs. We given incidence matrices and list out the parameters of newly constructed balanced and partially balanced n-aryt-designs.

Keywords: Balanced n-ary t-design, BSP Methodology, Incidence matrix, Polynomial, t-design

1. Introduction

Balanced n-ary designs were defined by Tocher (1952). The construction of balanced n-ary designs using a set of mutually orthogonal Latin squares is provided by Murthy and Das (1967), and using differences of sets by Saha and Dey (1973), as well as through BIB and two associated PBIB-triangular designs by Agarwal and Das (1987). Agarwal and Sharma (1976) obtained a series of balanced n-ary designs by collapsing certain (n-1) tuplets of blocks, suitably picked from the blocks of a BIBD. The method of constructing balanced ternary (3-ary) designs is given by Saha (1975). The use of n-ary block designs in diallel cross evaluations is provided by Agarwal and Das (1990). A recursive method for the construction of balanced n-ary designs is given by Gheribi-Aoulmi and Bousseboua (2005). The construction of balanced and partially balanced n-ary t-designs using BSP Methodology on 2-design and PBIBD(2) was attempted by Phad and Pawar (2016, 2017)

Definition 1.1 : A Balanced Incomplete Block Design (BIBD) is a set X of V (≥ 2) elements called treatments and a collection of B(> 0) subsets of X, called blocks, such that the following conditions are satisfied:

1. Each block contains exactly K treatments.
2. Each treatment appears in exactly R blocks.
3. Each pair of treatments appears simultaneously in exactly λ blocks.

Definition 1.2: A t-(V, K, Λ_t) block design (abbreviated t-design) is an incidence structure of treatments and blocks such that the following holds:

1. There are V treatments
2. Each block contains K treatments
3. For any t treatments there are exactly Λ_t blocks that contain all these treatments.

A 2-design is called Balanced Incomplete Block Designs (BIBD) and was first used as a statistical design by Yates (1936).

Definition 1.3 : Balanced n-ary t-design is an arrangement of V treatments in B blocks such that:

i^{th} treatment occurs in the j^{th} block n_{ij} times, i=1,2,….,V; j=1,2,….,B

n_{ij} can take 0 or 1 or 2…..or (n-1) value.

$$\sum_{i=1}^{V} n_{ij} = K, \qquad \sum_{j=1}^{B} n_{ij} = R, \qquad \sum_{j=1}^{B} n_{lj} n_{mj} = \begin{cases} \Delta & if \quad l = m \\ \Lambda_2 & if \quad l \neq m \end{cases}$$

For any t treatments there are exactly Λ_t blocks that contain all these treatments.

Further let R_i be the number of blocks in which any treatment occurs i-times and K_i be the number of treatments which occurs i-times in each block. Then the following relations hold:

$$\sum_{i=0}^{n-1} R_i = B, \qquad \sum_{i=0}^{n-1} K_i = V$$

$$\sum_{i=0}^{n-1} iR_i = R, \qquad \sum_{i=0}^{n-1} iK_i = K, \qquad \sum_{i=0}^{n-1} i^2 R_i = \Delta,$$

$$\Delta = RK - \Lambda_2 (V-1)$$

We used a tool Block Sum and Product (BSP) Methodology (2007) on a 3-(4, 3, 1) and 3-(5, 3, 1) designs and constructed new balanced n-ary t-designs. BSP Methodology gives number of designs, according to incidence matrix and parameters of newly constructed design it classified into balanced n-ary t-designs.

2. Construction of balanced n-ary t-designs:

2.1 Construction of balanced n-ary t-designs by BSP Methodology on 3-(4, 3, 1) design:

We consider 3-(4, 3, 1) design with parameters V=4, B=4, R=3, K=3, Λ_t=1. To apply BSP Methodology replace 1, 2, 3, 4 treatments by X_1, X_2, X_3, X_4. Take block sum B_i then consider product of B_i.

Table 1: The notation for BSP Methodology in 3-(4, 3, 1) design

Block Number (i)	Treatment content in block i	Treatment replaced for BSP	Block sum (B_i) for BSP
1	1 2 3	X_1 X_2 X_3	$B_1 = X_1 + X_2 + X_3$
2	1 2 4	X_1 X_2 X_4	$B_2 = X_1 + X_2 + X_4$
3	1 3 4	X_1 X_3 X_4	$B_3 = X_1 + X_3 + X_4$
4	2 3 4	X_2 X_3 X_4	$B_4 = X_2 + X_3 + X_4$

Product of Bi will be polynomial of degree 4 of variables X_1, X_2, X_3, X_4. This polynomial contains $K^B = 3^4$ (=81) terms. Similar types of terms of this polynomial are classified according to powers and new three designs are constructed.

Consider,

$$D_1 = \prod_{i=1}^{4} B_i = B_1.B_2.B_3.B_4 = 1\left[12 \quad \text{terms of the type} \quad X_i^3.X_j^1.X_k^0.X_l^0\right] + \; 4\left[12 \quad \text{terms of the type} \quad X_i^2.X_j^1.X_k^1.X_l^0\right]$$
$$+ \; 2\left[6 \quad \text{terms of the type} \quad X_i^2.X_j^2.X_k^0.X_l^0\right] + \; 9\left[X_1.X_2.X_3.X_4\right]$$

In the polynomial D_1, there are 12 terms of the type of the power 3100 and each term is repeated 1 time, the powers of these 12 terms give columns of the incidence matrix of design no.1 with V=4 and B=12 ; 12 terms of the type of the power 2110 and each term is repeated 4 times, the powers of these 12 terms give columns of the incidence matrix of design no.2 with V=4 and B=12 ; 6 terms of the type of the power 2200 and each term is repeated 2 times, the powers of these 6 terms give columns of the incidence matrix of design no.3 with V=4 and B=6. Last term is constant so it does not give any design. According to incidence matrix and parameters of newly constructed design it classified into balanced n-ary t-designs.

Table 2: Type of design and the parameters of newly constructed design using 3-(4, 3, 1) design

Design No.	Type of Design	Parameters of newly constructed design						
1	Balanced 4-ary 2-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**B**	**R**	**Λ**
		6	3	0	3	12	12	30
		K_0	**K_1**	**K_2**	**K_3**	**V**	**K**	**Λ_2**
		2	1	0	1	4	4	6
2	Balanced 3-ary 3-disgin	**R_0**	**R_1**	**R_2**	**B**	**R**	**Λ**	**Λ_3**
		3	6	3	12	12	18	6
		K_0	**K_1**	**K_2**	**V**	**K**	**Λ_2**	
		1	2	1	4	4	10	
3	Balanced 3-ary 2-disgin	**R_0**	**R_1**	**R_2**	**B**	**R**	**Λ**	
		3	0	3	6	6	12	
		K_0	**K_1**	**K_2**	**V**	**K**	**Λ_2**	
		2	0	2	4	4	4	

After applying BSP Methodology on 3-(4, 3, 1) design gives 3 balanced n-ary t-designs with parameters listed in table 2.

2.2 Construction of balanced n-ary t-designs by BSP Methodology on 3-(5, 3, 1) design

Let us consider 3-(5, 3, 1) design with parameters V=5, B=10, R=6, K=3, Λ_t=1. To apply BSP Methodology replace 1, 2, …, 5 treatments by X_1, X_2, …,X_5. Take block sum B_i then consider product of B_i.

Table 3: The notation for BSP Methodology in 3-(5, 3, 1) design

Block Number (i)	Treatment content in block i	Treatment replaced for BSP	Block sum (B_i) for BSP
1	1 2 3	X_1 X_2 X_3	$B_1=X_1 + X_2 + X_3$
2	1 2 4	X_1 X_2 X_4	$B_2=X_1 + X_2 + X_4$
3	1 2 5	X_1 X_2 X_5	$B_3=X_1 + X_2 + X_5$

4	1 3 4	X_1 X_3 X_4	$B_4=X_1+X_3+X_4$
5	1 3 5	X_1 X_3 X_5	$B_5=X_1+X_3+X_5$
6	1 4 5	X_1 X_4 X_5	$B_6=X_1+X_4+X_5$
7	2 3 4	X_2 X_3 X_4	$B_7=X_2+X_3+X_4$
8	2 3 5	X_2 X_3 X_5	$B_8=X_2+X_3+X_5$
9	2 4 5	X_2 X_4 X_5	$B_9=X_2+X_4+X_5$
10	3 4 5	X_3 X_4 X_5	$B_{10}=X_3+X_4+X_5$

Product of B_i will be polynomial of degree 10 of variables $X_1, X_2, \ldots, X_5$. This polynomial contains $K^B=3^{10}$ (=59049) terms. Similar types of terms of this polynomial are classified according to powers and new twenty designs are constructed.

$$\begin{aligned}
D_2 = \prod_{i=1}^{10} B_i = B_1.B_2....B_{10} = & 1\left[60 \text{ terms of the type } X_i^6.X_j^3 X_k^1.X_l^0.X_m^0\right] + 18\left[60 \text{ terms of the type } X_i^5.X_j^3 X_k^1.X_l^1.X_m^0\right] \\
& + 3\left[60 \text{ terms of the type } X_i^5.X_j^4.X_k^1.X_l^0.X_m^0\right] + 30\left[60 \text{ terms of the type } X_i^5.X_j^2.X_k^2.X_l^1.X_m^0\right] \\
& + 4\left[60 \text{ terms of the type } X_i^6.X_j^2.X_k^1.X_l^1.X_m^0\right] + 9\left[60 \text{ terms of the type } X_i^5.X_j^3.X_k^2.X_l^0.X_m^0\right] \\
& + 14\left[30 \text{ terms of the type } X_i^4.X_j^4.X_k^2.X_l^0.X_m^0\right] + 108\left[20 \text{ terms of the type } X_i^3.X_j^3.X_k^3.X_l^1.X_m^0\right] \\
& + 172\left[30 \text{ terms of the type } X_i^3.X_j^3.X_k^2.X_l^2.X_m^0\right] + 114\left[20 \text{ terms of the type } X_i^4.X_j^2.X_k^2.X_l^2.X_m^0\right] \\
& + 2\left[30 \text{ terms of the type } X_i^6.X_j^2.X_k^2.X_l^0.X_m^0\right] + 147\left[20 \text{ terms of the type } X_i^4.X_j^3.X_k^1.X_l^1.X_m^1\right] \\
& + 22\left[30 \text{ terms of the type } X_i^4.X_j^3.X_k^3.X_l^0.X_m^0\right] + 588\left[20 \text{ terms of the type } X_i^4.X_j^3.X_k^1.X_l^1.X_m^1\right] \\
& + 240\left[30 \text{ terms of the type } X_i^4.X_j^2.X_k^2.X_l^1.X_m^1\right] + 63\left[20 \text{ terms of the type } X_i^5.X_j^2.X_k^1.X_l^1.X_m^1\right] \\
& + 28\left[30 \text{ terms of the type } X_i^4.X_j^4.X_k^1.X_l^1.X_m^0\right] + 9\left[5 \text{ terms of the type } X_i^6.X_j^1.X_k^1.X_l^1.X_m^1\right] \\
& + 363\left[30 \text{ terms of the type } X_i^3.X_j^3.X_k^2.X_l^1.X_m^1\right] + 71\left[120 \text{ terms of the type } X_i^4.X_j^3.X_k^2.X_l^1.X_m^0\right] \\
& + 954\left[X_1^2.X_2^2.X_3^2.X_4^2.X_5^2\right]
\end{aligned}$$

Consider,

In the polynomial D_2, there are 60 terms of the type of the power 63100 and each term is repeated 1 time, the powers of these 60 terms give columns of the incidence matrix of design no.1 having V=5 and B=60. Likewise this polynomial gives 20 incidence matrices of new designs. Last term is constant so it does not give a new design. According to incidence matrix and parameters of newly constructed design it classified into balanced n-ary t-design.

Table 4: Type of design and the parameters of newly constructed design using 3-(5, 3, 1) design

<table>
<tr><th>Design No.</th><th>Type of Design</th><th>Parameters of newly constructed design</th></tr>
<tr><td>1</td><td>Balanced 7-ary 3-design</td><td>
<table>
<tr><th>R₀</th><th>R₁</th><th>R₂</th><th>R₃</th><th>R₄</th><th>R₅</th><th>R₆</th><th>B</th><th>R</th><th>Λ</th><th>Λ₃</th></tr>
<tr><td>24</td><td>12</td><td>0</td><td>12</td><td>0</td><td>0</td><td>12</td><td>60</td><td>120</td><td>552</td><td>108</td></tr>
<tr><th>K₀</th><th>K₁</th><th>K₂</th><th>K₃</th><th>K₄</th><th>K₅</th><th>K₆</th><th>V</th><th>K</th><th>Λ₂</th><th></th></tr>
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<tr><td>2</td><td>Balanced 6-ary 4-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>R_5</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td><td>Λ_4</td></tr>
<tr><td>12</td><td>24</td><td>0</td><td>12</td><td>0</td><td>12</td><td>60</td><td>120</td><td>432</td><td>228</td><td>180</td></tr>
<tr><td>K_0</td><td>K_1</td><td>K_2</td><td>K_3</td><td>K_4</td><td>K_5</td><td>V</td><td>K</td><td>Λ_2</td></tr>
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<tr><td>3</td><td>Balanced 6-ary 3-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>R_5</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td></tr>
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<tr><td>K_0</td><td>K_1</td><td>K_2</td><td>K_3</td><td>K_4</td><td>K_5</td><td>V</td><td>K</td><td>Λ_2</td></tr>
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</table></td></tr>
<tr><td>4</td><td>Balanced 6-ary 4-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>R_5</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td><td>Λ_4</td></tr>
<tr><td>12</td><td>12</td><td>24</td><td>0</td><td>0</td><td>12</td><td>60</td><td>120</td><td>408</td><td>264</td><td>240</td></tr>
<tr><td>K_0</td><td>K_1</td><td>K_2</td><td>K_3</td><td>K_4</td><td>K_5</td><td>V</td><td>K</td><td>Λ_2</td></tr>
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</table></td></tr>
<tr><td>5</td><td>Balanced 7-ary 4-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>R_5</td><td>R_6</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td><td>Λ_4</td></tr>
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</table></td></tr>
<tr><td>6</td><td>Balanced 6-ary 3-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>R_5</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td></tr>
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<tr><td>K_0</td><td>K_1</td><td>K_2</td><td>K_3</td><td>K_4</td><td>K_5</td><td>V</td><td>K</td><td>Λ_2</td></tr>
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</table></td></tr>
<tr><td>7</td><td>Balanced 5-ary 3-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td></tr>
<tr><td>12</td><td>0</td><td>6</td><td>0</td><td>12</td><td>30</td><td>60</td><td>216</td><td>96</td></tr>
<tr><td>K_0</td><td>K_1</td><td>K_2</td><td>K_3</td><td>K_4</td><td>V</td><td>K</td><td>Λ_2</td></tr>
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<tr><td>4</td><td>4</td><td>0</td><td>12</td><td>20</td><td>40</td><td>112</td><td>108</td><td>108</td></tr>
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<tr><td>1</td><td>1</td><td>0</td><td>3</td><td>5</td><td>10</td><td>72</td></tr>
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<tr><td>9</td><td>Balanced 4-ary 4-design</td><td><table>
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<tr><td>10</td><td>Balanced 5-ary 4-design</td><td><table>
<tr><td>R_0</td><td>R_1</td><td>R_2</td><td>R_3</td><td>R_4</td><td>B</td><td>R</td><td>Δ</td><td>Λ_3</td><td>Λ_4</td></tr>
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<tr><th>R_0</th><th>R_1</th><th>R_2</th><th>R_3</th><th>R_4</th><th>B</th><th>R</th><th>Λ</th><th>Λ_3</th><th>Λ_4</th><th>Λ_5</th></tr>
<tr><td>0</td><td>12</td><td>0</td><td>4</td><td>4</td><td>20</td><td>40</td><td>112</td><td>116</td><td>172</td><td>240</td></tr>
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<tr><td>0</td><td>4</td><td>12</td><td>4</td><td>20</td><td>40</td><td>88</td><td>148</td><td>272</td><td>480</td></tr>
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<tr><td>19</td><td>Balanced 4-ary 5-design</td><td><table>
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After applying BSP Methodology on 3-(5, 3, 1) design gives 20 balanced n-ary t-designs with parameters listed in table 4.

3. Discussion

In this paper we used Block Sum and Product (BSP) Methodology on 3-desings and constructed new balanced n-ary t-designs. The simple 3-(4, 3, 1) and 3-(5, 3, 1) designs are used as a parent 3-designs for the procedure. After applying BSP Methodology on 3-(4, 3, 1) design gives three balanced n-ary t-designs. Also after applying BSP Methodology on 3-(5, 3, 1) design gives 20 balanced n-ary t-designs. We list out the parameters of newly constructed designs. BSP Methodology tool can be used on any 3-design for the construction of balanced n-ary t-designs.

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