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Doubly Restrained Domination in Graphs

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Abstract

Let G be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$. A set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. A nonempty subset $S \subseteq V(G)$ is doubly restrained dominating set of G, if S is a dominating set and both $\langle S \rangle$ and $\langle V(G) \setminus S \rangle$ have no isolated vertices. The minimum cardinality of a doubly restrained dominating set of G, denoted by $\gamma_{rr}(G)$, is called the doubly restrained domination number of G. In this paper, we initiate the study of the concept and give the domination number of some special graphs and the corona of two graphs.

Keywords: dominating set, restrained dominating set, doubly restrained dominating set

1. Introduction

Domination in graph theory was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Following an article [2] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The domination number $\gamma(G)$ is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13].

Other type of domination parameter is the doubly connected domination number in a graph. A graph G is connected if there is at least one path that connects every two vertices $x, y \in V(G)$, otherwise, G is disconnected. A set $S \subseteq V(G)$ is a doubly connected dominating set of G if it is dominating and both $\langle S \rangle$ and $\langle V(G) \setminus S \rangle$ are connected. The cardinality of a minimum doubly connected dominating set of G is the doubly connected domination number of G and is denoted by $\gamma_{cc}(G)$. The concept of doubly connected domination in graphs was introduced by Cyman and Lemańska [14]. Variant of doubly connected domination in graphs can be found in [15], [16], [17], [18].

The restrained domination in graphs was introduced by Telle and Proskurowski [19] indirectly as a vertex partitioning problem, and Domke, Hattingh, Hedetniemi, Laskar, Markus [20] "Restrained domination in graphs". Accordingly, a set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. Alternately, a subset S of $V(G)$ is a restrained dominating set if $N[S] = V(G)$ and $\langle V(G) \setminus S \rangle$ is a subgraph without isolated vertices. The minimum cardinality of a restrained dominating set of G, denoted by $\gamma_r(G)$, is called the restrained domination number of G. A restrained dominating set of cardinalities $\gamma_r(G)$ is called an γ_r-set. Restrained domination in graphs was also found in the papers [21], [22], [23], [24], [25], [26], [27], [28], [29], [30].

The doubly connected domination in graphs and restrained domination in graphs have motivated the researchers to introduce a new domination in graphs – the doubly restrained domination in graphs. A nonempty subset $S \subseteq V(G)$ is doubly restrained dominating set of G, if S is a dominating set and both $\langle S \rangle$ and $\langle V(G) \setminus S \rangle$ have no isolated vertices. The minimum cardinality of a doubly restrained dominating set of G, denoted by $\gamma_{rr}(G)$, is called the doubly restrained domination number of G. In this paper, we initiate the study of the concept and give the domination number of some special graphs and the corona of two graphs.

For the general terminology in graph theory, readers may refer to [31]. A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $|E(G)| = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \bigcup_{v \in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \bigcup_{v \in X} N_G[v]$.

2. Results

The following definitions will be used throughout the study.

Definition 2.1 A simple graph G is an undirected graph with no loop edges or multiple edges.

Definition 2.2 The cycle $C_n = \{a_1 a_2 a_3 \cdots a_n a_1\}$ is the graph with $V(C_n) = \{a_1, a_2, a_3, \cdots, a_n\}$ and $E(C_n) = \{a_1a_2, a_2a_3, \cdots, a_na_1\}$.

Definition 2.3 The wheel W_n is the graph with $V(W_n) = \{a_0, a_1, a_2, a_3, \cdots, a_n\}$ and $E(W_n) = \{a_1a_2, a_2a_3, \cdots, a_{n-1}a_n, a_na_1\} \cup \{a_0a_i : i = 1,2, \cdots, n\}$ where $n \geq 3$.

Definition 2.4 The fan F_n is the graph with $V(F_n) = \{a_0, a_1, a_2, a_3, \cdots, a_n\}$ and $E(F_n) = \{a_1a_2, a_2a_3, \cdots, a_{n-1}a_n\} \cup \{a_0a_i : i = 1,2, \cdots, n\}$.

Definition 2.5 The complete graph K_n is the graph of order n where every pair of vertices is adjacent.

Definition 2.6 A complete bipartite graph is a graph whose vertex set can be partitioned into V_1 and V_2 such that every edge joins a vertex in V_1 with a vertex in V_2, and every vertex in V_1 is adjacent with every vertex in V_2.

Note: If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denoted by $K_{m,n}$.

The following results shows the doubly restrained domination number of some special graphs.

Theorem 2.7 Let G be a special graph. Then $\gamma_{rr}(G) = 2$ if one of the following is satisfied

1. $G = W_n$ or $G = F_n$ for all integer $n \geq 3$.
2. $G = K_n$ for all integer $n \geq 4$.
3. $G = K_{m,n}$ for all integers $m, n \geq 2$.

Proof. (i) Suppose that $G = W_n$ or $G = F_n$ for all integer $n \geq 3$. Consider $G = W_n = [x, v_1, v_2, \ldots, v_n]$ where $K_1 = [x]$ and $C_n = [v_1, v_2, \ldots, v_n]$ for all integers $n \geq 3$. The set $S = \{x, v_1\}$ and $V(G) \setminus S = \{v_2, v_3, \ldots, v_n\}$ are restrained dominating sets of G. Hence, S is a doubly restrained dominating set of G. Since $S \setminus \{v_1\} = \{x\}$ is a set, whose element is an isolated vertex, $V(G) \setminus V(C_n) = \{x\}$ is not a restrained dominating set of G, it follows that S is a minimum doubly restrained dominating set of G. Hence,

$\gamma_{rr}(G) = |S| = 2$. Next, consider $G = F_n = [x, v_1, v_2, \dots, v_n]$ where $K_1 = [x]$ and $P_n = [v_1, v_2, \dots, v_n]$ for all integers $n \geq 3$. The set $S = \{x, v_1\}$ and $V(G) \setminus S = \{v_2, v_3, \dots, v_n\}$ are restrained dominating sets of G. Hence, S is a doubly restrained dominating set of G. Since $S \setminus \{v_1\} = \{x\}$ is a set, whose element is an isolated vertex, $V(G) \setminus V(P_n) = \{x\}$ is not a restrained dominating set of G, it follows that S is a minimum doubly restrained dominating set of G. Hence, $\gamma_{rr}(G) = |S| = 2$.
(ii) Suppose that $G = K_n$ for all integer $n \geq 4$. Let $G = K_n = [v_1, v_2, \dots, v_n]$ for all integers $n \geq 4$. The set $S = \{v_1, v_2\}$ and $V(G) \setminus S = \{v_3, \dots, v_n\}$ are restrained dominating sets of G. Hence, S is a doubly restrained dominating set of G. Since $S \setminus \{v_2\} = \{v_1\}$ is a set, whose element is an isolated vertex, $V(G) \setminus \{v_2, v_3, \dots, v_n\} = \{v_1\}$ is not a restrained dominating set of G, it follows that S is a minimum doubly restrained dominating set of G. Hence, $\gamma_{rr}(G) = |S| = 2$.
(iii) Suppose that $G = K_{m,n}$ for all integers $m, n \geq 2$. Let $G = K_{m,n} = [u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_n]$ for all integers $m, n \geq 2$. The set $S = \{u_1, v_1\}$ and $V(G) \setminus S = \{u_2, u_3, \dots, u_m, v_2, v_3, \dots, v_n\}$ are restrained dominating sets of G. Hence, S is a doubly restrained dominating set of G. Since $S \setminus \{v_1\} = \{u_1\}$ is a set, whose element is an isolated vertex, $V(G) \setminus \{u_2, u_3, \dots, u_m, v_1, v_2, \dots, v_n\} = \{u_1\}$ is not a restrained dominating set of G, it follows that S is a minimum doubly restrained dominating set of G. Hence, $\gamma_{rr}(G) = |S| = 2$. ∎

Theorem 2.8 Let $G = C_n$ where $n = 4k$ for any positive integer k. Then $\gamma_{rr}(G) = \frac{n}{2}$.
Proof. Let $G = C_n = [v_1, v_2, \dots, v_n]$ for all integers $n \geq 4$. Suppose that $n \equiv 0 \ (mod\ 4)$. Then $n = 4k$, where k is a positive integer. The set $S = \left\{v_{4i-3}: i = 1,2,3, \dots, \frac{n}{4}\right\} \cup \left\{v_{4i-2}: i = 1,2,3, \dots, \frac{n}{4}\right\}$ and $V(G) \setminus S = \left\{v_{4i-1}: i = 1,2,3, \dots, \frac{n}{4}\right\} \cup \left\{v_{4i}: i = 1,2,3, \dots, \frac{n}{4}\right\}$ are restrained dominating sets of G. Hence, S is a doubly restrained dominating set of G. Since $S \setminus \{v\}$ is not a dominating set of G for any $v \in S$, it follows that S is a minimum doubly restrained dominating set of G. Hence, $\gamma_{rr}(G) = |S| = \frac{n}{4} + \frac{n}{4} = \frac{n}{2}$. ∎
The following definition is useful for our next results.
Definition 2.9 Let G and H be graphs of order m and n, respectively. The corona of G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G and m copies of H, and then joining the i-th vertex of G to every vertex of the i-th copy of H. For every $v \in V(G)$, denote by H^v the copy of H whose vertices are joined or attached to the vertex v.
Theorem 2.10 Let G be a connected nontrivial graph of order $m \geq 3$ and $H = P_n$ of order $n \geq 4$. Then a subset $S \subset V(G \circ H)$ is a doubly restrained dominating set of $G \circ H$ if one of the following is satisfied.

1. $S = V(G)$,
2. $S = \bigcup_{v \in V(G)} V(H^v)$,
3. $S = \bigcup_{v \in V(G)} S_v$, where $S_v = V(H^v) \setminus \{v_1, v_n\}$, or
4. $S = \left(\bigcup_{v \in V(G) \setminus S_G} S_H^v\right) \cup S_G$, where $S_H^v = V(H^v) \setminus \{v_1, v_n\}$ for each $v \in V(G) \setminus S_G$ and $\langle S_G \rangle$ is a nontrivial connected subgraph of G.

Proof. Suppose that statement i) is satisfied. Then $S = V(G)$. Clearly S is a dominating set of $G \circ H$. Since $H = P_n$ of order $n \geq 4$, H is connected. This implies that $\langle V(G \circ H) \setminus S \rangle = \langle \bigcup_{v \in V(G)} V(H^v) \rangle$ have no isolated vertices. Further, since G is a connected nontrivial graph of order $m \geq 3$, $\langle S \rangle = G$ has no isolated vertex. Thus, S is a dominating set and both $\langle S \rangle$ and $\langle V(G \circ H) \setminus S \rangle$ have no isolated vertices. By definition, $S = V(G)$ is doubly restrained dominating set of $G \circ H$.

Suppose that statement ii) is satisfied. Then $S = \bigcup_{v \in V(G)} V(H^v)$. Clearly is S a dominating set of $G \circ H$. Since G is connected, $\langle V(G \circ H) \setminus S \rangle = G$ has no isolated vertex. Further, since $H = P_n$ is connected, $\langle S \rangle = \langle \bigcup_{v \in V(G)} V(H^v) \rangle$ have no isolated vertices. Thus, S is a dominating set and both $\langle S \rangle$ and $\langle V(G \circ H) \setminus S \rangle$ have no isolated vertices. By definition, $S = \bigcup_{v \in V(G)} V(H^v)$ is doubly restrained dominating set of $G \circ H$.

Suppose that statement iii) is satisfied. Then $S = \bigcup_{v \in V(G)} S_v$, where $S_v = V(H^v) \setminus \{v_1, v_n\}$. Since $H = P_n = [v_1, v_2, \dots, v_n]$, for each $v \in V(G)$, $S_v = \{v_2, v_3, \dots, v_{n-1}\}$ is a dominating set of H^v. Thus, $S = \bigcup_{v \in V(G)} S_v$ is a dominating set of $G \circ H$. Now, for each $v \in V(G)$, $\langle V(H^v) \setminus S_v \rangle = \langle \{v_1, v_n\} \rangle$. Since $v_1 v, v_n v \in E(G \circ H)$ for some $v \in V(G)$, it follows that $\langle V(G \circ H) \setminus S \rangle = \langle V(G) \cup \left(\bigcup_{v \in V(G)} (V(H^v) \setminus S_v) \right) \rangle$ have no isolated vertices. Further, since $S_v = \{v_2, v_3, \dots, v_{n-1}\}$, $\langle S \rangle = \langle \bigcup_{v \in V(G)} S_v \rangle$ have no isolated vertices. Thus, S is a dominating set and both $\langle S \rangle$ and $\langle V(G \circ H) \setminus S \rangle$ have no isolated vertices. By definition, $S = \bigcup_{v \in V(G)} S_v$ is doubly restrained dominating set of $G \circ H$.

Finally, suppose that statement iv) is satisfied. Then $S = \left(\bigcup_{v \in V(G) \setminus S_G} S_H^v \right) \cup S_G$, where $S_H^v = V(H^v) \setminus \{v_1, v_n\}$ for each $v \in V(G) \setminus S_G$ and $\langle S_G \rangle$ is nontrivial connected subgraph of G. Let $x \in V(G \circ H) \setminus S$. Then $x \in \bigcup_{v \in S_G} V(H^v)$, or $x \in V(G) \setminus S_G$, or $x \in \bigcup_{v \in V(G) \setminus S_G} (V(H^v) \setminus S_H^v)$.

Case 1. If $x \in \bigcup_{v \in S_G} V(H^v)$, then $x \in V(H^v)$ for some $v \in S_G \subset S$ and $xv \in E(G \circ H)$ is clear. Since $H = P_n$ of order $n \geq 4$, $xy \in E(H^v)$ for some $y \in V(H^v) \subset V(G \circ H) \setminus S$.

Case 2. If $x \in V(G) \setminus S_G$, then $xv \in E(G)$ for some $v \in S_G \subset S$ since G is a connected nontrivial graph of order $m \geq 3$. Clearly, $xv_1 \in E(G \circ H)$ where $v_1 \in V(H^x) \setminus S_H^x \subset V(G \circ H) \setminus S$.

Case 3. If $x \in \bigcup_{v \in V(G) \setminus S_G} (V(H^v) \setminus S_H^v)$, then $x = v_1 \in V(H^v) \setminus S_H^v$ or $x = v_n \in V(H^v) \setminus S_H^v$ for some $v \in V(G) \setminus S_G \subset V(G \circ H) \setminus S$. If $x = v_1$, then $xv_2 \in E(G \circ H)$ where $v_2 \in S_H^v \subset S$. If $x = v_n$, then $xv_{n-1} \in E(G \circ H)$ where $v_{n-1} \in S_H^v \subset S$. Further, $xv \in E(G \circ H)$ for some $v \in V(G) \setminus S_G$.

In any case, S is a restrained dominating set of $G \circ H$, that is, $\langle V(G \circ H) \setminus S \rangle$ have no isolated vertices.

Now, let $y \in S = \left(\bigcup_{v \in V(G) \setminus S_G} S_H^v \right) \cup S_G$.

Case 1, If $y \in S = \bigcup_{v \in V(G) \setminus S_G} S_H^v$, then $y \in S_H^v$ for some $v \in V(G) \setminus S_G$. Since $H = P_n$ of order $n \geq 4$ and $S_H \subset V(H)$, it follows that $yu \in E(H)$ for some $u \in V(H) = \{v_1, v_n\} \cup S_H$. If $u \in \{v_1, v_2\}$, then $u \in V(G \circ H) \setminus S$. If $u \in S_H$, then $u \in S$.

Case 2. If $y \in S_G$, then $yu \in E(G)$ for some $u \in S_G$ since $\langle S_G \rangle$ is a nontrivial connected subgraph of G and $yu' \in E(G \circ H)$ for all $u' \in V(H^y) \subset V(G \circ H) \setminus S$. Thus, for every $y \in S$, $yu \in E(G \circ H)$, $u \in S$ and $yu' \in E(G \circ H)$ with $u' \in V(G \circ H) \setminus S$.

In any case, $\langle S \rangle$ have no isolated vertices. Thus, S is a dominating set and both $\langle S \rangle$ and $\langle V(G \circ H) \setminus S \rangle$ have no isolated vertices. By definition, $S = \left(\bigcup_{v \in V(G) \setminus S_G} S_H^v \right) \cup S_G$ is doubly restrained dominating set of $G \circ H$ (where $S_H^v = V(H^v) \setminus \{v_1, v_n\}$ for each $v \in V(G) \setminus S_G$ and $\langle S_G \rangle$ is a nontrivial connected subgraph of G).

This completes the proofs. ■

The following result is an immediate consequence of Theorem 2.10.

Corollary 2.11 Let G be a connected nontrivial graph of order $m \geq 3$ and $H = P_n$ of order $n \geq 4$. Then $\gamma_{rr}(G \circ H) = m$.

3. Conclusion and Recommendations

In this work, we introduced a new parameter of domination on graphs - the doubly restrained domination of graphs. The doubly restrained domination number of some special graphs were computed. The doubly restrained dominating sets in the corona of the two connected nontrivial graphs were presented and proved. Further, the doubly restrained domination number of the corona of two graphs, was also given. This study will pave a way to new researches such as bounds and other binary operations of two graphs – join, Cartesian product, etc. Other parameters involving doubly restrained domination in graphs may also be explored. Finally, the characterization of a doubly restrained domination in graphs may be the subject of further study.

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