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Modeling the Impact of Alcohol Consumption on Road Accidents in Tanzania

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Abstract
This study employs a nonlinear mathematical model to investigate traffic accidents resulting from drivers' alcohol consumption while operating vehicles. The model categorizes drivers into five distinct groups: N_u (non-alcohol users), E_u (individuals at high risk of alcohol use), Mu (moderate alcohol users), A_u (addicted alcohol users), and R_u (recovering alcohol users). To analyze the system, the stability theory of differential equations was applied, with particular focus on the alcohol-free and endemic equilibrium states. The basic reproduction number (R_0) was derived to establish the threshold conditions for the elimination or persistence of alcohol use among drivers. A sensitivity analysis was performed to evaluate the impact of key parameters on the model's dynamics. Subsequently, numerical simulations were conducted and presented graphically to demonstrate the model's behavior. The findings highlight that moderate alcohol consumption plays the most significant role in increasing the risk of road accidents.

Keywords: Alcohol Consumption, Road Accident, Deterministic Model, Qualitative Analysis, Numerical simulation

1. Introduction

A road accident is an unforeseen incident that occurs during vehicular travel on a roadway, often resulting in injuries and, in many instances, fatalities. These accidents frequently involve collisions between multiple vehicles or between a vehicle and external objects such as trees or buildings. Skidding is another common cause, which can lead to severe bodily harm or death. Such incidents typically involve impacts with other vehicles, animals, or stationary obstacles, leading to substantial property damage and loss of life [1]. Numerous factors contribute to the occurrence of road accidents. Mechanical issues, such as malfunctioning brakes, worn-out tires, and steering failures, can significantly impair a driver's ability to maintain control. Infrastructure-related problems also play a critical role especially in Tanzania where poorly maintained roads, debris, oil spills, potholes, non-functional highway lights, and muddy surfaces present major hazards in various regions. Natural disasters, including floods, heavy rains, strong winds, fog, and unstable terrains, further elevate the risk of accidents [2]. Nonetheless, human error is widely recognized as the leading cause of road accidents. As noted in [3], such errors are strongly associated with incidents involving poor handling of vehicles and misjudgment of the distance of oncoming traffic, particularly during overtaking maneuvers. Reckless behavior such as speeding, disobeying traffic regulations, fatigue, abrupt lane changes, and misinterpretation of road signs significantly heighten the risk of accidents. Moreover, prolonged driving without sufficient rest, especially over long distances, exacerbates driver fatigue and raises accident likelihood, as highlighted in [4]–[8]. Other critical risk factors include driving while sleep deprived, nighttime travel without adequate lighting, and the use of medications that induce drowsiness.

Alcohol consumption is recognized as one of the most critical factors contributing to road accidents. It impairs brain function, diminishing a driver's ability to react promptly, accurately judge speed and distance, and make sound decisions. Additionally, it fosters a false sense of confidence, often leading drivers to engage in risky behaviours, as noted in [9]–[12]. Worldwide, alcohol-related crashes are responsible for a considerable proportion of traffic fatalities, with a particularly severe impact observed in developing nations. According to the National Highway Traffic Safety Administration (NHTSA), approximately 37 individuals die each day in alcohol-impaired driving incidents in the United States equivalent to one death every 39 minutes and totalling 13,384 deaths in 2021. In Tanzania, it is common for long distance drivers to take evening breaks, during which many consume alcohol instead of sleeping. This leads to inadequate rest and results in fatigue and drowsiness behind the wheel. As a consequence, a notable number of accidents tend to occur in the early hours of the morning, typically between 3:00 AM and 11:00 AM.

This paper introduces a mathematical model to assess the impact of alcohol consumption on driving behavior and to identify the most influential parameters contributing to accident occurrence. Through this model, we aim to enhance understanding of alcohol-induced driving impairment and offer strategic recommendations to reduce related road safety risks.

2. Model Formulation

In this section, a mathematical model is developed and describes the accidents influenced by the alcohol user population of size N [13]-[15]. The model is divided into five compartments and is given by: $N_u(t)$, the fraction of non-alcohol users, $E_u(t)$ Exposed to the risk of being an alcohol user $M_u(t)$, fraction of moderate alcohol users $A_u(t)$, fraction of addicted alcohol users and $R_u(t)$ Individuals who have changed their habit of alcohol use. The total number of the human population is constant, that is, $N(t) = N_u(t) + E_u(t) + M_u(t) + A_u(t) + R_u(t)$. From the above reflections, we then have the following model flow diagram for an alcohol user:

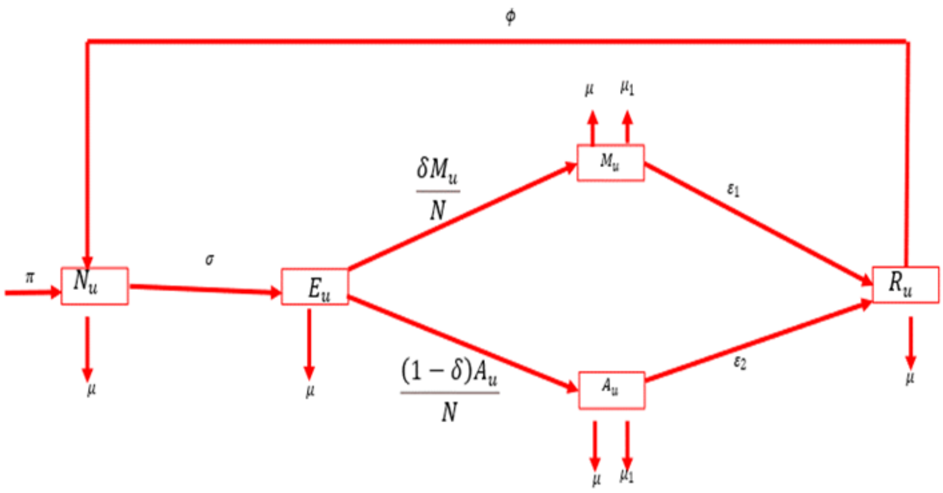

Figure 1: Modelling framework for alcohol induced accident.

From Figure 1, the model is defined by an initial value problem with five differential equations given as:

$$\begin{aligned}
\frac{dN_u}{dt} &= \pi + \phi R_u - (\mu + \sigma) N_u \\
\frac{dE_u}{dt} &= \sigma N_u - \mu E_u - (1-\delta)\frac{A_u}{N} E_u - \delta \frac{M_u}{N} E_u \\
\frac{dM_u}{dt} &= \delta \frac{M_u}{N} E_u - (\mu + \mu_1 + \varepsilon_1) M_u \\
\frac{dA_u}{dt} &= (1-\delta)\frac{A_u}{N} E_u - (\mu + \mu_2 + \varepsilon_2) A_u \\
\frac{dR_u}{dt} &= \varepsilon_1 M_u + \varepsilon_2 A_u - (\mu + \phi) R_u
\end{aligned} \qquad (1)$$

with initial conditions

$N(0) = N_0$, $E(0) = E_0$, $M(0) = M_0$, $A(0) = A_0$, $R(0) = R_0$,

where $N(t) = N_u(t) + E_u(t) + M_u(t) + A_u(t) + R_u(t)$.

The non-alcohol users $N_u(t)$ represent divers who are driving in cars, motorcycles, tricycles, who are not taking alcohol at all, but may take alcohol when exposed to an alcohol user. This compartment is increased by the recruitment rate π that means drivers who have received education and been given a license, and recovered individuals who become non-alcohol users ϕ, $N_u(t)$ decreased by natural accidents caused by road infrastructure, mechanical factors, or Natural calamitous μ, being forced to increase speed and change behaviour after interacting with alcohol users in social events such as picnics, graduations, weekend gatherings, visiting relatives, meeting points, and Christmas celebrations with relatives σ. The exposed $E_u(t)$, are the drivers who may take alcohol due to friends or relatives when they meet at a place where they sell alcohol. They may be tempted to take a small amount of alcohol, and after some time, they also become frequent alcohol users. Other instances occur when a boss's drivers can be given alcohol with their bosses if the boss has a habit of drinking after work before going home σ. $E_u(t)$ may be decreased by natural accidents caused by road infrastructure, mechanical factors, Natural calamitous, and exposure that leads to alcoholism users δ, moderate alcohol users $M_u(t)$, are drivers who can control the amount of alcohol consumption when they are expected to drive after an occasion like graduation, a wedding, meeting. This compartment $M_u(t)$ can be reduced by natural accidents caused by road infrastructure, mechanical factors, or natural calamitous μ, accidents due to moderate alcohol users μ_1, and the conversion rate of moderate alcohol users to recover ε_1. The addicted alcohol user $A_u(t)$ is increased by exposed non-alcohol users and becomes an alcohol user δ and decreased by natural accidents caused by road infrastructure, mechanical factors, or natural calamitous μ, accident due to an alcohol-addicted user μ_2 and the conversion rate of an addicted alcohol user to recover ε_2. The compartment $R_u(t)$ are individuals who have changed their habit of alcohol use and are increasing by the conversion rate of moderate alcohol users to recover ε_1, conversion rate of an addicted alcohol user to recover ε_2 and decreased by natural accidents caused by road infrastructure, mechanical factors, or natural calamitous μ and the rate of recovered individuals who become non-alcohol users ϕ.

Table 1. Description of parameters

Parameters	**Clarification**
π	Recruitment rate
σ	Interaction rate of non- alcohol users with alcohol users
δ	Fraction of exposed that become alcohol users
μ	Road infrastructure, mechanical factors, or natural calamitous
μ_1	Moderate alcohol user
μ_2	Addicted alcohol user
ε_1	Conversion rate of moderate alcohol users to recover
ε_2	Conversion rate of addicted alcohol users to recover
ϕ	Proportion of recovered individuals who become non-alcohol users

2.1 Positivity and Invariant Region of Solutions

Theorem 1. Given that the initial conditions for the model (1) are $N_u(0) \geq 0$, $E_u(0) \geq 0$, $M_u(0) \geq 0$, $A_u(0) \geq 0$, and $R_u(0) \geq 0$, then solution ($N_u(t)$, $E_u(t)$, $M_u(t)$, $A_u(t)$, $R_u(t)$) will remain positive for all $t > 0$.

Proof: To prove **t**heorem1, we shall use all the equations of the differential equation (1)

From the first equation of the differential equation (1), we have

$$\frac{dN_u}{dt} = \pi + \phi R_u - (\mu + \sigma) N_u$$
$$\frac{dN_u}{dt} \geq \pi - (\mu + \sigma) N_u$$
$$\frac{dN_u}{dt} - (\mu + \sigma) N_u \geq \pi$$
$$I.F = e^{\int_0^t (\mu+\sigma)dt}$$
$$e^{(\mu+\sigma)t} \frac{dN_u}{dt} - (\mu + \sigma) e^{(\mu+\sigma)t} N_u \geq \pi e^{(\mu+\sigma)t}$$
$$\int_0^t d\left(e^{(\mu+\sigma)t} N_u(t)\right) \geq \int_0^t \pi e^{(\mu+\sigma)t} dt$$
$$e^{(\mu+\sigma)t} N_u(t) - N_u(0) \geq \frac{\pi e^{(\mu+\sigma)t}}{(\mu+\sigma)} + \frac{\pi}{(\mu+\sigma)}$$
$$N_u(t) \geq N_u(0) e^{-(\mu+\sigma)t} + \frac{\pi}{(\mu+\sigma)}\left(1 + e^{-(\mu+\sigma)t}\right) > 0$$

From the second equation of model (1), we have

$$\frac{dE_u}{dt} = \sigma N_u - \mu E_u - (1-\delta)\frac{A_u}{N} E_u - \delta \frac{M_u}{N} E_u$$
$$\frac{dE_u}{dt} \geq -\left(\mu + (1-\delta)\frac{A_u}{N} + \delta \frac{M_u}{N}\right) E_u$$

separate variable

$$\int_0^t \frac{dE_u}{E_u(t)} \geq \int_0^t -\mu(t) - \left\{(1-\delta)\frac{A_u}{N} + \delta\frac{M_u}{N}\right\} dt$$
$$\ln E_u(t) - \ln E_u(0) \geq -\mu t - \int_0^t \left\{(1-\delta)\frac{A_u}{N} + \delta\frac{M_u}{N}\right\} dt$$
$$\ln \frac{E_u(t)}{E_u(0)} \geq -\mu t - \int_0^t \left\{(1-\delta)\frac{A_u}{N} + \delta\frac{M_u}{N}\right\} dt$$
$$\frac{E_u(t)}{E_u(0)} \geq e^{-\mu t - \int_0^t \left\{(1-\delta)\frac{A_u}{N} + \delta\frac{M_u}{N}\right\} dt}$$
$$E_u(t) \geq E_u(0) e^{-\mu t - \int_0^t \left\{(1-\delta)\frac{A_u}{N} + \delta\frac{M_u}{N}\right\} dt} > 0$$

other equations will be done Similarly to obtains:

$$M_u(t) \geq M_u(0) e^{-(\mu+\mu_1)t + \int_0^t \left(\delta\frac{E_u}{N} - \varepsilon_1\right) dt} > 0$$

The fourth equation of (1)

$$A_u(t) \geq A_u(0) e^{-(\mu+\mu_2)t + \int_0^t \left((1-\delta)\frac{E_u}{N} - \varepsilon_2\right) dt} > 0$$

And the last equation

$$R_u(t) \geq R_u(0) e^{-(\mu+\phi)t} > 0$$

Hence proved

Theorem 2: The feasible solutions of system (1) are bounded in the region Ω, where

$\Omega = \{ (N_u, E_u, M_u, A_u, R_u) \in \Re^5{}_+ \mid 0 \le N_u + E_u + M_u + A_u + R_u = N \le \pi / \mu \}$.

Proof: Consider the analysis of the system (1) in the feasible region $\Omega \subseteq R^5{}_+$ such that

The invariance region Ω is obtained by adding all equations in system (1), and the simplified equation is written as follows:

$$\frac{dN}{dt} = \pi - \mu N - \mu_1 M_u - \mu_2 A_u \le \pi - \mu N$$

$$\frac{dN}{dt} \le \pi - \mu N$$

$$\frac{dN}{dt} + \mu N \le \pi$$

$$e^{ut}\frac{dN}{dt} + \mu e^{ut} N \le e^{ut}\pi$$

$$d\left(e^{ut} N(t)\right) \le e^{ut}\pi dt$$

$$e^{ut} N(t) \le e^{ut}\frac{\pi}{\mu} + C$$

$$N(t) \le \frac{\pi}{\mu} + Ce^{-ut}$$

$$t \to 0$$

$$N(0) \le \frac{\pi}{\mu} + C$$

$$N(0) - \frac{\pi}{\mu} \le C$$

$$0 \le N(t) \le N(0)e^{-ut} + \frac{\pi}{\mu}\left(1 - \frac{\pi}{\mu}e^{-ut}\right)$$

where $N(0)$ represent sum of initial value of the variables $as\ t \to \infty, 0 \le N \le \frac{\pi}{\mu}\ if\ N(0) \le \frac{\pi}{\mu}\ then \lim_{t\to\infty} N(t) \le \frac{\pi}{\mu}$ This means that $\frac{\pi}{\mu}$ is an upper bound of N. On the other side if $N(0) \ge \frac{\pi}{\mu}$ then the solution ($N_u(t)$, $E_u(t)$, $M_u(t)$, $A_u(t)$, $R_u(t)$) enters Ω.

Hence, since all state variables remain non negative and bounded we therefore conclude that the feasible region Ω is positively invariant.

3 Model Analysis

Steady State Solutions

In this section, the model differential equations (1) are qualitatively analyzed by determining the model's stability, carrying out their stability analysis, and interpreting the results. Let E_0 be the alcohol-free equilibrium of the differential equations (1). Then, setting the right-hand side of the differential equations (1) to zero, one obtains:

$$\begin{aligned}
&\pi+\phi R_u-(\mu+\sigma)N_u=0\\
&\sigma N_u-\mu E_u-(1-\delta)\frac{A_u}{N}E_u-\delta\frac{M_u}{N}E_u=0\\
&\delta\frac{M_u}{N}E_u-(\mu+\mu_1+\varepsilon_1)M_u=0\\
&(1-\delta)\frac{A_u}{N}E_u-(\mu+\mu_2+\varepsilon_2)A_u=0\\
&\varepsilon_1 M_u+\varepsilon_2 A_u-(\mu+\phi)R_u=0
\end{aligned} \tag{2}$$

For alcohol free equilibrium, it is assumed that there is no alcohol user $A_u=0,\ M_u=0$ and $R_u=0$ substituting $A_u=0,\ M_u=0\ and\ R_u=0$ in the equation (2) as [15]

$$\pi+\phi R_u-(\mu+\sigma)N_u=0$$

$$\pi-(\mu+\sigma)N_u=0.$$

$$N_u=\frac{\pi}{(\mu+\sigma)}$$

$$\sigma N_u-\mu E_u=0$$

$$E_u=\frac{\sigma N_u}{\mu}\ but\quad N_u=\frac{\pi}{(\mu+\sigma)}$$

$$E_u=\frac{\sigma\pi}{\mu(\mu+\sigma)}$$

Therefore

$$N_u=\frac{\pi}{(\mu+\sigma)},\ E_u=\frac{\sigma\pi}{\mu(\mu+\sigma)}$$

Thus, the alcohol-free equilibrium E_0 of the differential equation 2 is given by

$$E_0=(N_u(t),E_u(t),M_u(t),A_u(t),R_u(t))=(N_u(t),E_u(t),0,0,0)=\left(\frac{\pi}{(\mu+\sigma)},\ \frac{\sigma\pi}{\mu(\mu+\sigma)},0,0,0\right) \tag{3}$$

4 The Basic Reproduction Number (R_0)

The dynamics of the accident are governed by the basic reproduction number R_0, a fundamental concept defined as the average number of new alcohol users generated by a single alcohol user introduced into a population of non-users. This value determines whether alcohol use will spread or decline: if the basic reproduction number is less than one ($R_0<1$), the system reaches an alcohol-free equilibrium; if it exceeds one ($R_0>1$), an alcohol-using equilibrium emerges. To better understand the impact of an alcohol-related accident, we will conduct a qualitative analysis of the differential equation system (1). The basic reproduction number for this system is calculated using the next generation matrix approach for ordinary differential equations, as outlined in [13] and [16]. Using the approach of [14] and [16], R_0 is obtained by taking the largest eigenvalue of $\left[\frac{\partial F_i(E_0)}{\partial X_j}\right]\left[\frac{\partial V_i(E_0)}{\partial X_j}\right]^{-1}$,

where, F_i is the rate of appearance of new alcohol users in the compartment i, V_i^+ is the transfer of individuals out of the compartment i by all other means and E_0 is the alcohol user free equilibrium,

$$F_i = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \delta \frac{M_u}{N} \\ (1-\delta)\frac{A_u}{N} \end{bmatrix}.$$

Using the linearization method, the associated matrix at the alcohol user free equilibrium is given by

$$\mathbf{F} = \begin{pmatrix} \frac{\partial F_1}{\partial M_u}(E_0) & \frac{\partial F_1}{\partial A_u}(E_0) \\ \frac{\partial F_2}{\partial M_u}(E_0) & \frac{\partial F_2}{\partial A_u}(E_0) \end{pmatrix}.$$

This implies that

$$\mathbf{F} = \begin{pmatrix} \frac{\delta}{N} & 0 \\ 0 & \frac{1-\delta}{N} \end{pmatrix}$$

The transfer of individuals out of the compartment i is given by

$$\mathbf{V}_i = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (\mu+\mu_1+\varepsilon_1)M_u \\ (\mu+\mu_2+\varepsilon_2)A_u \end{bmatrix}$$

Using the linearization method, the associated matrix alcohol user free equilibrium is given by,

$$\mathbf{V} = \begin{pmatrix} \frac{\partial V_1}{\partial M_u}(E_0) & \frac{\partial V_1}{\partial A_u}(E_0) \\ \frac{\partial V_2}{\partial M_u}(E_0) & \frac{\partial V_2}{\partial A_u}(E_0) \end{pmatrix}.$$

This gives $\mathbf{V} = \begin{pmatrix} \mu+\mu_1+\varepsilon_1 & 0 \\ 0 & \mu+\mu_2+\varepsilon_2 \end{pmatrix}$ with $\mathbf{V}^{-1} = \begin{pmatrix} \frac{1}{\mu+\mu_1+\varepsilon_1} & 0 \\ 0 & \frac{1}{\mu+\mu_2+\varepsilon_2} \end{pmatrix}$

Therefore,

$$\mathbf{FV}^{-1} = \begin{pmatrix} \frac{\delta}{N} & 0 \\ 0 & \frac{1-\delta}{N} \end{pmatrix} \begin{pmatrix} \frac{1}{\mu+\mu_1+\varepsilon_1} & 0 \\ 0 & \frac{1}{\mu+\mu_2+\varepsilon_2} \end{pmatrix} \tag{4}$$

The eigenvalues of the equation (4) are obtained by solving

$$\mathbf{FV}^{-1} = \begin{pmatrix} \frac{\delta}{(\mu+\mu_1+\varepsilon_1)N} & 0 \\ 0 & \frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$\mathbf{FV}^{-1} = \begin{pmatrix} \frac{\delta}{(\mu+\mu_1+\varepsilon_1)N} - \lambda & 0 \\ 0 & \frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} - \lambda \end{pmatrix}$$

$$det\left(\mathbf{FV}^{-1} - I\lambda\right) = \left(\frac{\delta}{(\mu+\mu_1+\varepsilon_1)N} - \lambda\right)\left(\frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} - \lambda\right) - 0$$

$$det\left(\mathbf{FV}^{-1} - I\lambda\right) = \frac{\delta}{(\mu+\mu_1+\varepsilon_1)N}\frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} - \frac{\delta}{(\mu+\mu_1+\varepsilon_1)N}\lambda - \lambda\frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} +$$

$$\lambda^2 \frac{\delta}{(\mu+\mu_1+\varepsilon_1)N}\frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} - \frac{\delta}{(\mu+\mu_1+\varepsilon_1)N}\lambda - \lambda\frac{1-\delta}{(\mu+\mu_2+\varepsilon_2)N} + \lambda^2 = 0$$

This gives

$$\lambda_1 = \frac{1}{2}\left(\frac{\frac{\delta}{1+\mu+\mu_1} + \frac{\delta-1}{1+\mu+\mu_2}}{N} - \sqrt{\frac{\frac{\delta^2}{(1+\mu+\mu_1)^2} + \frac{(\delta-1)^2}{(1+\mu+\mu_2)^2} + \frac{6(\delta-1)\delta}{(1+\mu+\mu_1)(1+\mu+\mu_2)}}{N^2}} \right)$$

$$\lambda_2 = \frac{1}{2}\left(\frac{\frac{\delta}{1+\mu+\mu_1} + \frac{\delta-1}{1+\mu+\mu_2}}{N} + \sqrt{\frac{\frac{\delta^2}{(1+\mu+\mu_1)^2} + \frac{(\delta-1)^2}{(1+\mu+\mu_2)^2} + \frac{6(\delta-1)\delta}{(1+\mu+\mu_1)(1+\mu+\mu_2)}}{N^2}} \right)$$

It follows that the Basic Reproductive number which is given by the largest Eigen value for model differential equation (1) denoted by R_0 and is given as

$$R_0 = \frac{1}{2}\left(\frac{\frac{\delta}{1+\mu+\mu_1} + \frac{\delta-1}{1+\mu+\mu_2}}{N} + \sqrt{\frac{\frac{\delta^2}{(1+\mu+\mu_1)^2} + \frac{(\delta-1)^2}{(1+\mu+\mu_2)^2} + \frac{6(\delta-1)\delta}{(1+\mu+\mu_1)(1+\mu+\mu_2)}}{N^2}} \right). \quad (5)$$

Model differential equation (1) has an alcohol user-free equilibrium E_0 if $R_0 < 1$, otherwise alcohol user equilibrium exists.

5. Tanzania Alcohol-Induced Accident Data Analysis

This study was conducted in the regions of Dar es Salaam, Coast, Tanga, and Kilimanjaro. These locations were chosen due to their positioning along the Dar es Salaam–Arusha highway, where a high rate of accidents involving private vehicles has been reported. Both quantitative and qualitative data were utilized in the research. Quantitative data were obtained by analyzing traffic police accident reports. To achieve the best fit between the observed and computed data, parameter values were estimated using the maximum likelihood estimation method. A statistical test was then performed in MATLAB to assess the relationship between the forecasted and actual data. Focus group discussions were employed to gather qualitative data due to their strong potential to provide in-depth insights into how alcohol consumption contributes to the rising incidence of road accidents. This method was specifically chosen for its effectiveness in uncovering nuanced perspectives and lived experiences. To maintain control over participants' responses and encourage meaningful engagement, each focus group was limited to between 4 and 8 individuals, resulting in a total of 17 respondents for the study. During the discussions, researchers took notes and recorded conversations, which were later transcribed for analysis. To enhance the reliability of the findings, triangulation was used, aligning with [17]'s assertion that no single research method is entirely self-sufficient, and combining methods can strengthen the validity of results. Accordingly, data obtained from traffic police reports were analyzed using descriptive statistics, while the qualitative data from the focus group discussions were examined through content analysis. In this section, information is presented based on [18]. As such, this work is devoted to fitting the predicted data of the models to the Tanzania data as obtained from focus group discussions, traffic police, and [19]. To get the best fit between the computed and observed data, parameter values were estimated by using the maximum likelihood estimator. A statistical test was carried out to establish the relationship between forecasted and observed data using MATLAB

5.1 Data Analysis, Results, and Discussion

As presented in Table 2, the original data set involved a 9-year accident report that emerged between 2016 to 2024.

Table 2: Accidents occurred between 2016 – 2024.

Source: Traffic Police, and [19]

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Period	Number of cases
2016	61
2017	93
2018	61
2019	492
2020	15
2021	34
2022	26
2023	19
2024	193

During the analysis procedure, the collected information was sorted. Therefore, in Figure 2, the raw data on accidents is displayed as collected from the field.

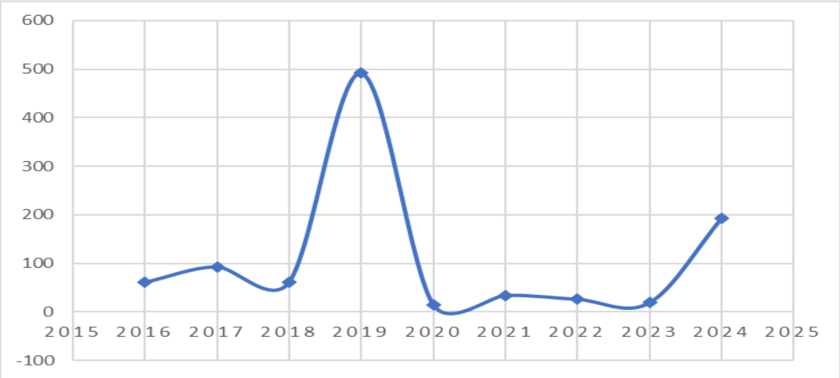

Figure 2: Raw data of road accidents caused by drunk driving

From Figure 2, it is observed that the number of accidents as a result of alcohol consumption decreased from 2016 to 2018. However, an increase in road accidents was later observed in 2019, then decreased in 2020-2023, and again increased in the year 2024, where most of the accidents were caused by private cars, especially at the end of the year during the Christmas season. This is because the majority prefer to use private cars from Dar es Salaam City to other regions. During the instances of taking driving breaks, the respective drivers pass by hotels or restaurants for drinks and food consumption.

Hence, it was shown that this is the time when we are losing control on the roads and cause accidents, as the Minister of Finance reported while presenting the annual budget 2024/2025. Explaining how this contributes to accidents, he stated that: *"Tumezoea kuona wakati wa sikukuu mwishoni mwa mwaka au katika matukio ya kijamii yanayopelekea wanafamilia husafiri chombo kimoja, kwa kuwa wanakuwa wanafahamiana umakini barabarani mara nyingi hupungua."Unakuta wanapita hoteli mbalimbali wanapata chakula au vinywaji vinavyowaondolea umakini barabarani hasa safari ndefu wanaona ni sehemu ya kufurahi na kunywa bia". During the end of the end of the year, it has been usual to find members of the family travelling in single private cars. But as they are familiar with each other, keen driving becomes low on the road; as they take alcoholic drinks such as beer, driving carelessly increases, and when the journeys are long, along with risky driving, inevitably the rate of accidents has been raising.*

5.2 Model Estimation and Evaluation

The highest possibility of the estimate was used to determine the best fit of the model. Here, the idea was to have a parsimonious model that captures as much variation in the data as possible. According to [18], the simple graph model captures most of the variability in most stabilized data. We used model system (1) to detect the observed data on accidents caused by drunk driving. The model was fitted to the data for accidents caused by drunk driving. Parameter values were estimated to vary within realistic means. Thus, the results as shown below

$\pi = 100$, $\sigma = 0.2$, $\delta = 0.9$, $\mu = 0.8$, $\mu_1 = 0.6$, $\mu_2 = 0.4$, $\varepsilon_1 = 0.23$, $\varepsilon_2 = 0.19$ and $\phi = 0.5$

The dynamic behaviour of the predicted and observed data using model system (1) and actual data was as presented in Figure 3.

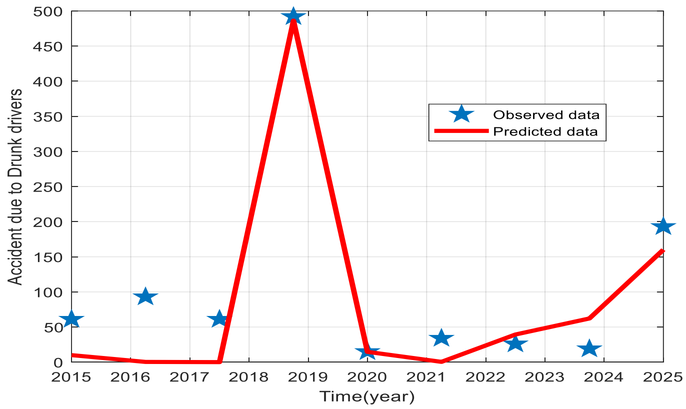

Figure 3: The dynamic behaviour of the predicted and observed data using model system (1) and actual data, respectively.

From Figure 3, it is apparent that various actual data values lie within the limits. Few lie exactly, meaning that the predicted data relates closely to the actual data. The coefficient of correlation (R) is +0.8980; hence, this suggests that there is a strong relationship between the predicted data and observed data. Nevertheless, a strong relationship is evident since the value is between +1 and -1; so, +0.8980 is close to +1. Coefficient of determination (R Square) is +0.806 something which shows that there is a strong relationship between predicted data and observed data. The reason for the strong relationship based on the fact that it takes the value between 0 and +1; so, +0.806 is close to +1. Therefore, 81% of the total variability of the predicted data supports the observed data of the model. Parameters' values were estimated by using the maximum likelihood estimator to get the best fit between the computed and observed data. Therefore, the following parameters' values were obtained (see Table 3).

Table 3: Parameter values that give the best fit to the data in the model.

Parameters	Description	Estimated value
π	Recruitment rate	126.1726
σ	Interaction rate of non- alcohol users with alcohol users	0.9389
δ	Fraction of exposed that become alcohol users	1.3537
μ	Road infrastructure, mechanical factors, or Natural calamitous	0.8193
μ_1	moderate alcohol user	3.6680
μ_2	addicted alcohol user	0.6503
ε_1	Conversion rate of moderate alcohol users to recover	0.3207
ε_2	Conversion rate of an addicted alcohol user to recover	0.4417
ϕ	Proportion of recovered individuals who become non-alcohol users	1.2276

Based on these observations, it is essential for policymakers and planners addressing alcohol use to project the prevalence of alcohol-related issues over the next five years using the model. The model indicates a projected increase in the prevalence of alcohol users during this period. The parameter values applied were consistent with those presented in Table 4. Figure 4 illustrates the projected trend of alcohol-consuming drivers through the year 2030.

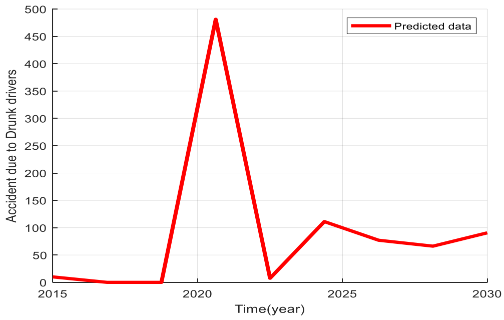

Figure 4: The projection of accidents caused by drunk driving in society

Figure 4 shows that the rate of accident occurrences initially declines slightly but then rises sharply over time. This trend highlights the urgent need to strengthen alcohol testing measures for drivers. One potential intervention could be the integration of in-vehicle alcohol detection systems capable of identifying intoxicated drivers. Such systems could automatically prevent the vehicle from starting until the driver is sober or an alternative, sober driver is available.

5.3 Sensitivity Analysis of Model Parameters

This section identifies various factors associated with alcohol use that contribute to reducing alcohol related accidents. To determine the most effective strategies for minimizing accidents caused by drunk driving, the sensitivity indices of the reproduction number for each model parameter were calculated, following the approaches outlined in [16] and [20]. These sensitivity indices indicate which parameters have the greatest influence on the reproduction number and should therefore be prioritized in efforts to reduce accident rates. The normalized sensitivity index on the R_0 for each parameter is defined as

$$X_r^{R_0} = \frac{\partial R_0}{\partial r} \times \frac{r}{R_0}. \tag{6}$$

Equation (6) is employed to compute the sensitivity index of each variable influencing the basic reproduction number, R_0. A positive sensitivity index indicates that increasing the corresponding parameter will raise R_0, thereby promoting the spread of road accidents. Conversely, a negative index implies that increasing the parameter will reduce R_0, which may contribute to curbing road accidents. As illustrated in Table 5 and Figure 5, the sensitivity indices highlight the relative impact of each parameter on the dynamics of alcohol induced road accidents. For instance, $X_\delta^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0} = +0.014751$, $X_\mu^{R_0} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0} = -0.00253676$.

Table 5: Sensitivity indices of model parameters to R_0

S/N	Parameter Symbol	Sensitivity Index
1	δ	+0.014751
2	μ_1	-0.00101099
3	μ_2	-0.00152572
4	μ	-0.00253676

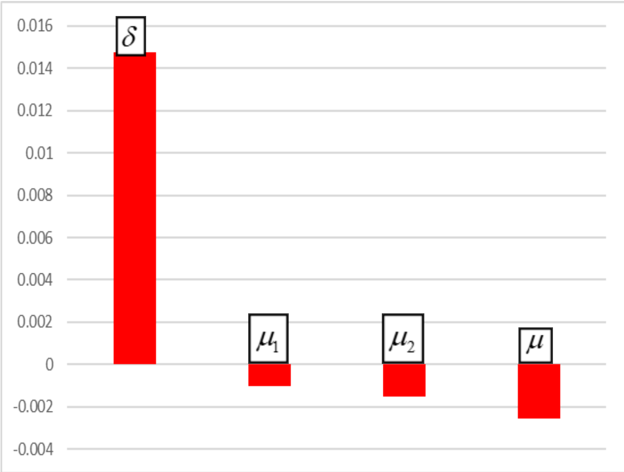

Figure 5: Chart of Sensitivity indices

6. Interpretation of Sensitivity Indices

The sensitivity analysis reveals a positive sensitivity index for parameter δ, while negative sensitivity indices are observed for parameters μ, μ1, and μ2. This indicates that an increase in parameter δ raises the basic reproduction number (R_0), thereby increasing the likelihood of alcohol-related road accidents. In contrast, increases in parameters μ, μ1, and μ2 lead to a reduction in R0, which contributes to the decline and potential elimination of such accidents. To effectively mitigate alcohol-induced accidents, it is crucial to focus on reducing parameter δ. This can be achieved by enhancing alcohol testing efforts, such as monitoring drivers who enter alcohol-selling establishments, or by incorporating advanced technology into vehicles (including cars, motorcycles, and tri-cycle detect intoxicated drivers. These systems could automatically disable the vehicle until the driver is sober or replaced by a sober individual. Implementing such measures would represent one of the most effective strategies for eliminating the risk of road accidents caused by alcohol consumption.

7. Stability Analysis

Local Stability of the Alcohol Users' Free Equilibrium Point

Theorem 3: The alcohol users-free equilibrium E_0 of the model system (1) is locally asymptotically stable if $R_0 < 1$ and is unstable if $R_0 > 1$.

Remark: This implies that there will be no accident in the community caused by a driver taking alcohol if $R_0 < 1$, and can remain in the community if $R_0 > 1$.

Proof: Local stability of the alcohol point is determined by the variational matrix $\mathbf{J}_{E_0}$ of the nonlinear differential equation (1) corresponding to E_0 as follows, let

$$\frac{dN_u}{dt} = \pi + \phi R_u - (\mu + \sigma) N_u = Q_1(N_u, E_u, M_u, A_u, R_u)$$

$$\frac{dE_u}{dt} = \sigma N_u - \mu E_u - (1-\delta)\frac{A_u}{N} E_u - \delta \frac{M_u}{N} E_u = Q_2(N_u, E_u, M_u, A_u, R_u)$$

$$\frac{dM_u}{dt} = \delta \frac{M_u}{N} E_u - (\mu + \mu_1 + \varepsilon_1) M_u = Q_3(N_u, E_u, M_u, A_u, R_u)$$
$$\frac{dA_u}{dt} = (1-\delta) \frac{A_u}{N} E_u - (\mu + \mu_2 + \varepsilon_2) A_u = Q_4(N_u, E_u, M_u, A_u, R_u)$$

$$\frac{dR_u}{dt} = \varepsilon_1 M_u + \varepsilon_2 A_u - (\mu + \phi) R_u = Q_5(N_u, E_u, M_u, A_u, R_u) \tag{7}$$

It follows that,

$$\mathbf{J}_{E_0} = \begin{bmatrix} \frac{\partial Q_1}{\partial N_u}(E_0) & \frac{\partial Q_1}{\partial E}(E_0) & \frac{\partial Q_1}{\partial M_u}(E_0) & \frac{\partial Q_1}{\partial A_u}(E_0) & \frac{\partial Q_1}{\partial R_u}(E_0) \\ \frac{\partial Q_2}{\partial N_u}(E_0) & \frac{\partial Q_2}{\partial E}(E_0) & \frac{\partial Q_2}{\partial M_u}(E_0) & \frac{\partial Q_2}{\partial A_u}(E_0) & \frac{\partial Q_2}{\partial R_u}(E_0) \\ \frac{\partial Q_3}{\partial N_u}(E_0) & \frac{\partial Q_3}{\partial E}(E_0) & \frac{\partial Q_3}{\partial M_u}(E_0) & \frac{\partial Q_3}{\partial A_u}(E_0) & \frac{\partial Q_3}{\partial R_u}(E_0) \\ \frac{\partial Q_4}{\partial N_u}(E_0) & \frac{\partial Q_4}{\partial E}(E_0) & \frac{\partial Q_4}{\partial M_u}(E_0) & \frac{\partial Q_4}{\partial A_u}(E_0) & \frac{\partial Q_4}{\partial R_u}(E_0) \\ \frac{\partial Q_5}{\partial N_u}(E_0) & \frac{\partial Q_5}{\partial E}(E_0) & \frac{\partial Q_5}{\partial M_u}(E_0) & \frac{\partial Q_5}{\partial A_u}(E_0) & \frac{\partial Q_5}{\partial R_u}(E_0) \end{bmatrix}$$

Thus, the variational matrix of the nonlinear model differential equation (7) is obtained as

$$\begin{pmatrix} -\mu-\sigma & 0 & 0 & 0 & \phi \\ \sigma & -\mu & \frac{-\pi\delta\sigma}{N\mu^2 + N\mu\sigma} & \frac{\pi(-1+\delta)\sigma}{N\mu(\mu+\sigma)} & 0 \\ 0 & 0 & -1-\mu+\frac{-\pi\delta\sigma}{N\mu^2+N\mu\sigma}-\mu_1 & 0 & 0 \\ 0 & 0 & 0 & -1-\mu+\frac{-\pi(-1+\delta)\sigma}{N\mu(\mu+\sigma)}-\mu_2 & 0 \\ 0 & 0 & 1 & 1 & -\mu-\phi \end{pmatrix} \text{.....(8)}$$

Therefore, the stability of the alcohol user free equilibrium point is clarified by studying the behaviour of $\mathbf{J}_{E_0}$ in which for local stability of alcohol user, we seek for its all eigenvalues to have negative real parts. It follows that, the characteristic function of the matrix (8) with λ being the eigenvalues of the Jacobian matrix, $\mathbf{J}_{E_0}$, then the Jacobian matrix has the following values:

$\lambda_1 = -\mu$, $\lambda_2 = -\mu - \sigma$, $\lambda_3 = -\mu - \phi$,

Then others are given as

$$\lambda_4 = \frac{-N\mu^2 - N\mu^3 + \pi\delta\sigma - N\mu\sigma - N\mu^2\sigma - N\mu^2\mu_1 - N\mu\sigma\mu_1}{N\mu(\mu+\sigma)}$$

$$\lambda_4 = -\frac{N\mu^2 + N\mu^3 - \pi\delta\sigma + N\mu\sigma + N\mu^2\sigma + N\mu^2\mu_1 + N\mu\sigma\mu_1}{N\mu(\mu+\sigma)}$$

$$\lambda_4 = \frac{-(\mu+\mu^2+\sigma+\mu\sigma+\mu\mu_1+\sigma\mu_1)}{\mu+\sigma} + \frac{\pi\delta\sigma}{N\mu(\mu+\sigma)} \text{ when } \frac{\pi\delta\sigma}{N\mu(\mu+\sigma)} < \frac{(\mu+\mu^2+\sigma+\mu\sigma+\mu\mu_1+\sigma\mu_1)}{\mu+\sigma}$$

$$\lambda_4 = -\left(\frac{(\mu+\mu^2+\sigma+\mu\sigma+\mu\mu_1+\sigma\mu_1)}{\mu+\sigma} - \frac{\pi\delta\sigma}{N\mu(\mu+\sigma)}\right) \text{ when } \frac{\pi\delta\sigma}{N\mu(\mu+\sigma)} < \frac{(\mu+\mu^2+\sigma+\mu\sigma+\mu\mu_1+\sigma\mu_1)}{\mu+\sigma}$$

$$\lambda_5 = \frac{-N\mu^2 - N\mu^3 - \pi\sigma(\delta-1) - N\mu\sigma - N\mu^2\sigma - N\mu^2\mu_2 - N\mu\sigma\mu_2}{N\mu(\mu+\sigma)} \text{ when } \delta \geq 1$$

Therefore, the system is stable since all five eigenvalues are negative. This implies that at $R_0 < 1$ the alcohol user-free Equilibrium point is locally asymptotically stable.

8. Existence and Local Stability of Endemic Equilibrium

Since we are concerned with the presence of alcohol users, we can reduce differential equation (1) to a 3-dimensional system by eliminating N_u & R_u respectively.

$$\begin{aligned} \frac{dE_u}{dt} &= \sigma N_u - \mu E_u - (1-\delta)\frac{A_u}{N}E_u - \delta\frac{M_u}{N}E_u \\ \frac{dM_u}{dt} &= \delta\frac{M_u}{N}E_u - (\mu+\mu_1+\varepsilon_1)M_u \\ \frac{dA_u}{dt} &= (1-\delta)\frac{A_u}{N}E_u - (\mu+\mu_2+\varepsilon_2)A_u \end{aligned} \qquad (9)$$

Then we set

$$\frac{dE_u}{dt} = \frac{dA_u}{dt} = \frac{dM_u}{dt} = 0.$$

At that juncture, the model of differential equation (9) has an exceptional endemic equilibrium given by $E^* = (E_u^*, M_u^*, A_u^*)$ in Ω, with

$$\sigma N_u - \mu E_u^* - (1-\delta)\frac{A^*_u}{N}E_u^* - \delta\frac{M^*_u}{N}E_u^* = 0 \qquad 9.1$$

$$\delta\frac{M^*_u}{N}E_u^* - (\mu+\mu_1+\varepsilon_1)M^*_u = 0 \qquad 9.2$$

$$(1-\delta)\frac{A^*_u}{N}E^*_u - (\mu+\mu_2+\varepsilon_2)A^*_u = 0 \qquad 9.3$$

9. Existence of Endemic Equilibrium

For the existence and uncommonness of endemic equilibrium $E^* = \left(E_u^*, M_u^*, A_u^*\right)$, should satisfy the conditions $E_u^* \neq 0$ or $M_u^* \neq 0$ or $A_u^* \neq 0$ *i.e.* $E_m^* > 0$ or $M_u^* > 0$ or $A_u^* > 0$ by using the idea of [13]

Adding equations (9.1) -(9.3), gives:

$$\sigma N_u - \mu E_u^* - (1-\delta)\frac{A^*_u}{N}E_u^* - \delta\frac{M^*_u}{N}E_u^* + \delta\frac{M^*_u}{N}E_u^* - (\mu+\mu_1+\varepsilon_1)M^*_u + (1-\delta)\frac{A^*_u}{N}E^*_u - (\mu+\mu_2+\varepsilon_2)A^*_u = 0$$

$$\sigma N_u - \mu E_u^* - (\mu+\mu_1+\varepsilon_1)M^*_u - (\mu+\mu_2+\varepsilon_2)A^*_u = 0$$

$$\sigma N_u - \mu E_u^* - (\mu+\mu_1+\varepsilon_1)M^*_u - (\mu+\mu_2+\varepsilon_2)A^*_u = 0$$

It follows that:

$$\mu E_u^* + (\mu+\mu_1+\varepsilon_1)M^*_u + (\mu+\mu_2+\varepsilon_2)A^*_u = \sigma N_u$$

Then,

$$\mu E_u^* > 0, (\mu+\mu_1+\varepsilon_1)M^*_u > 0 \textit{ and } (\mu+\mu_2+\varepsilon_2)A^*_u > 0$$

This implies that

$$\mu > 0 \textit{ and } E_u^* > 0, \quad (\mu+\mu_1+\varepsilon_1) > 0 \textit{ and } M^*_u > 0 \textit{ then}$$

$$(\mu+\mu_2+\varepsilon_2) > 0 \textit{ and } \quad A^*_u > 0$$

Thus, the endemicity of the alcohol user exists since $E_m^* > 0$ or $M_u^* > 0$ or $A_u^* > 0$ and then the accident rate due to alcohol use $\mu_1 > 0$ and $\mu_2 > 0$, means that the accident due to alcohol user exists, and then also there exists a conversion rate of moderate and addicted alcohol users recovering from using alcohol, i.e. $\varepsilon_1 > 0 \textit{ and } \varepsilon_2 > 0$ respectively.

10. Local Stability of the Endemic Equilibrium

Local stability of the endemic equilibrium point is determined by the variational matrix $J\left(E^*\right)$ of the nonlinear differential equations (9) corresponding to E^* as follows:

$$\begin{aligned}
\frac{dE_u}{dt} &= \sigma N_u - \mu E_u - (1-\delta)\frac{A_u}{N}E_u - \delta\frac{M_u}{N}E_u = a_1(E_u, M_u, A_u) \\
\frac{dM_u}{dt} &= \delta\frac{M_u}{N}E_u - (\mu+\mu_1+\varepsilon_1)M_u = a_2(E_u, M_u, A_u) \\
\frac{dA_u}{dt} &= (1-\delta)\frac{A_u}{N}E_u - (\mu+\mu_2+\varepsilon_2)A_u = a_3(E_u, M_u, A_u)
\end{aligned} \tag{10}$$

It follows that

$$J\left(E^*\right)=\begin{bmatrix} \frac{\partial a_1}{\partial E_u} & \frac{\partial a_1}{\partial M_u} & \frac{\partial a_1}{\partial A_u} \\ \frac{\partial a_2}{\partial E_u} & \frac{\partial a_2}{\partial M_u} & \frac{\partial a_2}{\partial A_u} \\ \frac{\partial a_3}{\partial E_u} & \frac{\partial a_3}{\partial M_u} & \frac{\partial a_3}{\partial A_u} \end{bmatrix}$$

Hence, the variation matrix of the nonlinear model equation (10) was obtained as presented below:

$$J\left(E^*\right)=\begin{bmatrix} -\mu-\frac{(1-\delta)A_u}{N}-\frac{\delta M_u}{N} & \frac{-\delta E_u}{N} & \frac{-(1-\delta)E_u}{N} \\ \frac{\delta M_u}{N} & -\mu+\frac{\delta E_u}{N}-\varepsilon_1-\mu_1 & 0 \\ \frac{(1-\delta)A_u}{N} & 0 & -\mu+\frac{\delta E_u}{N}-\varepsilon_2-\mu_2 \end{bmatrix} \qquad (11)$$

The following Lemma was stated and proved by [21] - [23] to demonstrate the local stability of the endemic equilibrium point E^*

Lemma 1: Let $J\left(E^*\right)$ be a 3x3 matrix, if tr $\left(J\left(E^*\right)\right)$, det $\left(J\left(E^*\right)\right)$ and $\left(J^{[2]}\left(E^*\right)\right)$ are all negative, hence from the Jacobian matrix in (11). We have

$$Tr\left(J\left(E^*\right)\right)=-\mu-\frac{(1-\delta)A_u}{N}-\frac{\delta M_u}{N}-\mu+\frac{\delta E_u}{N}-\varepsilon_1-\mu_1-\mu+\frac{\delta E_u}{N}-\varepsilon_2-\mu_2<0$$

$$Tr\left(J\left(E^*\right)\right)=-\mu-\frac{(1-\delta)A_u}{N}-\frac{\delta M_u}{N}-\mu+2\frac{\delta E_u}{N}-\varepsilon_1-\mu_1-\mu-\varepsilon_2-\mu_2<0 \; when \; 2\frac{\delta E_u}{N}=\frac{(1-\delta)A_u}{N}+\frac{\delta M_u}{N}$$

$$Tr\left(J\left(E^*\right)\right)=-\left(\mu+\frac{(1-\delta)A_u}{N}+\frac{\delta M_u}{N}+\mu-2\frac{\delta E_u}{N}+\varepsilon_1+\mu_1+\mu+\varepsilon_2+\mu_2\right)<0 \; when \; 2\frac{\delta E_u}{N}=\frac{(1-\delta)A_u}{N}+\frac{\delta M_u}{N}$$

$$\det\left(J\left(E^*\right)\right)=\begin{vmatrix} -\mu-\frac{(1-\delta)A_u}{N}-\frac{\delta M_u}{N} & \frac{-\delta E_u}{N} & \frac{-(1-\delta)E_u}{N} \\ \frac{\delta M_u}{N} & -\mu+\frac{\delta E_u}{N}-\varepsilon_1-\mu_1 & 0 \\ \frac{(1-\delta)A_u}{N} & 0 & -\mu+\frac{\delta E_u}{N}-\varepsilon_2-\mu_2 \end{vmatrix}$$

$$\det\left(J\left(E^*\right)\right)=\frac{-N(1-\delta)\mathrm{A}_u\left(-\delta\,\mathrm{E}_u+\mathrm{NQ}\right)P-N\left(-\delta\mu E_u+\left(N\mu+\delta M_u\right)Q\right)\left(\left(-1+\delta\right)E_u+NP\right)}{N^3}$$

Therefore,

$$\det\left(J\left(E^*\right)\right)=\frac{-N(1-\delta)\mathrm{A}_u\left(\mathrm{NQ}-\delta\,\mathrm{E}_u\right)P-N\left(\left(N\mu+\delta M_u\right)Q-\delta\mu E_u\right)\left(\left(-1+\delta\right)E_u+NP\right)}{N^3}$$

$$\det\left(J\left(E^*\right)\right)=-\left(\frac{(1-\delta)\mathrm{A}_u\left(\mathrm{NQ}-\delta\,\mathrm{E}_u\right)P+\left(\left(N\mu+\delta M_u\right)Q-\delta\mu E_u\right)\left(\left(-1+\delta\right)E_u+NP\right)}{N^2}\right)<0$$

$$where \; Q=\mu+\varepsilon_1+\mu_1 \; and \; P=\mu+\varepsilon_2+\mu_2$$

Hence, the trace and determinant of the Jacobian matrix J(E)are all negative

The second additive compound matrix is obtained from Lemma 2

Lemma 2: Let $[r_{gq} \mid c_{gq}]$ be a subset of matrix P. The entry $V_{ij}J^{[2]u}(E^*)$ is obtained by taking the coefficient of b in the calculation of the determinant of the sub-matrix of P index by row r_{gq} and column c_{gq} *but* $g \neq q$

$$P = \left[J\left(E^*\right) + Ib\right] = \begin{bmatrix} -\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N} & \frac{-\delta E_u}{N} & \frac{-(1-\delta)E_u}{N} \\ \frac{\delta M_u}{N} & -\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1 & 0 \\ \frac{(1-\delta)A_u}{N} & 0 & -\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2 \end{bmatrix} + \begin{bmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$$

where I is identity matrix

$$P = \begin{bmatrix} -\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N} + b & \frac{-\delta E_u}{N} & \frac{-(1-\delta)E_u}{N} \\ \frac{\delta M_u}{N} & -\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1 + b & 0 \\ \frac{(1-\delta)A_u}{N} & 0 & -\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2 + b \end{bmatrix}$$

Then V_{11} is

$$V_{11} = DetP[r_{12} \mid c_{12}] = \begin{bmatrix} -\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N} + b & \frac{-\delta E_u}{N} \\ \frac{\delta M_u}{N} & -\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1 + b \end{bmatrix}$$

$$V_{11} = DetP[r_{12} \mid c_{12}] = \left(-\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N} + b\right)\left(-\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1 + b\right) + \frac{\delta M_u}{N} \times \frac{\delta E_u}{N}$$

coefficient of b which is $-\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N} - \mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1$

Others will be calculated following the same procedure and obtained as:

$$V_{12} = \det P[r_{12} \mid c_{13}] \begin{bmatrix} -\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N} + b & \frac{-(1-\delta)E_u}{N} \\ \frac{\delta M_u}{N} & 0 \end{bmatrix}$$

$$\therefore V_{12} = 0$$

$$V_{13} = \det P[r_{12} \mid c_{23}] = \begin{bmatrix} \frac{-\delta E_u}{N} & \frac{-(1-\delta)E_u}{N} \\ \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1\right) + b & 0 \end{bmatrix}$$

$$V_{13} = \frac{(1-\delta)E_u}{N}$$

$$V_{21} = \det P[r_{13} \mid c_{12}] = \begin{bmatrix} \left(-\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N}\right) + b & \frac{-\delta E_u}{N} \\ \frac{(1-\delta)A_u}{N} & 0 \end{bmatrix}$$

$$V_{21} = 0$$

$$V_{22} = \det P[r_{13} \mid c_{13}] = \begin{bmatrix} \left(-\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N}\right) + b & \frac{-(1-\delta)E_u}{N} \\ \frac{(1-\delta)A_u}{N} & \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2\right) + b \end{bmatrix}$$

$$V_{22} = \left(-\mu - \frac{(1-\delta)A_u}{N} - \frac{\delta M_u}{N}\right) + \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2\right)$$

$$V_{23} = \det P[r_{13} \mid c_{23}] = \begin{pmatrix} \frac{-\delta E_u}{N} & \frac{-(1-\delta)E_u}{N} \\ 0 & \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2\right) + b \end{pmatrix}$$

$$V_{23} = \frac{-\delta E_u}{N}$$

$$V_{31} = \det P[r_{23} \mid c_{12}] = \begin{bmatrix} \frac{\delta M_u}{N} & \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1\right) + b \\ \frac{(1-\delta)A_u}{N} & 0 \end{bmatrix}$$

$$V_{31} = -\frac{(1-\delta)A_u}{N}$$

$$V_{32} = \det P[r_{23} \mid c_{13}] = \begin{bmatrix} \frac{\delta M_u}{N} & 0 \\ \frac{(1-\delta)A_u}{N} & \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2\right) + b \end{bmatrix}$$

$$V_{32} = \frac{\delta M_u}{N}$$

$$V_{33} = \det P[r_{23} \mid c_{23}] = \begin{bmatrix} \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1\right) + b & 0 \\ 0 & \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2\right) + b \end{bmatrix}$$

$$V_{33} = \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_1 - \mu_1\right) + \left(-\mu + \frac{\delta E_u}{N} - \varepsilon_2 - \mu_2\right)$$

$$J^{[2]}\left(E^*\right)=\begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix}$$

$$\det\left(J^{[2]}\left(E^*\right)\right)=\begin{vmatrix} -\mu-\frac{(1-\delta)A_u}{N}-\frac{\delta M_u}{N}-\mu+\frac{\delta E_u}{N}-\varepsilon_1-\mu_1 & 0 & \frac{(1-\delta)E_u}{N} \\ 0 & -\mu-\frac{(1-\delta)A_u}{N}-\frac{\delta M_u}{N}-\mu+\frac{\delta E_u}{N}-\varepsilon_2-\mu_2 & \frac{-\delta E_u}{N} \\ -\frac{(1-\delta)A_u}{N} & \frac{\delta M_u}{N} & -\mu+\frac{\delta E_u}{N}-\varepsilon_1-\mu_1-\mu+\frac{\delta E_u}{N}-\varepsilon_2-\mu_2 \end{vmatrix}$$

This gives

$$\det\left(J^{[2]}\left(E^*\right)\right)=-\frac{1}{N^3}\begin{pmatrix} \delta^2 E_u M_u\left(-(-1+\delta)A_u-\delta E_u+\delta M_u+NQ\right)+ \\ \left(-(-1+\delta)A_u-\delta E_u+\delta M_u+NG\right)\times \\ \left((-1+\delta)^2 A_u E_u+\left(-(-1+\delta)A_u-\delta E_u+\delta M_u+NQ\right)\left(-2\delta E_u+NP\right)\right) \end{pmatrix}$$

$$P=\left(2\mu+\varepsilon_1+\varepsilon_2+\mu_1+\mu_2\right)$$
$$Q=\left(2\mu+\varepsilon_1+\mu_1\right)$$
$$G=\left(2\mu+\varepsilon_2+\mu_2\right)$$

It follows that:

$$\det\left(J^{[2]}\left(E^*\right)\right)=-\frac{1}{N^3}\begin{pmatrix} \delta^2 E_u M_u\left(-(-1+\delta)A_u-\delta E_u+\delta M_u+NQ\right)+ \\ \left(-(-1+\delta)A_u-\delta E_u+\delta M_u+NG\right)\times \\ \left((-1+\delta)^2 A_u E_u+\left(-(-1+\delta)A_u-\delta E_u+\delta M_u+NQ\right)\left(-2\delta E_u+NP\right)\right) \end{pmatrix}$$

$$\textit{Hence}\ \det\left(J^{[2]}\left(E^*\right)\right)<0$$

Thus, from the lemma 2, the prevalent equilibrium E^* of the model Differential equation (10) is locally asymptotically stable in Ω.

11. Numerical Simulations and Discussion of Results.

Figures 6, 7, and 8 present a detailed analysis of the influence of various factors namely μ, μ_1, and μ_2, on the prevalence of road accidents, emphasizing systemic issues such as inadequate road infrastructure, insufficient vehicle maintenance, exposure to natural disasters, and alcohol consumption by drivers. In particular, Figure 6 underscores the substantial risk posed by poor road conditions and the neglect of routine vehicle inspections. The lack of proper infrastructure, such as pothole-ridden roads or the absence of clear signage, along with vehicles that are not regularly serviced, creates an unsafe driving environment. Natural events like floods or landslides further compound these risks, especially when transportation systems lack the resilience to withstand such disruptions. These conditions collectively elevate the probability of accidents by increasing the unpredictability and danger of the driving environment.

As demonstrated in Figure 7, the issue of alcohol consumption among drivers significantly magnifies the risks, particularly during periods of social interaction such as weekends, lunch breaks, or after-work gatherings. Even at low levels, alcohol impairs cognitive function, reducing a driver's ability to make quick and sound decisions. Reaction times are notably slowed, and attention spans become compromised, making it difficult for drivers to respond effectively to sudden changes in road conditions or traffic patterns. This becomes more problematic in scenarios where drivers engage in regular, casual drinking, assuming it has little to no effect on their driving capabilities. However, evidence shows that even minor intoxication can result in dangerous misjudgements, such as miscalculating distances or failing to recognize hazards, thereby increasing the likelihood of collisions and fatal accidents.
Figure 8 expands the discussion by focusing on chronic alcohol use and its direct connection to some of the most severe road accidents. Drivers who habitually consume alcohol, particularly those who do so heavily during the late-night to early-morning hours, are at a heightened risk due to the compounded effects of intoxication and sleep deprivation. In countries like Tanzania, a significant number of road accidents occur between 3:00 AM and 11:00 AM, a period closely associated with reduced visibility, lower traffic enforcement presence, and driver fatigue. These drivers often operate vehicles in a physically and mentally impaired state, drastically reducing their ability to stay alert or react appropriately in critical situations. The confluence of these factors makes them a major contributor to traffic-related fatalities. Therefore, mitigating these issues calls for urgent and targeted interventions. Strategies must include not only infrastructure development and vehicle inspection policies, but also rigorous enforcement of drunk driving laws, public education campaigns on the dangers of impaired driving, and rehabilitation programs for habitual offenders. Such multi-faceted efforts are essential to reducing the incidence of alcohol induced accidents and improving overall road safety.

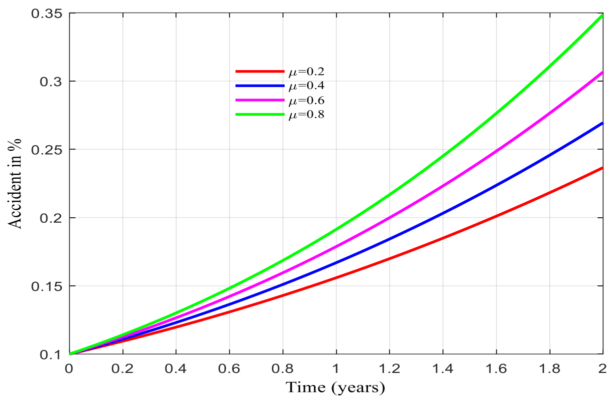

Figure 6: Impact of μ on alcohol induced road accidents.

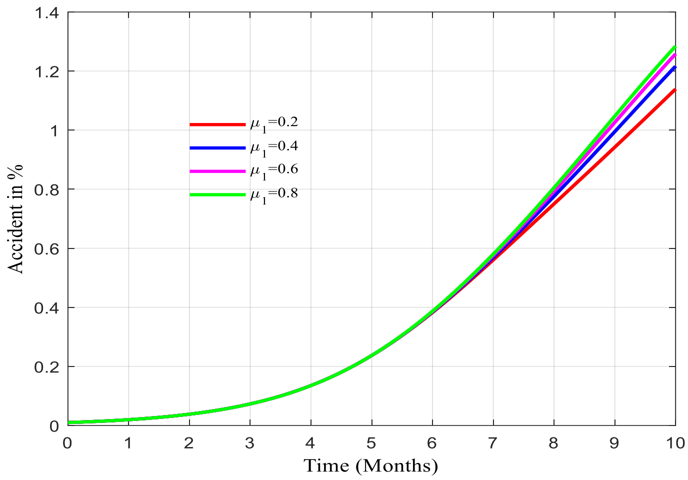

Figure 7: Impact of increasing μ_1 on alcohol induced road accidents.

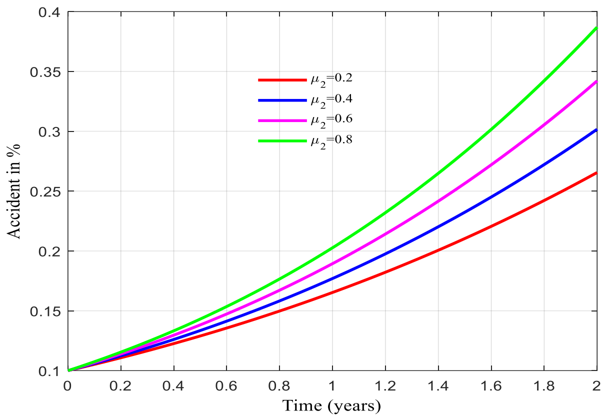

Figure 8: Impact of increasing μ_2 on alcohol induced road accidents

12. Conclusion

A compartmental model was developed to analyse the impact of alcohol consumption on road accidents. This involved categorizing individuals into five groups such as the non-alcohol users, exposed to alcohol users, moderate alcohol users, addicted alcohol users, and individuals who have recovered from using alcohol. The sensitivity analysis identified the fraction of exposed alcohol users as the most influential factor in accident occurrence. This was followed by moderate alcohol use, addicted alcohol users. Interestingly, road infrastructure conditions, mechanical failures, and natural disasters was found to have the least significant effect on accident rates. To mitigate these risk factors, proactive measures should be implemented at the county level. Strengthening alcohol testing for drivers is crucial. One potential solution is mandatory installation of alcohol testers in all vehicles. This should include cars, motorbikes and tricycle vehicles. So, if a driver tests positive to alcohol consumption, then his or her vehicle should be programmed to the automatically disable to the movement until such the driver sobers up or an alternative driver is assigned the driving task. This study is of the view that the approach could be effective strategy to reducing alcohol related accidents.

Further, the numerical simulations revealed that negligence in maintaining road infrastructure, inadequate vehicle maintenance, and natural disasters contribute to an increase in accidents. Moreover, a rising tendency of consuming alcoholic drinks among drivers, particularly those observed in moderate alcohol use was shown to significantly escalate accident rates. Therefore, interventions by introducing advanced technological devices and establishment of policies that restrict consumption or alcoholic drinks on roads and proper enforcement of the existing traffic laws should be prioritised for they are essential in minimizing road accidents and enhancing overall traffic safety.

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