# The Properties of Intuitionistic Multi L–Fuzzy Cosets And Pseudo Intuitionistic Multi L-Fuzzy Cosets of an Intuitionistic Multi L- Fuzzy Subgroup Under Homomorphism and Anti – Homomorphism

**C.Gowrishankar[1], M.Akilesh[2]**

[1]Research Scholar, Department of Mathematics, SRMV College of Arts and Science, Coimbatore-641 020, India.
[2]Assistant Professor, Department of Mathematics, SRMV College of Arts and Science, Coimbatore-641 020, Tamil Nadu, India.

**Abstract**
In this paper, we introduce and study the concepts of Intuitionistic Multi L - fuzzy cosets and Pseudo Intuitionistic Multi L – fuzzy cosets of an Intuitionistic Multi L – fuzzy subgroup under group homomorphisms and anti-homomorphisms. We investigate the structural properties of these newly defined cosets and explore their relationships with Intuitionistic Multi L – fuzzy subgroups. Furthermore, we analyze the preservation of Intuitionistic Multi L- fuzzy membership and non-membership functions under homomorphic and anti-homomorphic mappings, establishing several fundamental results. Our findings extend classical coset theory to the framework of Intuitionistic Multi L- fuzzy algebraic structures, providing deeper insights into their algebraic behaviour.

**Keywords:** Intuitionistic Multi L-Fuzzy Subgroup (IMLFSG), Intuitionistic Multi L–Fuzzy Coset (IMLFC), Pseudo Intuitionistic Multi L – Fuzzy Coset (PIMLFC), Homomorphism – Anti-homomorphism.

## 1. Introduction

Fuzzy set theory and its generalizations have played a crucial role in extending classical algebraic structures to handle uncertainty and imprecision. Among these extensions, intuitionistic fuzzy sets, multi L- fuzzy sets, and their combinations have been widely studied to capture vagueness more effectively. In particular, intuitionistic multi L-fuzzy subgroups provide a more refined approach for analyzing algebraic structures under uncertainty.

The concept of fuzzy sets, first proposed by L.A. Zadeh in 1965, is a method for representing uncertainty mathematically in a regular setting. An element’s membership function in fuzzy set theory is a single value between 0 and 1. Consequently, K.T. Attanassov (1986) presented intuitionistic fuzzy sets, a generalization of fuzzy sets that deal with the degree of hesitation and the degree of non-membership function. The theory of multi-fuzzy sets was first presented by Sabu Sebastian and T.V. Ramakrishnan using a multi-dimensional membership function. The idea of the multi-anti fuzzy ideal of a ring under homomorphism was first presented by R. Muthuraj and S. Balamurugan.

In this paper, we explore the concept of intuitionistic multi L - fuzzy cosets and pseudo intuitionistic multi L – fuzzy cosets within the framework of group homomorphisms and anti-homomorphisms. The study of fuzzy cosets extends classical coset theory by incorporating degrees of membership and non-membership, allowing for a more flexible algebraic analysis.

## 2. Preliminaries

The basic definitions used in the sequel are listed in this section.

**2.1 Definition**

Let X be a non-empty set. A fuzzy set $A$ of X is defined by $A: X \to [0,1]$.

**2.2 Definition**

Let X be non-empty set. Let $A = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x)\rangle : x \epsilon X\}$ in X is defined as a set of ordered sequences.

ie., $A = \{\langle x, (\mu_{A_1}(x), \mu_{A_2}(x), \ldots., \mu_{A_i}(x), \ldots.), (\gamma_{A_1}(x), \gamma_{A_2}(x), \ldots., \gamma_{A_i}(x), \ldots.)\rangle : x \epsilon X\}$.

Where $\mu_{A_i}: X \to [0,1]$, $\gamma_{A_i}: X \to [0,1]$ and $0 \leq \mu_{A_i}(x) + \gamma_{A_i}(x) \leq 1$ for all $i$.

Here, $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq . \ldots . \geq \mu_{A_i}(x) \geq . \ldots$ , for all $x \epsilon X$ are decreasingly ordered sequence. Then the set $A$ is said to be an Intuitionistic Multi L-fuzzy subset (IMLFS) of X.

**Remark**

Since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

**2.3 Definition**

The Intuitionistic Multi L-fuzzy subset $A = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x)\rangle : x \epsilon X\}$ of a group G is said to be Intuitionistic Multi L-fuzzy subgroup of G (IMLFSG) if it satisfies the following: For all $x, y \epsilon G$,

1. $\mu_{A_i}(xy) \geq min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ and $\gamma_{A_i}(xy) \leq max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$,
2. $\mu_{A_i}(x^{-1}) = \mu_{A_i}(x)$ and $\gamma_{A_i}(x^{-1}) = \gamma_{A_i}(x)$.

Or Equivalently, if $A$ is IMLFSG of G iff

$\mu_{A_i}(xy^{-1}) \geq min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ and $\gamma_{A_i}(xy^{-1}) \leq max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$.

**2.4 Definition**

Let A be an Intuitionistic Multi L-fuzzy subgroup of a group G. For any $a \epsilon G$, the Intuitionistic Multi L–fuzzy (left)coset (IMLFC) $(aA)$ of G is defined by, $(aA) = \left\{\left(x, \mu_{(aA_i)}(x), \gamma_{(aA_i)}(x)\right) / x \in G\right\}$ where

1. $\mu_{(aA_i)}(x) = \mu_{A_i}(a^{-1}x)$
2. $\gamma_{(aA_i)}(x) = \gamma_{A_i}(a^{-1}x)$, for all $x \epsilon G$.

**2.5 Definition**

Let A be an Intuitionistic Multi L-fuzzy subgroup of a group G. For any $a, b \epsilon G$, the Intuitionistic Multi L–fuzzy middle coset (IMLFMC) $(aAb)$ of G is defined by,

$(aAb) = \left\{\left(x, \mu_{(aA_ib)}(x), \gamma_{(aA_ib)}(x)\right) / x \in G\right\}$where

1. $\mu_{(aA_ib)}(x) = \mu_{A_i}(a^{-1}xb^{-1})$
2. $\gamma_{(aA_ib)}(x) = \gamma_{A_i}(a^{-1}xb^{-1})$, for all $x \epsilon G$.

**Remarks**

1. If $a = e$ in G, then the Intuitionistic Multi L-fuzzy coset (aA) of G is defined by,

1. $\mu_{(aA_i)}(x) = \mu_{A_i}(x)$
2. $\gamma_{(aA_i)}(x) = \gamma_{A_i}(x)$, where A is an IMLFSG of G.

2. The Intuitionistic Multi L-fuzzy middle coset of an Intuitionistic Multi L-fuzzy group A of the group G determined by the element $a, b \in G$ is an IMLFSG of G if $b = a^{-1}$.

**2.6 Definition**

Let A be an Intuitionistic Multi L-fuzzy subgroup of a group G and an element $a\epsilon G$. Then Pseudo Intuitionistic Multi L–fuzzy coset (PIMLFC) $(aA)^p$ of G is defined by, $(aA)^p = \left\{\left(x, \mu_{(aA_i)^p}(x), \gamma_{(aA_i)^p}(x)\right)/x \in G\right\}$ where

1. $\mu_{(aA_i)^p}(x) = p(a)\mu_{A_i}(x)$
2. $\gamma_{(aA_i)^p}(x) = p(a)\gamma_{A_i}(x)$, for all $x\epsilon G$ and $p\epsilon P$.

**3. The Properties of IMLFC And PIMLFC of an IMLFSG Under Homomorphism and Anti-Homomorphism**

In this section, we discuss some properties of Intuitionistic Multi L- fuzzy cosets and Pseudo Intuitionistic Multi L – fuzzy cosets of an Intuitionistic Multi L- fuzzy subgroup of G under homomorphism and anti-homomorphism.

**3.1 Definition**

The function $f: G \to G'$ is said to be a homomorphism if $f(xy) = f(x)f(y)$,for all $x, y \in G$.

**3.2 Definition**

The function $f: G \to G'$ $(G\ and\ G'\ are\ not\ necessarily\ commutative)$ is said to be an anti-homomorphism if $f(xy) = f(y)f(x)$,for all $x, y \in G$.

**3.3 Definition**

Let $G\ and\ G'$ be any two groups. Let $f: G \to G'$ be a homomorphism and onto. Let $\mu_{A_i}: G \to L^k$ and $\gamma_{A_i}: G \to L^k$ be an Intuitionistic Multi L-fuzzy subgroups of $G$.

1. Then $f(\mu_{A_i})$ is an Intuitionistic Multi L-fuzzy subgroup of a group $G'$, if $\mu_{A_i}$ has a sup property and $\mu_{A_i}$ is $f - invariant$.
2. Then $f(\gamma_{A_i})$ is an Intuitionistic Multi L-fuzzy subgroup of a group $G'$, if $\gamma_{A_i}$ has a inf property and $\gamma_{A_i}$ is $f - invariant$.

**3.4 Theorem**

Let $G\ and\ G'$ be any two groups. Let $f: G \to G'$ be a homomorphism and onto. Let $A: G \to L^k$ be an IMLFSG of G and (aA) be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup A of the group G determined by the element $a \in G$. Then $f(aA)$ is an Intuitionistic Multi L-fuzzy coset of an IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$ and $f(aA) = f(a)f(A)$, if (i) $\mu_{A_i}$ has a sup property and $\mu_{A_i}$ is $f - invariant$ and (ii) $\gamma_{A_i}$ has a inf property and $\gamma_{A_i}$ is $f - invariant$.

**Proof**

Let $A$ be an IMLFSG of G and (aA) be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy group A of the subgroup G determined by the element $a \in G$.

Clearly, $f(A)$ is an IMLFSG of $G'$ and $f(aA)$ is an Intuitionistic Multi L- fuzzy coset of a IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$.

To prove that $f(aA) = f(a)f(A)$.

Now, for any $x \in G, f(x) \in G'$.

(i) Let, $\mu_{f(a)f(A_i)}(f(x)) = \mu_{f(A_i)}(f(a)^{-1}f(x))$ [$\because$ by definition 2.4]

$= \mu_{f(A_i)}(f(a^{-1})f(x))$

$= \mu_{f(A_i)}(f(a^{-1}x))$

$= \mu_{f(aA_i)}(f(x))$

i.e., $\mu_{f(a)f(A_i)}(f(x)) = \mu_{f(aA_i)}(f(x))$

(ii) Similarly, $\gamma_{f(a)f(A_i)}(f(x)) = \gamma_{f(A_i)}(f(a)^{-1}f(x))$ [$\because$ by definition 2.4]

$= \gamma_{f(A_i)}(f(a^{-1})f(x))$

$= \gamma_{f(A_i)}(f(a^{-1}x))$

$= \gamma_{f(aA_i)}(f(x))$

i.e., $\gamma_{f(a)f(A_i)}(f(x)) = \gamma_{f(aA_i)}(f(x))$

Hence, by (i) & (ii) gives, $f(aA) = f(a)f(A)$.

**3.5 Theorem**

Let $G$ *and* $G'$ be any two groups. Let $f: G \to G'$ be a homomorphism. Let $B: G' \to L^k$ be an IMLFSG of $G'$ and $(bB)$ be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup B of the group $G'$ determined by the element $b \in G'$. Then $f^{-1}(bB)$ is an Intuitionistic Multi L- fuzzy coset of an IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$ and $f^{-1}(bB) = f^{-1}(b)f^{-1}(B)$.

**Proof**

Let $B$ be an IMLFSG of $G'$ and $(bB)$ be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup $B$ of the group $G'$ determined by the element $b \in G'$.

Clearly, $f^{-1}(B)$ is an IMLFSG of $G$ and $f^{-1}(bB)$ is an Intuitionistic Multi L- fuzzy coset of a IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$.

To prove that $f^{-1}(bB) = f^{-1}(b)f^{-1}(B)$.

Now, for any $y \in G', f^{-1}(y) \in G$.

(i) Let, $\mu_{f^{-1}(bB_i)}(f^{-1}(y)) = \mu_{(bB_i)}(y)$

$= \mu_{(bB_i)}(f(x))$, there exists $x \in G$ such that $f(x) = y$.

$= \mu_{f^{-1}(b)f^{-1}(B_i)}(x)$

$= \mu_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

i.e., $\mu_{f^{-1}(bB_i)}(f^{-1}(y)) = \mu_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

(ii) Similarly, $\gamma_{f^{-1}(bB_i)}(f^{-1}(y)) = \gamma_{(bB_i)}(y)$

$= \gamma_{(bB_i)}(f(x))$, there exists $x \in G$ such that $f(x) = y$.

$= \gamma_{f^{-1}(b)f^{-1}(B_i)}(x)$

$= \gamma_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

i.e., $\gamma_{f^{-1}(bB_i)}(f^{-1}(y)) = \gamma_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

Hence, by (i) & (ii) gives, $f^{-1}(bB) = f^{-1}(b)f^{-1}(B)$.

**3.6 Theorem**

Let $G$ *and* $G'$ be any two groups. Let $f: G \to G'$ be an anti-homomorphism and onto. Let $A: G \to L^k$ be an IMLFSG of G and (aA) be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup A of the group G determined by the element $a \in G$. Then $f(aA)$ is an Intuitionistic Multi L-fuzzy coset of an IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$ and $f(aA) = f(a)f(A)$, if (i) $\mu_{A_i}$ has a sup property and $\mu_{A_i}$ is $f - invariant$ and (ii) $\gamma_{A_i}$ has a inf property and $\gamma_{A_i}$ is $f - invariant$.

**Proof**

Let $A$ be an IMLFSG of G and (aA) be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup A of the group G determined by the element $a \in G$.

Clearly, $f(A)$ is an IMLFSG of $G'$ and $f(aA)$ is an Intuitionistic Multi L- fuzzy coset of a IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$.

To prove that $f(aA) = f(a)f(A)$.

Now, for any $x \in G, f(x) \in G'$.

(i) Let, $\mu_{f(a)f(A_i)}(f(x)) = \mu_{f(A_i)}(f(a)^{-1}f(x))$ [∵ by definition 2.4]

$= \mu_{f(A_i)}(f(a^{-1})f(x))$

$= \mu_{f(A_i)}(f(x)f(a^{-1}))$ [ ∵ by definition 3.2]

$= \mu_{f(A_i)}(f(xa^{-1}))$

$= \mu_{A_i}(xa^{-1}) = \mu_{A_i}((xa^{-1})^{-1})$ [ ∵ A is an IMLFSG of G]

$= \mu_{A_i}(ax^{-1})$

$= \mu_{(a^{-1}A_i)}(x^{-1}) = \mu_{(aA_i)}(x)$

i.e., $\mu_{f(a)f(A_i)}(f(x)) = \mu_{f(aA_i)}(f(x))$

(ii) Similarly, $\gamma_{f(a)f(A_i)}(f(x)) = \gamma_{f(A_i)}(f(a)^{-1}f(x))$ [∵ by definition 2.4]

$= \gamma_{f(A_i)}(f(a^{-1})f(x))$

$= \gamma_{f(A_i)}(f(x)f(a^{-1}))$ [ ∵ by definition 3.2]

$= \gamma_{f(A_i)}(f(xa^{-1}))$

$= \gamma_{A_i}(xa^{-1}) = \gamma_{A_i}((xa^{-1})^{-1})$ [ ∵ A is an IMLFSG of G]

$= \gamma_{A_i}(ax^{-1})$

$= \gamma_{(a^{-1}A_i)}(x^{-1}) = \gamma_{(aA_i)}(x)$

i.e., $\gamma_{f(a)f(A_i)}(f(x)) = \gamma_{f(aA_i)}(f(x))$

Hence, by (i) & (ii) gives, $f(aA) = f(a)f(A)$.

**3.7 Theorem**

Let $G\ and\ G'$ be any two groups. Let $f: G \to G'$ be an anti-homomorphism. Let $B: G' \to L^k$ be an IMLFSG of $G'$ and $(bB)$ be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup B of the group $G'$ determined by the element $b \in G'$. Then $f^{-1}(bB)$ is an Intuitionistic Multi L- fuzzy coset of an IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$ and $f^{-1}(bB) = f^{-1}(b)f^{-1}(B)$.

**Proof**

Let $B$ be an IMLFSG of $G'$ and $(bB)$ be a Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup $B$ of the group $G'$ determined by the element $b \in G'$.

Clearly, $f^{-1}(B)$ is an IMLFSG of $G$ and $f^{-1}(bB)$ is an Intuitionistic Multi L- fuzzy coset of a IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$.

To prove that $f^{-1}(bB) = f^{-1}(b)f^{-1}(B)$.

Now, for any $y \in G', f^{-1}(y) \in G$.

(i) Let, $\mu_{f^{-1}(bB_i)}(f^{-1}(y)) = \mu_{(bB_i)}(y)$

$= \mu_{(bB_i)}(f(x))$, there exists $x \in G$ such that $f(x) = y$.

$= \mu_{f^{-1}(b)f^{-1}(B_i)}(x)$

$= \mu_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

i.e., $\mu_{f^{-1}(bB_i)}(f^{-1}(y)) = \mu_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

(ii) Similarly, $\gamma_{f^{-1}(bB_i)}(f^{-1}(y)) = \gamma_{(bB_i)}(y)$

$= \gamma_{(bB_i)}(f(x))$, there exists $x \in G$ such that $f(x) = y$.

$= \gamma_{f^{-1}(b)f^{-1}(B_i)}(x)$

$= \gamma_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

i.e., $\gamma_{f^{-1}(bB_i)}(f^{-1}(y)) = \gamma_{f^{-1}(b)f^{-1}(B_i)}(f^{-1}(y))$

Hence, by (i) & (ii) gives, $f^{-1}(bB) = f^{-1}(b)f^{-1}(B)$.

## 4. Properties of Pseudo Intuitionistic Multi L-Fuzzy Cosets of an IMLFSG Under Homomorphism and Anti-Homomorphism

In this section, we discuss the properties of pseudo Intuitionistic Multi L- fuzzy cosets of an IMLFSG of G under homomorphism and anti-homomorphism.

### 4.1 Theorem

Let $G$ $and$ $G'$ be any two groups. Let $f: G \to G'$ be a homomorphism and onto. Let $A: G \to L^k$ be an IMLFSG of G and $(aA)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup A of the group G determined by the element $a \in G$. Then $f((aA)^p)$ is a Pseudo Intuitionistic Multi L- fuzzy coset of an IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$ and $f((aA)^p) = (f(a)f(A))^p$, if (i) $\mu_{A_i}$ has a sup property and $\mu_{A_i}$ is $f - invariant$ and (ii) $\gamma_{A_i}$ has a inf property and $\gamma_{A_i}$ is $f - invariant$.

**Proof**

Let $A$ be an IMLFSG of G and $(aA)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a IMLFSG A of the group G determined by the element $a \in G$.

Clearly, $f(A)$ is an IMLFSG of $G'$ and $f((aA)^p)$ is an Pseudo Intuitionistic Multi L- fuzzy coset of an IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$.

To prove that $f((aA)^p) = (f(a)f(A))^p$.

Now, for any $x \in G, f(x) \in G'$.

(i) Let, $\mu_{(f(a)f(A_i))^p}(f(x)) = p(f(a))\mu_{f(A_i)}(f(x))$ [∵ by definition 2.6]

$= p(a)\mu_{A_i}(x)$

$= \mu_{(aA_i)^p}(x)$

$= \mu_{f((aA_i)^p)}(f(x))$

i.e., $\mu_{(f(a)f(A_i))^p}(f(x)) = \mu_{f((aA_i)^p)}(f(x))$

(ii) Similarly, $\gamma_{(f(a)f(A_i))^p}(f(x)) = p\big(f(a)\big)\gamma_{f(A_i)}(f(x))$ [∵ by definition 2.6]

$= p(a)\,\gamma_{A_i}(x)$

$= \gamma_{(aA_i)^p}(x)$

$= \gamma_{f((aA_i)^p)}(f(x))$

i.e., $\gamma_{(f(a)f(A_i))^p}(f(x)) = \gamma_{f((aA_i)^p)}(f(x))$

Hence, by (i) & (ii) gives, $f((aA)^p) = (f(a)f(A))^p$.

**4.2 Theorem**

Let $G$ *and* $G'$ be any two groups. Let $f: G \to G'$ be a homomorphism. Let $B: G' \to L^k$ be an IMLFSG of $G'$ and $(bB)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup B of the group $G'$ determined by the element $b \in G'$. Then $f^{-1}((bB)^p)$ is a Pseudo Intuitionistic Multi L-fuzzy coset of an IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$ and $f^{-1}((bB)^p) = (f^{-1}(b)f^{-1}(B))^p$.

**Proof**

Let $B$ be an IMLFSG of $G'$ and $(bB)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup $B$ of the group $G'$ determined by the element $b \in G'$.

Clearly, $f^{-1}(B)$ is an IMLFSG of $G$ and $f^{-1}((bB)^p)$ is a Pseudo Intuitionistic Multi L- fuzzy coset of a IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$.

To prove that $f^{-1}((bB)^p) = (f^{-1}(b)f^{-1}(B))^p$.

Now, for any $y \in G', f^{-1}(y) \in G$.

(i) Let, $\mu_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \mu_{(bB_i)^p}(y)$

$= \mu_{(bB_i)^p}(f(x))$, there exists $x \in G$ such that $f(x) = y$.

$= p(b)\mu_{B_i}(f(x))$

$= p(f^{-1}(b))\, \mu_{f^{-1}(B_i)}(x)$

$= \mu_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$

i.e., $\mu_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \mu_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$

(ii) Similarly, $\gamma_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \gamma_{(bB_i)^p}(y)$

$= \gamma_{(bB_i)^p}(f(x))$, there exists $x \in G$ such that $f(x) = y$.

$= p(b)\, \gamma_{B_i}(f(x))$

$= p(f^{-1}(b))\, \gamma_{f^{-1}(B_i)}(x)$

$= \gamma_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$

i.e., $\gamma_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \gamma_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$

Hence, by (i) & (ii) gives, $f^{-1}((bB)^p) = (f^{-1}(b)f^{-1}(B))^p$.

**4.3 Theorem**

Let $G$ *and* $G'$ be any two groups. Let $f: G \to G'$ be an anti-homomorphism and onto. Let $A: G \to L^k$ be an IMLFSG of G and $(aA)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup A of the group G determined by the element $a \in G$. Then $f((aA)^p)$ is a Pseudo Intuitionistic Multi L- fuzzy coset of an IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$ and $f((aA)^p) = (f(a)f(A))^p$, if (i) $\mu_{A_i}$ has a sup property and $\mu_{A_i}$ is $f - invariant$ and (ii) $\gamma_{A_i}$ has a inf property and $\gamma_{A_i}$ is $f - invariant$.

**Proof**

Let $A$ be an IMLFSG of G and $(aA)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a IMLFSG A of the group G determined by the element $a \in G$.

Clearly, $f(A)$ is an IMLFSG of $G'$ and $f((aA)^p)$ is an Pseudo Intuitionistic Multi L- fuzzy coset of a IMLFSG $f(A)$ of the group $G'$ determined by the element $f(a) \in G'$.

To prove that $f((aA)^p) = (f(a)f(A))^p$.
Now, for any $x \in G, f(x) \in G'$.
(i) Let, $\mu_{(f(a)f(A_i))^p}(f(x)) = p(f(a))\mu_{f(A_i)}(f(x))$ [$\because$ by definition 2.6]
$$= p(a)\mu_{A_i}(x)$$
$$= \mu_{(aA_i)^p}(x)$$
$$= \mu_{f((aA_i)^p)}(f(x))$$
i.e., $\mu_{(f(a)f(A_i))^p}(f(x)) = \mu_{f((aA_i)^p)}(f(x))$
(ii) Similarly, $\gamma_{(f(a)f(A_i))^p}(f(x)) = p\big(f(a)\big)\gamma_{f(A_i)}(f(x))$ [$\because$ by definition 2.6]
$$= p(a)\, \gamma_{A_i}(x)$$
$$= \gamma_{(aA_i)^p}(x)$$
$$= \gamma_{f((aA_i)^p)}(f(x))$$
i.e., $\gamma_{(f(a)f(A_i))^p}(f(x)) = \gamma_{f((aA_i)^p)}(f(x))$
Hence, by (i) & (ii) gives, $f((aA)^p) = (f(a)f(A))^p$.

**4.4 Theorem**

Let $G$ *and* $G'$be any two groups. Let $f: G \to G'$ be an anti-homomorphism. Let $B: G' \to L^k$ be an IMLFSG of $G'$ and$(bB)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup B of the group $G'$ determined by the element $b \in G'$. Then $f^{-1}((bB)^p)$ is a Pseudo Intuitionistic Multi L- fuzzy coset of an IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$ and $f^{-1}((bB)^p) = (f^{-1}(b)f^{-1}(B))^p$.

**Proof**

Let $B$ be an IMLFSG of $G'$ and $(bB)^p$ be a Pseudo Intuitionistic Multi L- fuzzy coset of a Intuitionistic Multi L-fuzzy subgroup $B$ of the group $G'$ determined by the element $b \in G'$.
Clearly, $f^{-1}(B)$ is an IMLFSG of $G$ and $f^{-1}((bB)^p)$ is a Pseudo Intuitionistic Multi L- fuzzy coset of a IMLFSG $f^{-1}(B)$ of the group $G$ determined by the element $f^{-1}(b) \in G$.
To prove that $f^{-1}((bB)^p) = (f^{-1}(b)f^{-1}(B))^p$.
Now, for any $y \in G', f^{-1}(y) \in G$.
(i) Let, $\mu_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \mu_{(bB_i)^p}(y)$
$$= \mu_{(bB_i)^p}(f(x)), \text{ there exists } x \in G \text{ such that } f(x) = y.$$
$$= p(b)\mu_{B_i}(f(x))$$
$$= p(f^{-1}(b))\, \mu_{f^{-1}(B_i)}(x)$$
$$= \mu_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$$
i.e., $\mu_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \mu_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$
(ii) Similarly, $\gamma_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \gamma_{(bB_i)^p}(y)$
$$= \gamma_{(bB_i)^p}(f(x)), \text{ there exists } x \in G \text{ such that } f(x) = y.$$
$$= p(b)\, \gamma_{B_i}(f(x))$$
$$= p(f^{-1}(b))\, \gamma_{f^{-1}(B_i)}(x)$$
$$= \gamma_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$$
i.e., $\gamma_{f^{-1}((bB_i)^p)}(f^{-1}(y)) = \gamma_{(f^{-1}(b)f^{-1}(B_i))^p}(f^{-1}(y))$
Hence, by (i) & (ii) gives, $f^{-1}((bB)^p) = (f^{-1}(b)f^{-1}(B))^p$.
Hence the theorem.

**Remark**

In a similar manner, we can extend these ideas to discuss the properties of Intuitionistic Multi L-fuzzy middle cosets and Intuitionistic Multi L-fuzzy double cosets of an Intuitionistic Multi L- fuzzy (normal) subgroup of G determined by the elements a and b in G under homomorphism and anti-homomorphism.

## 5. Conclusion

In this study, we explored the properties of Intuitionistic Multi L- fuzzy cosets and Pseudo Intuitionistic Multi L- fuzzy cosets within the framework of an Intuitionistic Multi L- fuzzy subgroup under homomorphism and anti-homomorphism. While homomorphisms generally preserve their structure, anti-homomorphisms may alter relationships due to operational reversals. These findings contribute to the broader understanding of fuzzy algebraic structures and their applications. Future work may explore their implications in topological and categorical settings.

## 6. References

1. Akilesh.M and Muthuraj.R, “Some Properties of Multi L- Fuzzy Cosets”, International Journal of Mathematical Archive, Vol.9, Issue 5, pp. 211-215, 2018.
2. Atanassov. K.T, “Intuitionistic fuzzy sets”, Fuzzy Sets and Systems, Vol. 20, no.1, pp. 87-96, 1986.
3. Choudhury. F.P, Chakraborty. A.B and Khare .S.S, “A note on fuzzy subgroups and fuzzy homomorphism”, Journal of mathematical analysis and applications 131, pp. 537-553, 1988.
4. Muthuraj.R and Balamurugan.S, “ A Study on Intuitionistic Multi-Fuzzy Subgroups”, International Journal of Applications of Fuzzy sets and Artificial Intelligence, Vol.4, pp. 153-172, 2014.
5. Nagarajan.R and Solairaju.A, “On Pseudo Fuzzy Cosets of Fuzzy Normal Subgroups”, IJCA (0975-8887), Volume 7, No.6, pp. 34-37, 2010.
6. Palaniappan.N andMuthuraj.R, “The homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy groups, Varahmihir Journal of Mathematics Sciences, Vol. 4, No. 2, pp. 387-399, 2004.
7. Rosenfeld.A, "Fuzzy groups", J. Math. Anal. Appl, Vol. 35, pp. 512-517, 1971.
8. Roventa.A and Spircu.T., "Groups operating on fuzzy sets", Fuzzy sets and systems, Vol.120, 543 –548,2001.
9. Sabu Sebastian and Ramakrishnan.T.V, “Multi Fuzzy Sets”, International Mathematical Forum, Vol. 5, No. 50, pp.2471-2476, 2010.
10. Sharma.P.K., “Homomorphism of Intuitionistic fuzzy groups”, International Mathematics Forum , Vol. 6, no. 64, pp. 3169-3178, 2011.
11. Sharma.P.K and Sandhu.M.K, “On Pseudo Intuitionistic Fuzzy Cosets”, International Journal of MathematicalSciences and Applications, Vol.6, Issue 2, pp.611-618, 2016.
12. Shuka.S, “Pseudo Fuzzy Cosets”, IJSRP, Vol.3, Issue 1, pp. 1-2, 2013.
13. Sunderrajan.K andSuresh.M , “Homomorphism of Multi L-Fuzzy Subgroup”, Gen. Math. Notes, ISSN: 2219-7184, Vol. 22, No.1, pp. 86-92, 2014.
14. Zadeh.L.A, “Fuzzy sets”, Information and control, Vol. 8, pp. 338-353, 1965.