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Some Properties of b#-Frontier in Intuitionistic Topological Spaces

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Abstract

Using the idea of intuitionistic b#-open sets and intuitionisticb#-closed sets, several characteristics of intuitionistic topological concepts of b#-Interior and b#-Closure have been shown. Moreover, the qualities of intuitionistic b# Frontier have been presented and examined, as well as the properties of intuitionistic b# Frontier. Many counter-examples for the relevant classifications have been provided. The relation between intuitionistic b#-Interior, b#-Closure and b#-Frontier have been studied.

Keywords: Intuitionistic $b^{\#}$-Interior, Intuitionistic $b^{\#}$-Closure and Intuitionistic $b^{\#}$-Frontier

1. INTRODUCTION

Coker[2] developed the notion of intuitionistic fuzzy topological spaces by building on intuitionistic fuzzy sets [1]. Healso developed the concept of intuitionistic set in topological space. He examined some fundamental topological characteristics of intuitionistic sets. Sasikala[7][8][9] and Navaneetha Krishnan studied semi open sets, preopen sets, α -open sets in intuitionistic topological spaces. Velraj[10] has deliberated intuitionistic b#-open sets.The characteristics of intuitionistic b#-open sets are presented and described in this study.

In this paper, we defined b#-Frontier and some of its properties in intuitionistic topological spaces. We list some concepts and results introduced in [1,2,3].

Definition1.1 [cited from 3]: A familyτ of an intuitionistic subset (IS for short) in$\tilde{S}$satisfyingthefollowingaxioms is called anintuitionistictopology τ(ITforshort)on $\tilde{S}$:

(T1) $\tilde{\phi}, \tilde{S} \in \tau$,
(T2) $\tilde{P}_1 \cap \tilde{P}_2 \in \tau$ for any $\tilde{P}_1, \tilde{P}_2 \in \tau$ and
(T3) $\cup \tilde{P}_i \in \tau$ for any arbitrary family $\{\tilde{P}_i: i \in J\} \subseteq \tau$.

Definition 1.2[cited 4]:[4] Let $(\tilde{S},\tau)$ be an ITS and $\tilde{L}$ = <S, L^1,L^2> be an IS in $\tilde{S}$. Then the interior and closure of $\tilde{L}$ are defined by

$cl(\tilde{L}) = \cap\{\tilde{P}: \tilde{P}$ is an intuitionistic closed set in $\tilde{S}$ and $\tilde{L} \subseteq \tilde{P}\}$ and

$int(\tilde{L}) = \cup\{\tilde{P}: \tilde{P}$ is an intuitionistic open set in $\tilde{S}$ and $\tilde{P}\subseteq\tilde{L}\}$.

Definition 1.3 [cited from 6,7,10]: A subset $\tilde{L}$ of an ITS$(\tilde{S},\tau)$ is called

- *Intuitionistic b-open* (IbO for short) if $\tilde{L}\subseteq$int(cl($\tilde{L}$)) $\cup$ cl(int($\tilde{L}$)) and intuitionistic b-closed (IbC for short) if int(cl($\tilde{\square}$))$\cap$ cl(int($\tilde{\square}$)) $\subseteq\tilde{\square}$
- *Intuitionistic regular open* (IRO for short) if $\tilde{\square}$ = □nt(cl($\tilde{\square}$)) and intuitionistic regular closed (IRC for short) if cl(int($\tilde{\square}$)) = $\tilde{\square}$
- *Intuitionistic b#-open* (Ib#O for short) if $\tilde{\square}$ = cl(int($\tilde{\square}$))$\cup$int(cl($\tilde{\square}$)) and intuitionisticb#-closed (Ib#C) if int(cl($\tilde{\square}$)) $\cap$cl(int($\tilde{\square}$))= $\tilde{\square}$.

Proposition 1.4[cited from 10]. An IS $\tilde{\square}$ is an intuitionistic b#closed set if and only if $\tilde{\square}^c$ is Intuitionistic b# -open.

Proposition 1.5[cited from 10]. Let ($\tilde{\square}$, τ) be an ITS. For any intuitionistic subsets $\tilde{\square}$ and $\tilde{\square}$ of $\tilde{\square}$, we have

(i) Ib#int($\tilde{\square}$) $\subseteq\tilde{\square}$
(ii) $\tilde{\square}$ is the intuitionistic b#open set in $\tilde{\square}\Leftrightarrow$Ib#int($\tilde{\square}$) = $\tilde{\square}$
(iii) Ib#int(Ib#int($\tilde{\square}$)) = Ib#int($\tilde{\square}$)
(iv) If $\tilde{\square}\subseteq\tilde{\square}$ then Ib#int($\tilde{\square}$) $\subseteq$Ib#int($\tilde{\square}$)

Remark 1.6[cited from 10]. Since the intersection of intuitionistic b# -closed sets in $\tilde{\square}$ is also intuitioistic b# -closed set in $\tilde{\square}$, Ib#□□($\tilde{\square}$) is a intuitionistic b#closed set in $\tilde{\square}$.

Preposition 1.7[cited from 10]. Let ($\tilde{\square}$, τ) be an ITS and let $\tilde{\square}$ and $\tilde{\square}$ be the subsets of $\tilde{\square}$. Then

(i) (Ib#int($\tilde{\square}$))^c = Ib#cl($\tilde{\square}^c$)
(ii) (Ib#cl($\tilde{\square}$))^c = Ib#int($\tilde{\square}^c$)
(iii) $\tilde{\square}\subseteq$Ib#cl($\tilde{\square}$)
(iv) $\tilde{\square}$ is the intuitionistic b#-closed set in $\tilde{\square}\Leftrightarrow$Ib#cl($\tilde{\square}$) = $\tilde{\square}$
(v) Ib#cl(Ib#cl($\tilde{\square}$)) = Ib#cl($\tilde{\square}$)

Preposition 1.8[cited from 10]. Let ($\tilde{\square}$, τ) be an ITS. For any intuitionistic subsets $\tilde{\square}$ and $\tilde{\square}$ of $\tilde{\square}$,if $\tilde{\square}\subseteq\tilde{\square}$ then Ib#cl($\tilde{\square}$) $\subseteq$Ib#cl($\tilde{\square}$).

Proposition 1.9[cited from 10]. Let $\tilde{\square}$ be a subset of on ITS ($\tilde{\square}$, □). Then *Iint*($\tilde{\square}$) $\subseteq$*Ib#int*($\tilde{\square}$) $\subseteq\tilde{\square}\subseteq$*Icl*($\tilde{\square}$) $\subseteq$*Ib#cl*($\tilde{\square}$).

Theorem 1.10.[cited from 10] Let($\tilde{\square}$, τ) be an ITS and let $\tilde{\square}$ be a subset of $\tilde{\square}$. Then

(i) Ib#int($\tilde{\square}$) is an intuitionistic b#-open set.
(ii) Ib#int($\tilde{\square}$) is the largest open set contained in $\tilde{\square}$
(iii) $\tilde{\square}$ is Ib#open if and only if Ib#-int($\tilde{\square}$)=$\tilde{\square}$

Proposition 1.11[cited from 10].Let ($\tilde{\square}$, τ) be an ITS. For any intuitionistic subsets $\tilde{\square}$ and $\tilde{\square}$ of $\tilde{\square}$, we have

(i) $Ib^{\#}cl(\tilde{\square}\cup\tilde{\square}) \supseteq Ib^{\#}cl(\tilde{\square})\cup Ib^{\#}cl(\tilde{\square})$.
(ii) $Ib^{\#}cl(\tilde{\square}\cap\tilde{\square}) \subseteq Ib^{\#}cl(\tilde{\square})\cap Ib^{\#}cl(\tilde{\square})$.

Theorem 1.12. For a IS$\tilde{\square}$ of ($\tilde{\square}$, τ), Ib#□□□(Ib#□□□($\tilde{\square}$)) = Ib#□□□(Ib#□□($\tilde{\square}$)) ⊇Ib#□□□($\tilde{\square}$).

2. INTUITIONISTIC b$^{\#}$-FRONTIER

We introducedintuitionistic b$^{\#}$-Frontierin intuitionistic topological spaces and go over their properties in this section.

Definition 2.1.Let $\tilde{\square}$ be a subset of an ITS ($\tilde{\square}$, τ). Then the intuitionistic b#-Frontier of$\tilde{\square}$is the intersection of intuitionistic b#-closure of $\tilde{\square}$ and intuitionistic b#-closure of $\tilde{\square}^c$andit is referred by Ib#Fr($\tilde{\square}$). That is Ib#Fr($\tilde{\square}$) = Ib#cl($\tilde{\square}$) ∩Ib#cl($\tilde{\square}^c$).

Example 2.2.Let $\tilde{\square}$= {1,2,3} and consider the family τ={$\tilde{\phi}$, $\tilde{\square}$, $\tilde{\square}_1$, $\tilde{\square}_2$, $\tilde{\square}_3$} where $\tilde{\square}_1$= <*S*, {3},{1,2}>, $\tilde{\square}_2$= <*S*,{1},{2,3}>, $\tilde{\square}_3$= <*X*, {1,3},{2}>. Let $\tilde{\square}$ = <S, {1,2}, {3}> and $\tilde{\square}$ = <*S*, {2,3}, {1}> be the intuitionistic subsets of ($\tilde{\square}$, τ). Then $\tilde{\square}$and $\tilde{\square}$are intuitionistic b#-open sets. Let $\tilde{\square}$ =< □,{ϕ},{*1,2* } >. $\tilde{\square}^c$ =<S,{1,2},{ϕ}. Now Ib#cl($\tilde{\square}$) = <S, {3},{1,2}>.Ib#cl($\tilde{\square}^c$) = $\tilde{\square}$. Then Ib#Fr($\tilde{\square}$) = Ib#cl($\tilde{\square}$)∩Ib#cl($\tilde{\square}^c$) = <S,{3}, {1,2}>.

Theorem 2.3. For an IS $\tilde{\square}$ in the ITS ($\tilde{\square}$, τ), Ib#□□($\tilde{\square}$) =Ib#□□($\tilde{\square}^c$).

Proof. Let $\tilde{\square}$ be an IS in the ITS($\tilde{\square}$, τ).

Then by Definition 2.1, Ib#□□($\tilde{\square}$) = Ib#□□($\tilde{\square}$)∩Ib#□□($\tilde{\square}^c$) = Ib#□□($\tilde{\square}^c$)∩Ib#□□($\tilde{\square}$)
= Ib#□□($\tilde{\square}^c$)∩(Ib#□□(($\tilde{\square}^c$)^c)).

Again by Definition 2.1, this is equal to Ib#□□($\tilde{\square}^c$).

Hence Ib#□□($\tilde{\square}$)=Ib#□□($\tilde{\square}^c$).

Theorem 2.3. Let $\tilde{\square}$ be an IS in the ITS ($\tilde{\square}$, τ). Then Ib#□□($\tilde{\square}$) = Ib#□□($\tilde{\square}$) – Ib#□□□($\tilde{\square}$).

Proof. Let $\tilde{\square}$ be an IS in the ITS($\tilde{\square}$, τ).

By Proposition 1.7 (ii), (Ib#□□($\tilde{\square}^c$))^c = Ib#□□□($\tilde{\square}$) and by Definition 2.1, Ib#□□($\tilde{\square}$) = Ib#□□($\tilde{\square}$) ∩ (Ib#□□($\tilde{\square}^c$)) = Ib#□□($\tilde{\square}$) ∩ (Ib#□□□($\tilde{\square}^c$))^c.

By using $\tilde{\square}$ – $\tilde{\square}$=$\tilde{\square}$∩$\tilde{\square}^c$, Ib#□□($\tilde{\square}$)= Ib#□□($\tilde{\square}$) – Ib#□□□($\tilde{\square}$).

Hence Ib#□□($\tilde{\square}$)=Ib#□□($\tilde{\square}$) – Ib#□□□($\tilde{\square}$).

Theorem 2.4. An IS $\tilde{\square}$ is intuitionistic b#closed set in ($\tilde{\square}$, τ) iffIb#□□($\tilde{\square}$) ⊆$\tilde{\square}$.

Proof. Let $\tilde{\square}$ be an intuitionistic b#-closed set in the ITS($\tilde{\square}$, τ).

Then by Definition 2.1, Ib#□□($\tilde{\square}$)=Ib#□□($\tilde{\square}$)∩Ib#□□($\tilde{\square}^c$) ⊆Ib#□□($\tilde{\square}$). By using Proposition 1.7 (iv),Ib#□□($\tilde{\square}$)= $\tilde{\square}$.

Hence Ib#□□($\tilde{\square}$) ⊆$\tilde{\square}$, if $\tilde{\square}$ is intuitionistic b#-closed in $\tilde{\square}$.

Conversely, Assume that, Ib#□□($\tilde{\square}$) ⊆$\tilde{\square}$. Then Ib#□□($\tilde{\square}$) − Ib#□□□($\tilde{\square}$)⊆$\tilde{\square}$.

Since Ib#□□□($\tilde{\square}$) ⊆$\tilde{\square}$, we conclude that Ib#□□($\tilde{\square}$) = $\tilde{\square}$.

Hence □̃ is intuitionistic b#-closed.

Theorem 2.5. If □̃ is an intuitionistic b#-open set in (□̃, τ), then Ib#□□(□̃) ⊆□̃^{c}.
Proof. Let □̃ be an intuitionistic b#-open set in the ITS(□̃, τ).
By Proposition 1.4,□̃$^□$ is intuitionistic b#-closed set in □̃. By Theorem 2.3, Ib#□□(□̃$^□$)⊆□̃$^□$and by Definition 2.1., we get Ib#□□(□̃) ⊆□̃^{c}.

Theorem 2.6. Let □̃⊆□̃and □̃ be any intuitionistic b#-closed set in (□̃, τ). Then Ib#□□(□̃) ⊆□̃.
Proof. By Proposition 1.8 (i),□̃⊆□̃, Ib#□□(□̃) ⊆Ib#□□(□̃).
By Definition 2.1, Ib#□□(□̃)=Ib#□□(□̃)∩Ib#□□(□̃^{c}) ⊆Ib#□□(□̃)∩Ib#□□(□̃^{c}) ⊆Ib#□□(□̃).
Then by Remark 1.6**,** this is equal to □̃. Hence Ib#□□(□̃) ⊆□̃.

Theorem 2.7. Let □̃ be an IS in the ITS(□̃, τ). Then (Ib#□□(□̃))$^□$= Ib#□□□(□̃)∪Ib#□□□(□̃^{c}).
Proof. Let □̃ be an IS in the ITS(□̃, τ).
Then by Definition 2.1, (Ib#□□(□̃))$^□$=(Ib#□□(□̃)∩Ib#□□(□̃$^□$))$^□$ = ((Ib#□□(□̃))$^□$∪(Ib#□□(□̃$^□$))$^□$.
By Proposition 1.7 (ii), which is equal to Ib#□□□(□̃^{c}) ∪Ib#□□□(□̃).
Hence (Ib#□□(□̃))$^□$= Ib#□□□(□̃)∪Ib#□□□(□̃$^□$).

Theorem 2.8. For an IS □̃ in the ITS(□̃, τ), Ib#□□(□̃) ⊆IF□(□̃).
Proof. Let □̃ be an IS in the ITS(□̃, τ).
Then by Proposition 1.9, Ib#□□(□̃) ⊆I□□(□̃) and Ib#□□(□̃^{c}) ⊆I□□(□̃^{c}).
By Definition 2.1, Ib#□□(□̃)=Ib#□□(□̃)∩Ib#□□(□̃^{c}) ⊆I□□(□̃)∩I□□(□̃^{c}), = IF□(□̃).
Hence Ib#□□(□̃) ⊆IF□(□̃).

The example that follows demonstrates that the preceding Theorem's converse is not true.

Example 2.9. Let □̃= {1,2,3,4} and consider the family τ={$\tilde{\phi}$, □̃, □̃$_1$, □̃$_2$, □̃$_3$ } where □̃$_1$= <*S*,{1,2},{3,4}>, □̃$_2$= <*S*,{1,2,4},{3}>, □̃$_3$= <S, {1,2,3},{4}>. Let □̃ = <*S*, {1,2,4}, {3}> and □̃ = <*S*, {3,4}, {1,2}> be the ISs of (□̃, τ). Then □̃and □̃are intuitionistic b#-open sets. Let □̃ =< □,{*1*},{*2,3,4*} >. □̃^{c} =<S,{2,3,4},{1}. Now Ib#cl(□̃) = <S, {1,2},{3,4}>.Ib#cl(□̃^{c}) = □̃. Then Ib#Fr(□̃) = <S,{1,2}, {3,4}> Now Icl(□̃) = <S, {1,2,4},{3}>.Icl(□̃^{c}) = □̃. Then IFr(□̃) = <S,{1,2,4}, {3}>. Hence Ib#□□(□̃) ⊆IF□(□̃) but I□□(□̃) ⊈ Ib#F□(□̃).

Theorem 2.10. For an IS □̃ in the ITS(□̃, τ), Ib#□□(Ib#□□(□̃)) ⊆Ib#□□(□̃).
Proof. Let □̃ be the IS in the ITS(□̃, τ).
Then by Definition 2.1, Ib#□□(Ib#□□(□̃)) =Ib#□□(Ib#□□(□̃) ∩(Ib#□□(□̃^{c}))) ⊆(Ib#□□(Ib#□□(□̃))) ∩ (Ib#□□(Ib#□□(□̃^{c}))).
ByProposition 1.8 (iii),Ib#□□(Ib#□□(□̃)) =Ib#□□(□̃) ∩ (Ib#□□(□̃^{c})). By Definition 2.1, this is equal to Ib#□□(□̃)

Theorem 2.11. For an IS □̃ in the ITS(□̃, τ), Ib#□□(Ib#□□□(□̃)) ⊆Ib#□□(□̃).
Proof. Let □̃ be the IS in the ITS(□̃, τ).
Then by Definition 2.1, Ib#□□(Ib#□□□(□̃)) = Ib#□□(I□□□(□̃)) ∩ (Ib#□□(Ib#□□□(□̃))^c).

By Proposition 1.7 (i),Ib#□□(Ib#□□□(□̃)) = Ib#□□(Ib#□□□(□̃)) ∩ (Ib#□□ (Ib#□□ (□̃^{c}))).
= Ib#□□(Ib#□□□(□̃))∩(Ib#□□(□̃^{c}) . (By Proposition 1.7 (v))
⊆Ib#□□(□̃)∩Ib#□□(□̃^{c}) (By Proposition 1.5 (i))
= Ib#□□(□̃) (By Definition 2.1)
Hence Ib#□□(Ib#□□□(□̃)) ⊆(Ib#□□(□̃))

The example that follows demonstrates that the preceding Theorem's converse is not true.

Example 2.12. Consider the ITS in Example 2.2, Let□̃ =< □,{*1*},{*2,3*} >. Ib#int(□̃) = ϕ. Ib#Fr(Ib#int(□̃)) = ϕ. Now □̃^{c} =<S,{2,3},{1}. Ib#cl(□̃) = <S, {1},{2,3}>. Ib#cl(□̃^{c}) =□̃. Hence Ib#Fr(□̃) = <S,{1}, {2,3}>⊈ ϕ = Ib#Fr(Ib#int(□̃)).

Theorem 2.13. For an IS □̃ in the ITS(□̃, τ), Ib#□□(Ib#□□(□̃)) ⊆Ib#□□(□̃).
Proof. Let□̃ be an IS in the ITS(□̃, τ).
Then by Theroem2.4, Ib#□□(Ib#□□(□̃)) = Ib#□□(Ib#□□(□̃)) ∩ (Ib#□□(Ib#□□(□̃))^c).
= Ib#□□(□̃) ∩ (Ib#□□(Ib#□□□(□̃^{c})) (By Proposition 1.7 (ii) & (v) and Proposition 1.8)
⊆Ib#□□(□̃) ∩Ib#□□(□̃^{c}) (By Proposition 1.5 (i))
= Ib#□□(□̃) (By Definition 2.1)
Hence Ib#□□(Ib#□□(□̃)) ⊆Ib#□□(□̃).

The example that follows demonstrates that the preceding Theorem's converse is not true.

Example 2.14.Examine the ITS presented in Example 2.2, Let □̃ =< □,{*1*},{*2,3* } >. Ib#cl(□̃) = <S, {1},{2,3}> and (Ib#cl(□̃))^c =<S,{2,3},{1}>. Ib#cl(Ib#cl(□̃)) = <S, {1},{2,3}>. Ib#cl((Ib#cl(□̃))^c) =<S,{3},{1,2}>Ib#Fr(Ib#cl(□̃)) = ϕ. Now □̃^{c} =<S,{2,3},{1}. Ib#cl(□̃) = <S, {1},{2,3}>. Ib#cl(□̃^{c}) = □̃. Hence Ib#Fr(□̃) = <S,{1}, {2,3}>⊈ϕ= Ib#Fr(Ib#cl(□̃)).

Theorem 2.15. Let □̃ be an IS in the ITS(□̃, τ). Then Ib#□□□(□̃) ⊆□̃– Ib#□□(□̃).
Proof. Let □̃ be an IS in the ITS(□̃, τ).
Now by Definition 2.1, □̃ – Ib#□□(□̃) = □̃∩(Ib#□□(□̃))$^□$
=□̃∩[Ib#□□(□̃) ∩Ib#□□(□̃$^□$)]$^□$= □̃∩[Ib#□□□(□̃$^□$) ∪Ib#□□□(□̃)]
= [□̃∩Ib#□□□(□̃$^□$)] ∪ [□̃∩Ib#□□□(□̃)]
= [□̃∩Ib#□□□(□̃$^□$)] ∪Ib#□□□(□̃) ⊇Ib#□□□(□̃)
Hence Ib#□□□(□̃) ⊇□̃ – Ib#□□(□̃).

The following Theorem shows the relation between intuitionistic b#-Interior, b#-Closure and b#-Frontier.

Theorem 2.16. Let □̃ be an IS in the ITS(□̃, τ). Then Ib#int(□̃) = (Ib#cl(□̃^{c}))^c.
Proof: By Theorem 1.12, Ib#Ext(□̃) = (Ib#cl(□̃))^c
Therefore Ib#int(□̃) = Ib#Ext(□̃^{c}) = (Ib#cl(□̃^{c}))^c.

Theorem 2.17. Let □̃ be an IS in the ITS(□̃, τ). Then the intuitionistic b#closure of the complement of □̃ is the complement of the intuitionistic b#interior of □̃. That is Ib#cl(□̃^{c}) = (Ib#int(□̃))^c.

Proof. Taking complements in Theorem 2.16, (Ib#int(□̃))^c = ((Ib#cl(□̃^{c}))^c)^c = Ib#cl(□̃^{c})

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