An Alternative Row Adaptation technique for Solving Assignment Problems

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Abstract

Assignment problems are a fundamental optimization challenge in operations management, where tasks, resources, or personnel must be assigned efficiently to minimize costs or maximize effectiveness. These problems often arise in various industries, including logistics, manufacturing, and workforce scheduling. The classical assignment problem is modeled as a linear programming problem, typically solved using the Hungarian algorithm, which guarantees an optimal assignment with polynomial-time complexity. This paper explores an alternative time-consuming technique to solve balanced and unbalanced assignment problems which is very effective and straight forward. The traditional technique subtracts the row or column minimum entries from all the entries of assignment matrix whereas technique proposed in this paper subtracts only rows minimum entry from each row of the cost matrix, and for adaptation added unit amount in each row. Finally, the numerical illustrations and comparisons of different real-world problems shows the effectiveness of the proposed technique.

Keywords: Assignment problem, Row Adaptation, Balanced and unbalanced problems, Cost matrix, Optimization.

1. Introduction

The Assignment Problem is a special case of the more general Transportation Problem in operations research aims to optimize costs, time, or efficiency in allocation or movement of resources. It is a type of linear programming problem where the goal is to assign a number of resources (like workers, machines, or vehicles) to an equal number of tasks (jobs, locations, or destinations) in such a way that the total cost is minimized (or total efficiency is maximized) In it each resource is assigned to exactly one task and each task is assigned to exactly one resource and usually represented by a cost matrix, where the rows represent agents (e.g., drivers or delivery trucks) and columns represent tasks or destinations.

Generally, Balanced and unbalanced assignment problems are available in the literature. If in the assignment matrix, the number of agents is not equal to the number of tasks then problems can be termed as unbalanced assignment problem. The effectiveness matrix in this case will not be a square matrix. In such problems, dummy row(s) or dummy column(s) are added in the matrix so as to complete it to form a square matrix. In traditional approach, costs (C_{ij}) in such rows or columns represented by dummy will be treated as zero. Having formed to balanced matrix, the problem is then solved by Hungarian method or other available method. In literature a good amount of research is available for solving unbalanced and balanced assignment problems. In this paper we propose an efficient approach to solve balanced and unbalanced assignment problem by adding one additional unit in every row of cost matrix of assignment problems. In unbalanced assignment problems, sometimes require to add some dummy rows to convert it to balanced one. In this case we follow the same process after making the problems balanced. Our paper includes eight sections, of these eight sections first one explains the introduction, later section shows the worked already done on this topic, mathematical formulation is on third part, next we have our proposed algorithm and technique. Following the next section explores the numerical examples applying our proposed technique then comparison have been presented. Finally, limitations along future plan have been written and in the last section displays the conclusion of the paper.

2. Literature Review

The roots of the assignment problem can be traced back to the 19th century, when the Hungarian mathematician Dénes Kőnig developed graph theory concepts that would later become foundational in modeling assignment tasks. The most significant breakthrough came in 1955, when Harold W. Kuhn, an American mathematician, published a paper titled *"The Hungarian Method for the Assignment Problem"*. He named the method in honor of Hungarian mathematicians Dénes Kőnig and Jenő Egerváry, whose earlier work on matrices and combinatorics inspired the algorithm. Kuhn's algorithm provided the first efficient, polynomial-time method to solve the assignment problem exactly and has since become the standard approach [1]. Later a number of articles has been published on this field. Derigs et al. introduces an efficient algorithm for the linear assignment problem using the shortest augmenting path method. The paper enhances computational performance for large, dense problems, making it suitable for job scheduling, production tasks, and logistics [2]. Rainer Burkard et al. focuses on rectangular assignment problems where the number of agents and tasks differ (not a square matrix). It provides transformations to reduce such problems to the classical linear assignment problem and discusses applications in personnel assignment and production planning [3]. Mingyu Gao et al. proposes a distributed algorithm using Alternating Direction Method of Multipliers (ADMM) to solve large-scale assignment problems. This is particularly relevant in cloud computing, resource allocation, and massive logistics networks [4]. The application of the Hungarian algorithm in solving real-world product distribution problems; a case study optimizing cost-effective transportation between supply and demand points in a logistics system has been demonstrated by Shahariar et al.[5] . Kenan Cetin et al. [6] Proposes a multi-objective model for task assignment using Linear Physical Programming (LPP). It optimizes staff allocation in the banking industry, ensuring both workload balance and task prioritization. A Greedy & Metaheuristic hybrid to solve task assignment problems efficiently. It offers quick, near-optimal solutions for use in production scheduling and personnel allocation has successfully carried out by Douglas, Allison M. et al. [7]. An optimal solution technique has been illustrated by D. Sridevi, Dr. G. Sumathi et al. [8]. A .N.M. Rezaul Karim proposed approach has been illustrated with some numerical examples to demonstrate its effectiveness. The programming language of Python 3.8 was used to implement this novel approach [9]. There are also some effective processes has been demonstrated. The simplex method for linear programming was modified to solve assignment problem [10],[11],[12]. M.L. Balinski presented an effective signature method for the assignment problems [13]. For solving an unbalanced problem Kore proposed a new approach [14]. A technique like Hungarian method was proposed by Basirzaden, called Ones assignment method for solving assignment problems [15]. H. D Afroz et al. also proposed an alternative approach for solving balanced assignment problems where, the least element of the cost matrix has been subtracted from every entry of the cost matrix. [16]. Including these Farzana et al. proposes a new algorithm for solving assignment problems, offering a simplified logical and arithmetic approach, with accompanying C programming code for implementation [17]. An alternative solving procedure of unbalanced assignment has been explained in Rashid et.al.[18] With these available techniques our paper also proposes an effective simple technique by adapting the row entries of cost matrix of assignment problems, whereas most of the available technique considered both row and columns of cost matrix. As we consider only rows of assignment problems, so it becomes simple easy understandable and completely a unique from available method of solving assignment problems.

3. Mathematical Formulation

Associate to each assignment problem there is a matrix called cost or effectiveness (C_{ij}), where C_{ij} is the cost associated with assigning i^{th}resource (worker) to j^{th}activity (project). The cost matrix for such a problem is given below:

Activity (Projects)

		P_1	P_2	……	P_n	$Capacity\ (a_j)$
Resource (Worker)	W_1	c_{11}	c_{12}	……	c_{1n}	1
	W_2	c_{21}	c_{22}	……	c_{2n}	1
	⋮	⋮	⋮	……	⋮	⋮
	W_n	c_{n1}	c_{n2}	……	c_{nn}	1
	$Requirement\ (b_j)$	1	1	……	1	

Thus, the cost matrix is same as that of transportation cost matrix except that capacity (or availability) at each of the resources and the requirement at each of the destinations is taken to be unity due to the fact that assignments are made on a one-to-one basis.

Let:

n = number of agents (workers, machines, etc.)

n = number of tasks (jobs, activities, etc.)

C_{ij} = cost (or profit) of assigning agent i to task j

x_{ij} = decision variable, defined as:

$$x_{ij} = \begin{cases} 1\ ;\ if\ agent\ i\ is\ assigned\ to\ task\ j \\ 0\ ;\ otherwise \end{cases}$$

For Cost Minimization:

$$Minimize\ \ Z = \sum_{i}^{n}\sum_{j}^{n} C_{ij}x_{ij}$$

Subject to the constraints:

Each agent is assigned to exactly one task:

$$\sum_{i}^{n} x_{ij} = 1 \quad ;\ \forall i = 1, 2,3,4,5, \dots \dots \dots . . n$$

Each task is assigned to exactly one agent:

$$\sum_{j}^{n} x_{ij} = 1 \quad ;\ \forall j = 1, 2,3,4,5, \dots \dots \dots . . n$$

Using the above-mentioned mathematical model the assignment problems assign it goods to different destinations keeping total cost minimum. As a result, supply chain gets the optimum network holding maximum profit. Keep this motivation in mind this paper proposes an effective simple hassle-free algorithm to solve any kind of assignment problems balanced or unbalanced. In the later section the theoretical development and numerical illustration has been presented sequentially.

4. Proposed Algorithm

To evaluate the optimal solution of the assignment problem we developed an efficient algorithm to solve assignment problems just applying very few steps which are as follows:

Step 1: Check the Assignment problem whether it is balanced or unbalanced. If it is unbalanced then make it balanced.

Step 2: Identify the smallest value (cost) in each row and subtract this value from every element in that row.

Step 3: Add 1 to all the elements in the cost matrix.

Step 4: For each column, find the smallest value and divide every element in that column by this smallest value.

Step 5: Create assignments based on ones in the matrix. If there are rows with no assignments, then the solution isn't optimal. In such cases, proceed to the next steps.

Step 6: Using the following method, draw the fewest number of lines important to cover every one in the matrix:

a) Mark (√) the rows that do not have any assignments.
b) Mark (√) the columns containing ones in the rows that have been marked.
c) Mark (√) which rows have assignments in the respective columns.
d) Continue to go within steps (ii) and (iii) until there are no more rows or columns to mark.
e) Through all of the selected columns and all of the unmarked rows, draw straight lines. The current technique is ideal if the total number of lines is equal to the number of rows or columns. If not, move on to the following step.

Step 7: Divide all of the uncovered elements by the smallest uncovered value found in the reduced matrix. The numbers that the lines cover don't change. This process will result in new ones appearing in the matrix. Now, proceed to make assignments again based on the newly created ones.

Step 8: If you still can't achieve an optimal assignment for each row, repeat Step 4 and the subsequent steps until the optimal solution is found.

5. Numerical Examples

In this section, we will illustrate two real-life examples using our proposed algorithm. Of these examples first one is balanced and second one is unbalanced problem. Here it is noticeable that the unbalanced problem has been solved here by adding dummy rows only.

Example 1 (Balanced Assignment Problem): An airline has four pilots to assign to four flights. The pilots estimated performance scores (higher is better) for each flight are presented here using a matrix:

$$\begin{array}{c} \\ pilot\ 1 \\ pilot\ 2 \\ pilot\ 3 \\ pilot\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} Flight\ \mathrm{A} & Flight\ \mathrm{B} & Flight\ \mathrm{C} & Flight\ \mathrm{D} \end{array} \\ \begin{pmatrix} 85 & 80 & 90 & 88 \\ 78 & 82 & 85 & 90 \\ 92 & 85 & 80 & 87 \\ 86 & 88 & 84 & 89 \end{pmatrix} \end{array}$$

We have to determine the best pilot assignment for the flight in order to minimize the overall scores.

Here we follow our proposed algorithm step by step:
Step 1: The problem is balanced one so we have the option to follow the next step.
Step 1: Determine each row's minimum element and reduce it from each of the row's elements. The smaller matrix then is like this:

$$\begin{array}{c} \\ pilot\ 1 \\ pilot\ 2 \\ pilot\ 3 \\ pilot\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \text{Flight A} & \text{Flight B} & \text{Flight C} & \text{Flight C} \end{array} \\ \begin{pmatrix} 5 & 0 & 10 & 8 \\ 0 & 4 & 7 & 12 \\ 12 & 5 & 0 & 7 \\ 2 & 4 & 0 & 5 \end{pmatrix} \end{array}$$

Step 2: Now, add 1 to each matrix element.

$$\begin{array}{c} \\ pilot\ 1 \\ pilot\ 2 \\ pilot\ 3 \\ pilot\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \text{Flight A} & \text{Flight B} & \text{Flight C} & \text{Flight D} \end{array} \\ \begin{pmatrix} 6 & 1 & 11 & 9 \\ 1 & 5 & 8 & 13 \\ 13 & 6 & 1 & 8 \\ 3 & 5 & 1 & 6 \end{pmatrix} \end{array}$$

Step-3: Determine which number in each column is the least. Next, divide each column's total number by its smallest value.

$$\begin{array}{c} \\ pilot\ 1 \\ pilot\ 2 \\ pilot\ 3 \\ pilot\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \text{Flight A} & \text{Flight B} & \text{Flight C} & \text{Flight D} \end{array} \\ \begin{pmatrix} 6 & 1 & 11 & 1.5 \\ 1 & 5 & 8 & 2.17 \\ 13 & 6 & 1 & 1.33 \\ 3 & 5 & 1 & 1 \end{pmatrix} \end{array}$$

Step-4: Set initial assignments.

$$\begin{array}{c} \\ pilot\ 1 \\ pilot\ 2 \\ pilot\ 3 \\ pilot\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \text{Flight A} & \text{Flight B} & \text{Flight C} & \text{Flight D} \end{array} \\ \begin{pmatrix} 6 & \boxed{1} & 11 & 1.5 \\ \boxed{1} & 5 & 8 & 2.17 \\ 13 & 6 & \boxed{1} & 1.33 \\ 3 & 5 & \cancel{1} & \boxed{1} \end{pmatrix} \end{array}$$

Hence the optimum assignment schedule is, pilot-1 $\rightarrow$ Flight B, Pilot-2 $\rightarrow$ Flight A, Pilot-3 $\rightarrow$ Flight C, Pilot-4 $\rightarrow$ Flight D.
$\therefore$ Total scores is = (80+78+80+89)
= 327

Example 2 (Unbalanced assignment problem): There are three workers (W1, W2, W3) and five jobs (J1, J2, J3, J4, J5). The cost of assigning each worker to each job are:

$$\begin{array}{c} \\ w1 \\ w2 \\ w3 \end{array}\begin{array}{c} \begin{array}{ccccc} J1 & J2 & J3 & J4 & J5 \end{array} \\ \begin{pmatrix} 9 & 7 & 8 & 5 & 6 \\ 6 & 4 & 7 & 9 & 8 \\ 3 & 5 & 4 & 6 & 7 \end{pmatrix} \end{array}$$

We wish to find the optimal assignment for the above assignment problem. Now we will apply our proposed algorithm to solve the assignment problem for fulfill our requirement.

Step 1: Here in the problem no of workers in less than the no. of jobs. So, it is an unbalanced problem. Now we introduce two dummy row W4 and W5 with zero cost and balance the problem. Therefore, the new cost matrix becomes:

$$\begin{array}{c} \\ w1 \\ w2 \\ w3 \\ w4 \\ w5 \end{array}\begin{array}{c} \begin{array}{ccccc} J1 & J2 & J3 & J4 & J5 \end{array} \\ \begin{pmatrix} 9 & 7 & 8 & 5 & 6 \\ 6 & 4 & 7 & 9 & 8 \\ 3 & 5 & 4 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Step 2: Determine each row's minimum element and reduce it from each of the row's elements. The smaller matrix then looks like this:

$$\begin{array}{c} \\ w1 \\ w2 \\ w3 \\ w4 \\ w5 \end{array}\begin{array}{c} \begin{array}{ccccc} J1 & J2 & J3 & J4 & J5 \end{array} \\ \begin{pmatrix} 4 & 2 & 3 & 0 & 1 \\ 2 & 0 & 3 & 5 & 4 \\ 0 & 2 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Step 3: Now add 1 to each entry of the assignment matrix element.

$$\begin{array}{c} \\ w1 \\ w2 \\ w3 \\ w4 \\ w5 \end{array}\begin{array}{c} \begin{array}{ccccc} J1 & J2 & J3 & J4 & J5 \end{array} \\ \begin{pmatrix} 5 & 3 & 4 & 1 & 2 \\ 3 & 1 & 4 & 6 & 5 \\ 1 & 3 & 2 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

Step-3: Determine which number in each column is the least. Next, divide each column's total number by its smallest value.

$$\begin{array}{c} \\ w1 \\ w2 \\ w3 \\ w4 \\ w5 \end{array}\begin{array}{c} \begin{array}{ccccc} J1 & J2 & J3 & J4 & J5 \end{array} \\ \begin{pmatrix} 5 & 3 & 4 & 1 & 2 \\ 3 & 1 & 4 & 6 & 5 \\ 1 & 3 & 2 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

Step-4: Set initial assignments.

$$\begin{array}{c|ccccc|}
 & J1 & J2 & J3 & J4 & J5 \\
w1 & 5 & 3 & 4 & \boxed{1} & 2 \\
w2 & 3 & \boxed{1} & 4 & 6 & 5 \\
w3 & \boxed{1} & 3 & 2 & 4 & 5 \\
w4 & \xcancel{1} & \xcancel{1} & \boxed{1} & \xcancel{1} & \xcancel{1} \\
w5 & \xcancel{1} & \xcancel{1} & \xcancel{1} & \xcancel{1} & \boxed{1}
\end{array}$$

Optimal assignment schedule is w1 →J4, w2 → J2, w3 →J1, w4 →J3, w5 →J5.
Total cost is = 5+4+3+0+0
=12

6. Comparison of Results

Using proposed algorithm, we solved numbers of assignment problems of various size of cost matrix and compare the results with that of obtained using Hungarian Method. The comparative study shows our technique is very convenient to solve all types of assignment problem presenting best optimal result which are shown in the table below:

Table 1: Comparison when the problems are balanced

Problem	Size of the matrix	Hungarian Method	New proposed method	optimum
1. $\begin{pmatrix} 9 & 2 & 7 \\ 6 & 4 & 3 \\ 3 & 8 & 5 \end{pmatrix}$	3×3	08	08	08
2. $\begin{pmatrix} 30 & 20 & 25 \\ 28 & 32 & 18 \\ 35 & 25 & 24 \end{pmatrix}$	3×3	73	72	72
3. $\begin{pmatrix} 12 & 15 & 11 \\ 14 & 10 & 13 \\ 13 & 16 & 12 \end{pmatrix}$	3×3	34	34	34
4. $\begin{pmatrix} 12 & 30 & 21 & 15 \\ 18 & 33 & 9 & 31 \\ 44 & 25 & 24 & 21 \\ 23 & 30 & 28 & 14 \end{pmatrix}$	4×4	60	60	60
5. $\begin{pmatrix} 8 & 26 & 17 & 11 \\ 13 & 28 & 4 & 26 \\ 38 & 19 & 18 & 15 \\ 19 & 26 & 24 & 10 \end{pmatrix}$	4×4	41	41	41

6. $\begin{pmatrix} 20 & 28 & 19 & 13 \\ 15 & 30 & 31 & 28 \\ 40 & 21 & 20 & 17 \\ 21 & 28 & 26 & 12 \end{pmatrix}$	4×4	75	61	61
7. $\begin{pmatrix} 120 & 130 & 140 & 110 \\ 100 & 120 & 150 & 130 \\ 110 & 100 & 120 & 140 \\ 130 & 140 & 110 & 120 \end{pmatrix}$	4×4	420	420	420
8. $\begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix}$	5×5	12	09	09
9. $\begin{pmatrix} 4 & 7 & 6 & 5 & 8 \\ 6 & 8 & 5 & 7 & 9 \\ 8 & 6 & 7 & 6 & 7 \\ 7 & 4 & 6 & 5 & 6 \\ 6 & 5 & 8 & 7 & 5 \end{pmatrix}$	5×5	25	24	24
10. $\begin{pmatrix} 9 & 6 & 7 & 8 & 5 \\ 8 & 9 & 6 & 7 & 6 \\ 7 & 8 & 6 & 5 & 9 \\ 6 & 7 & 8 & 6 & 7 \\ 5 & 6 & 9 & 7 & 8 \end{pmatrix}$	5×5	33	28	28

Table 2: Comparison when the problems are unbalanced

Problem	Size of the matrix	Hungarian Method	Proposed method	Optimum
1. $\begin{array}{cc} & \begin{matrix} J1 & J2 & J3 & J4 & J5 \end{matrix} \\ \begin{matrix} w1 \\ w2 \\ w3 \end{matrix} & \begin{pmatrix} 9 & 7 & 8 & 5 & 6 \\ 6 & 4 & 7 & 9 & 8 \\ 3 & 5 & 4 & 6 & 7 \end{pmatrix} \end{array}$	3×5	15	12	12
2. $\begin{array}{cc} & \begin{matrix} R1 & R2 & R3 & R4 \end{matrix} \\ \begin{matrix} s1 \\ s2 \end{matrix} & \begin{pmatrix} 10 & 7 & 8 & 6 \\ 12 & 5 & 9 & 11 \end{pmatrix} \end{array}$	2×4	11	11	11

3. $$\begin{array}{c} \\ J1 \\ J2 \\ J3 \end{array} \begin{array}{c} \begin{array}{ccccc} W1 & W2 & W3 & W4 & W5 \end{array} \\ \begin{pmatrix} 10 & 12 & 14 & 15 & 20 \\ 16 & 18 & 12 & 20 & 14 \\ 14 & 10 & 18 & 17 & 15 \end{pmatrix} \end{array}$$	3×5	34	32	32

7. Limitations and Scope of future Research

The proposed algorithm is efficient and applied to any forms of assignment problems that we faced in real-life situations. But our technique fails to provide accurate result when the problem is unbalanced and require to add more dummy columns to convert it to balanced one. As proposed algorithm consider adaptation only in the row of cost matrix by adding unit amount into every row of matrix. Still the operation is unknown to solve unbalanced assignment problem requires adding dummy columns and working to find more convenient way to solve these type problems in future.

8. Conclusions

In the highly competitive business world, every organization must be aware of about the goods it produces. In this race organizations have some promise to consumer to deliver produced goods in scheduled time by expending less. So, manager have to follow an appropriate optimization and assignment technique, in which assignees are given tasks to complete. The paper offers a different, simple and easy understandable method for determining the optimal results to the assignment problem. The result obtained using the proposed technique and comparing with the most frequent Hungarian method shows outcomes very relevant and some cases are better than others. As a result, the new approach will be impactful to the decision makers for its simple calculation and taking decision to apply it in any practical assignment problem.

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