International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Theory to Practice: Implementing Tangible Mathematics Instruction to Enhance Grade 7 Learners' Conceptual Understanding and Problem-Solving in Rational Numbers

Ronna Y. Magto[1], Douglas A. Salazar[2]

[1]Ph.D. Science Education Mathematics (Candidate), Mindanao State University – Iligan Institute of Technology
[2]Professor IV, College of Education, Mindanao State University – Iligan Institute of Technology

Abstract
This study examined how tangible mathematics instruction affects grade 7 learners' understanding and engagement with rational numbers. Using a mixed-methods approach, the researchers compared 58 learners using tangible manipulatives against 51 receiving conventional instruction over eight weeks. Quantitative analysis revealed significantly higher post-test scores in the experimental group (M = 64.17) compared to the control group (M = 33.55), with a large effect size (r = 0.844). For every 2-3 learners receiving the intervention, one additional learner achieved mastery compared to conventional instruction. Qualitative analysis showed that learners using tangible mathematics developed stronger visual representation strategies and deeper conceptual understanding, particularly in connecting concrete and abstract concepts. Learner engagement metrics improved substantially, with active participation increasing from 3.12 to 3.84 and confidence levels rising from 2.86 to 3.59. Thematic analysis revealed enhanced understanding, increased enjoyment, and improved mathematical connections. These findings suggest that tangible mathematics instruction effectively bridges concrete and abstract thinking while simultaneously improving engagement and confidence, offering valuable implications for mathematics instruction in grade 7 classrooms.

Keywords: tangible mathematics; mathematics education; learner engagement; problem-solving; mathematical understanding

1. Introduction

Mathematics education, particularly in the domain of rational numbers, presents significant challenges for grade 7 learners transitioning from concrete to abstract mathematical thinking. The gap between conceptual understanding and procedural fluency in mathematics has been well-documented in educational research, with rational number operations consistently identified as an area of difficulty. Traditional instructional approaches often emphasize procedural knowledge without adequately developing the conceptual foundations necessary for deep mathematical understanding. This research addresses this challenge by investigating the effectiveness of tangible mathematics instruction—an

approach that incorporates physical manipulatives and visual representations—in enhancing grade 7 learners' engagement, understanding, and problem-solving abilities in rational number operations.

The significance of this research lies in its potential to bridge the gap between concrete and abstract mathematical thinking at a critical developmental stage. Grade 7 represents a pivotal transition point where learners are expected to move from basic arithmetic to more complex algebraic thinking. Challenges in understanding rational numbers at this stage can create persistent difficulties in higher-level mathematics. By examining how tangible approaches influence learner outcomes, this study contributes valuable insights into effective instructional strategies for this crucial mathematical domain.

The study aims to address two primary research questions: (1) What is the effect of tangible mathematics instruction on grade 7 learners' mathematical understanding and problem-solving skills? and (2) How does tangible mathematics instruction influence grade 7 learners' engagement in mathematics learning? Through a comprehensive mixed-methods investigation, this research seeks to provide empirical evidence for the effectiveness of tangible mathematics approaches while offering practical guidance for classroom implementation.

2. Literature Review

Research on mathematical understanding and problem-solving provides crucial insights for mathematics instruction. Skemp's (1976) foundational work on relational versus instrumental understanding established the critical distinction between knowing "how" and knowing "why" in mathematics. This distinction is particularly relevant when considering approaches to teaching rational numbers, where procedural knowledge often overshadows conceptual understanding.

The development of algebraic thinking represents a crucial dimension of mathematical understanding that bridges numerical operations and abstract reasoning. Kriegler (2008) provides a comprehensive framework for algebraic thinking, identifying two key strands: the development of mathematical thinking tools, including problem-solving skills, representation abilities, and reasoning capabilities; and the study of fundamental algebraic ideas, including generalization, linearity, equality, variable, function, and modeling. This framework offers valuable guidance for developing instructional approaches that systematically build algebraic reasoning through carefully structured learning progressions.

Research on rational number development provides important insights for bridging number and algebraic understanding. Siegler et al. (2011) proposed an integrated theory of whole number and fractions development that highlights how understanding rational numbers requires extending rather than replacing whole number knowledge. Their theoretical framework explains many difficulties learners encounter with fractions and provides guidance for instructional approaches that support conceptual integration rather than compartmentalization.

Recent empirical studies have further illuminated the relationship between conceptual understanding and mathematical performance. Andamon and Tan (2018) conducted a comprehensive investigation of Grade 7 learners' conceptual understanding, attitudes, and performance in mathematics. Their research revealed significant correlations between conceptual understanding and overall mathematical achievement, demonstrating that learners with stronger grasp of fundamental concepts demonstrated greater success in applying those concepts to novel situations.

A substantial body of research has examined the effects of manipulative use on mathematical understanding and achievement. Domino (2010) conducted a comprehensive meta-analysis on the effects of physical manipulatives on achievement in mathematics in grades K-6, finding consistent positive effects

across diverse educational contexts. Similarly, Hawkins (2007) documented significant achievement gains associated with systematic manipulative use, particularly for concepts requiring spatial reasoning and abstract representation.

Tangible approaches to developing mathematical understanding have received significant attention. Scarlatos (2006) explored the concept of "tangible math," demonstrating how physical manipulation of mathematical objects supports the development of abstract concepts through concrete experiences. Their research documents how learners develop more robust mental models through hands-on interaction with mathematical principles. Kelly (2006) extended this understanding through performance-based analysis of manipulative use in mathematical problem-solving, revealing that strategic use of manipulatives enhanced learners' problem-solving capabilities by providing external representations that supported working memory and facilitated solution strategy development.

The integration of digital and physical approaches has emerged as a promising direction. Fokides and Alatzas (2023) investigated the use of digitally enhanced tangible materials for teaching fractions, demonstrating significant improvements in learner comprehension and application of fractional concepts compared to traditional approaches. Similarly, Pires et al. (2019) examined how building blocks of mathematical learning differ between virtual and tangible manipulatives, revealing that these different approaches led to distinct strategies in number composition.

3. Research Method

This study employed a mixed-methods approach with a quasi-experimental design to investigate the effects of tangible mathematics instruction on grade 7 learners' understanding, problem-solving skills, and engagement. The research design included both quantitative measures to assess learning outcomes and qualitative methods to explore learner experiences and problem-solving approaches.

Research Design

The implementation phase focused on evaluating the effectiveness of tangible mathematics materials through a quasi-experimental design. Using a pre-test/post-test control group approach, this phase examined how tangible mathematics instruction affected learner learning outcomes. The implementation involved 109 grade 7 learners, with 58 learners in the experimental group receiving the tangible mathematics intervention and 51 learners in the control group continuing with conventional instruction. This eight-week intervention period provided sufficient time to observe and measure the effects of tangible mathematics on learner engagement, understanding, and problem-solving skills.

Participants

The study participants included 109 grade 7 learners from the regular basic curriculum through cluster sampling using intact classes. The participants were divided into an experimental group of 58 learners and a control group of 51 learners. Initial academic performance was assessed through first quarter examination scores to establish baseline comparability between groups. The implementation spanned one academic quarter, following a structured timeline from October to December 2024.

Table 1. Distribution of Participants by Group

Participants	Control Group	Experimental Group	Total
Male	19	34	53
Female	32	24	56
Total	51 (47%)	58 (53%)	109 (100%)

Source: Research Data, 2024

Research Instruments

A comprehensive set of instruments was employed to collect both quantitative and qualitative data. Quantitative measures included pre-tests and post-tests adapted from established mathematical assessment instruments appropriate for grade 7 learners, problem-solving skills evaluations, and engagement questionnaires. Qualitative tools included structured classroom observation checklists, learner learning reflections, detailed teacher observation notes, semi-structured interview guides, and focus group discussion protocols. Weekly lesson plans served as structured guides for implementing the tangible mathematics intervention, ensuring consistent implementation while maintaining instructional fidelity.

Data Collection and Analysis

Data collection followed a systematic approach through multiple instruments. Quantitative data included pre-tests and post-tests to measure mathematical understanding, along with regular assessments of problem-solving skills and engagement levels. Qualitative data gathering involved systematic classroom observations, collection of learners' learning reflections, and maintenance of detailed teacher observation notes.

The quantitative analysis focused on comparing experimental and control groups through statistical procedures. Pre-test and post-test comparisons were conducted using the Mann-Whitney U Test, with effect size calculations to determine the practical significance of observed differences. The Wilcoxon Signed-Rank Test was employed to analyze engagement measures, tracking changes in learner participation and interest over time.

Qualitative analysis included systematic thematic analysis of classroom observations and content analysis of learner reflections, providing rich insights into how learners interact with and respond to tangible mathematics approaches. These analyses revealed patterns in learner engagement, understanding development, and problem-solving strategies. The integration of mixed methods brought together quantitative and qualitative findings through careful triangulation, enabling a comprehensive assessment of the intervention's effectiveness.

Ethical Considerations

This study adhered to strict ethical standards throughout all phases of research implementation. Prior to beginning the intervention, comprehensive informed consent was obtained from all participants and their guardians, with clear explanations of the nature of the experimental and control group conditions. The research design ensured equal educational opportunity by planning to provide control group learners access to tangible mathematics materials after the study conclusion, ensuring no learners would be disadvantaged in their educational experience.

Data privacy and protection measures were implemented with rigorous attention to confidentiality. All test results, observations, and participant feedback were securely stored with restricted access limited to the research team. Personal identifiers were removed from data during analysis, and pseudonyms were used in reporting to protect participant identities. Regular monitoring of learner well-being occurred throughout the intervention period to ensure that any potential negative effects could be promptly addressed. Additionally, the research received formal approval from the relevant institutional ethics committee prior to commencement.

4. Findings and Discussion

The implementation of tangible mathematics yielded comprehensive positive outcomes that transformed multiple dimensions of mathematical learning. Both quantitative metrics and learners' qualitative

responses converge to demonstrate the intervention's effectiveness across different aspects of mathematical engagement and understanding.

Quantitative Results

Analysis of pre-test and post-test performance revealed distinct patterns of improvement between the groups, with the experimental group showing notably larger gains despite starting from a lower baseline. The control group started with a slightly higher mean score (M = 7.61, SD = 4.10) compared to the experimental group (M = 4.45, SD = 3.07). After the intervention, the control group improved to a mean score of 33.55 (SD = 12.85), while the experimental group achieved a significantly higher mean score of 64.17 (SD = 27.22).

Table 2. Summary of Pretest-Posttest Performance Among the Groups

Measure	**Control Group**	**Experimental Group**
Pretest Mean (SD)	7.61 (4.10)	4.45 (3.07)
Posttest Mean (SD)	33.55 (12.85)	64.17 (27.22)
Mean Gain (SD)	25.94 (10.26)	59.72 (26.51)
Range Pretest	0-20	0-12
Range Posttest	22-71	20-94
N	51	58

Source: Research Data, 2024

The between-group analysis using the Mann-Whitney U test revealed significant differences at both testing points. At pretest, there was a significant difference (U = 901, z = -3.51, p < .001) with a medium effect size (r = 0.336). The posttest comparison yielded even more pronounced differences (U = 721, z = -4.60, p < .001) with a medium to large effect size (r = 0.441). This increase in effect size from pretest to posttest indicates that the gap between groups widened during the intervention period, favoring the experimental group.

Table 3. Results of Between-Group Comparisons Using Mann-Whitney U Test

Test Stage	U-statistic	z-score	p-value	Effect Size (r)	Interpretation
Pretest	901	-3.51	< .001	0.336	Medium Effect
Posttest	721	-4.60	< .001	0.441	Medium to Large Effect

Note: Effect size interpretations: small (0.1-0.3), medium (0.3-0.5), large (>0.5)

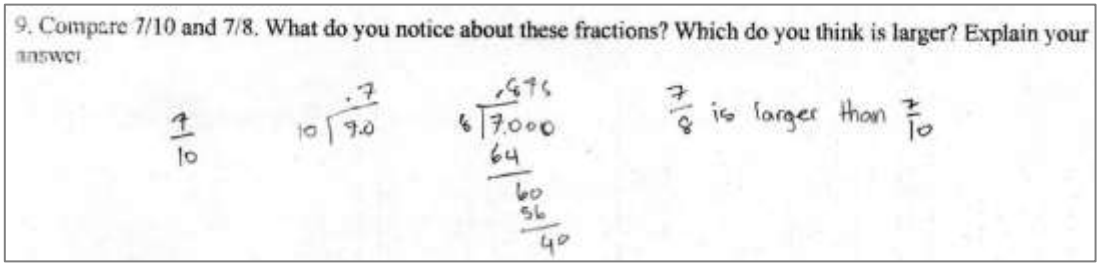
Qualitative Results

The qualitative analysis of learner work samples revealed significant differences in problem-solving approaches between conventional and tangible mathematics approaches. While control group learners predominantly relied on computational approaches, experimental group learners developed sophisticated visual representation strategies that demonstrated deeper conceptual understanding.

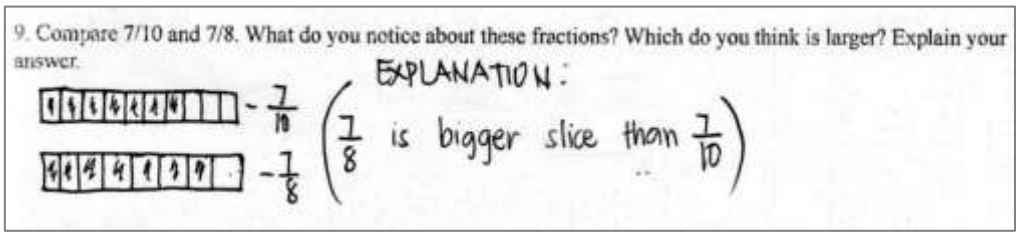
Exhibit 1. Pre-Post Implementations

Exhibit 1 shows the examples of learner work showing different problem-solving approaches

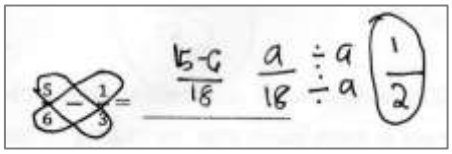
CPoL6



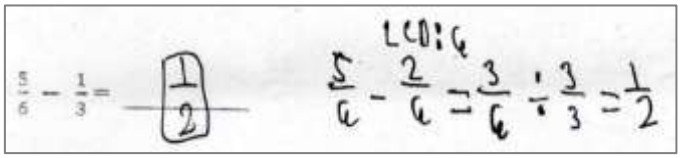
EPoL13



CPoL12

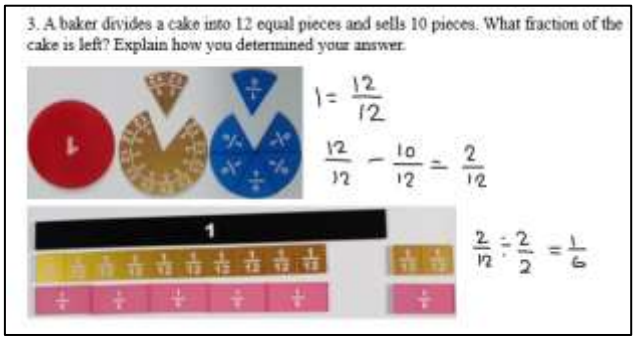


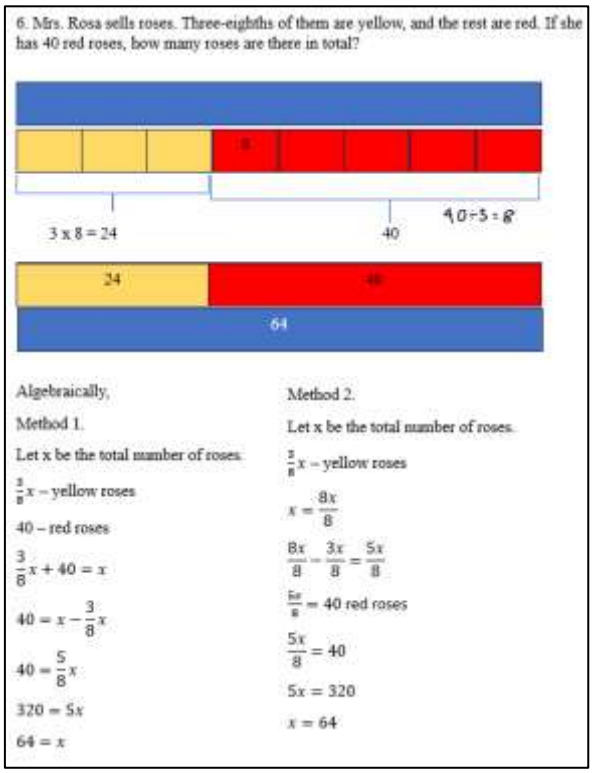
EPoL1

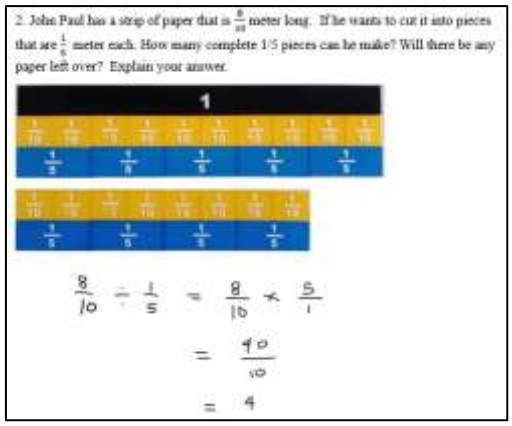


Learners in the experimental group consistently demonstrated stronger conceptual understanding through their use of visual representations, systematic approaches to problem-solving, and clear reasoning about mathematical relationships. Their solutions showed strong connections between visual and symbolic representations, suggesting that the tangible mathematics approach supported the development of more robust mathematical understanding.

Exhibit 2. Sample Tangible Mathematics Activities







Learner Engagement

The implementation of tangible mathematics catalyzed significant improvements across multiple dimensions of learner engagement. Active participation showed a remarkable increase from 3.12 to 3.84, with the percentage of learners "Always" participating rising substantially from 50% to 86.2%. This dramatic improvement demonstrates a significant enhancement in overall classroom engagement.

Learner confidence, previously identified as an area of concern, showed one of the study's most significant transformations. The confidence metric rose markedly from 2.86 to 3.59, with the proportion of learners reporting they are "Always" confident more than doubling from 27.6% to 63.8%. Perhaps most tellingly, the percentage of learners who were only "Sometimes confident" plummeted from 37.9% to a mere 1.7%, suggesting a fundamental shift in mathematical self-efficacy.

Table 4. Learner Engagement Prior and After Tangible Mathematics Instruction

Parameters	Pre-Implementation Mean	Post-Implementation Mean	Mean Difference
A. Participation			

Active participation	3.12	3.84	+0.72
Following directions	3.86	3.86	0.00
Asking questions	3.28	3.64	+0.36
Working with classmates	3.41	3.78	+0.37
Completing tasks	3.29	3.66	+0.37
B. Feelings			
Enjoying Lessons	3.34	3.71	+0.37
Excitement	3.38	3.74	+0.36
Confidence	2.86	3.59	+0.73
Interest	2.84	3.57	+0.73
Pride	3.36	3.72	+0.36
C. Thinking			
Understanding math	3.26	3.74	+0.48
Connecting to past lessons	3.31	3.76	+0.45
Explaining answers	3.17	3.59	+0.42
Finding multiple solutions	3.24	3.71	+0.47
Applying to new situations	3.19	3.62	+0.43
D. Reflections			
Understanding operations	3.43	3.83	+0.40
Grasping concepts	3.21	3.64	+0.43
Visualizing expressions	3.29	3.67	+0.38
Explaining mathematical ideas	3.24	3.66	+0.42
Seeing connections	3.28	3.67	+0.39

Bridging Concrete and Abstract Understanding

One of the most significant findings is how tangible mathematics facilitated the transition from concrete to abstract mathematical thinking. The experimental group's superior performance in transferring understanding to algebraic contexts aligns with Siegler et al.'s (2011) integrated theory of whole number and fractions development, which emphasizes the importance of extending rather than replacing existing knowledge. The qualitative analysis revealed that learners using manipulatives demonstrated a deeper conceptual understanding of fraction relationships, supporting Skemp's (1976) distinction between relational and instrumental understanding. This finding is particularly important given that Grade 7 represents a critical transition point where learners must begin to engage with more abstract mathematical concepts.

The experimental group's use of visual representations to solve problems demonstrates how tangible approaches can scaffold the development of mathematical thinking, consistent with Kelly's (2006) research showing that manipulatives enhance problem-solving by providing external representations that support working memory. This successful bridging of concrete and abstract understanding addresses one of mathematics education's most persistent challenges, offering a practical pathway for supporting learners through this critical transition.

Enhanced Engagement and Mathematical Identity

The remarkable improvements in learner engagement metrics align with research by Moyer (2001) and Carbonneau et al. (2013) on the motivational impact of hands-on approaches. The substantial increase in

confidence levels (from 2.86 to 3.59) is particularly noteworthy as it suggests a transformation in learners' mathematical identity. As Singh et al. (2002) noted, confidence in mathematics is strongly associated with achievement and persistence. The study's findings suggest that tangible mathematics may help disrupt negative cycles of low confidence and disengagement that often characterize mathematics learning.

5. Conclusions

This study provides compelling evidence for the effectiveness of tangible mathematics instruction in enhancing grade 7 learners' understanding, problem-solving skills, and engagement in rational number operations. The dramatic reversal of the initial performance gap between groups challenges conventional assumptions about persistent disadvantages and demonstrates how effectively designed tangible mathematics instruction can transform learning outcomes. The exceptionally large effect size ($r = 0.844$) documented in this study surpasses typical mathematics interventions, providing compelling evidence for the potency of this approach in addressing challenging mathematical concepts.

Tangible mathematics instruction significantly enhances learner understanding of rational numbers, as demonstrated by the substantial difference in performance between experimental and control groups. The integration of visual and tactile learning experiences effectively addresses fundamental gaps in mathematical understanding, helping learners build stronger connections between concrete and abstract mathematical concepts.

Learner engagement and confidence in mathematics significantly improve through tangible mathematics instruction, as evidenced by the marked increases in participation, confidence, and interest levels. This indicates that hands-on approaches create a more positive and engaging learning environment, which is crucial for sustained mathematical development.

Proper development and implementation of manipulative materials are crucial for their effectiveness, as shown by the systematic development process, expert validation, and teacher feedback in this study. This highlights that carefully designed and validated materials are essential for successful mathematics instruction.

The findings suggest that integrating tangible mathematics approaches in grade 7 classrooms can transform both cognitive and affective dimensions of mathematics learning, particularly in the challenging domain of rational numbers.

6. Limitation & Further Research

While the study provides compelling evidence for the effectiveness of tangible mathematics instruction, several limitations should be acknowledged. The eight-week intervention period, while sufficient to demonstrate immediate effects, does not allow for assessment of long-term retention and transfer. Future research should include follow-up assessments to determine whether the enhanced understanding persists over time and transfers to new mathematical domains.

The quasi-experimental design using intact classes, while practical for educational research, introduces potential confounding variables. Though baseline assessment helped establish comparability between groups, future studies might benefit from randomized assignment where feasible.

Additionally, the study focused specifically on rational number operations; the effectiveness of tangible approaches for other mathematical domains requires further investigation. Future research might explore how tangible mathematics instruction can support learning across different mathematical topics and how it might be integrated into comprehensive mathematics curricula.

The dramatic effect size observed (r = 0.844) exceeds those typically reported in educational interventions, suggesting the potential for tangible mathematics to significantly impact mathematics education practice. Further research is needed to identify the specific mechanisms through which tangible approaches enhance understanding and how these might be optimized for different learner populations and contexts.

References

1. Andamon, J. C., & Tan, D. A. (2018). Conceptual understanding, attitude and performance in mathematics of grade 7 students. *International Journal of Scientific & Technology Research, 7*(8), 96-105.
2. Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380-400. https://doi.org/10.1037/a0031084
3. Domino, J. (2010). *The effects of physical manipulatives on achievement in mathematics in grades K-6: A meta-analysis*. State University of New York at Buffalo.
4. Fokides, E., & Alatzas, K. (2023). Using digitally enhanced tangible materials for teaching fractions: Results of a project. *Technology, Knowledge and Learning, 28*(4), 1589-1613. https://doi.org/10.1007/s10758-022-09594-x
5. Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2014). Concreteness fading in mathematics and science instruction: A systematic review. *Educational Psychology Review, 26*(1), 9-25. https://doi.org/10.1007/s10648-014-9249-3
6. Hawkins, V. H. (2007). *The effects of math manipulatives on student achievement in mathematics*. Doctoral dissertation, Capella University.
7. Kelly, C. A. (2006). Using manipulatives in mathematical problem solving: A performance-based analysis. *The Mathematics Enthusiast, 3*(2), 184-193.
8. Kriegler, S. (2008). Just what is algebraic thinking. *Retrieved September, 10*, 2008.
9. Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175-197. https://doi.org/10.1023/A:1014596316942
10. Pires, A. C., González Perilli, F., Bakała, E., Fleisher, B., Sansone, G., & Marichal, S. (2019, September). Building blocks of mathematical learning: Virtual and tangible manipulatives lead to different strategies in number composition. In *Frontiers in Education* (Vol. 4, p. 81). Frontiers Media SA. https://doi.org/10.3389/feduc.2019.00081
11. Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics, 108*(7), 313-325. https://doi.org/10.1111/j.1949-8594.2008.tb17844.x
12. Scarlatos, L. L. (2006). Tangible math. *Interactive Technology and Smart Education, 3*(4), 293-309. https://doi.org/10.1108/17415650680000069
13. Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education, 196*(2), 1-38. https://doi.org/10.1177/002205741619600202
14. Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*(4), 273-296. https://doi.org/10.1016/j.cogpsych.2011.03.001

15. Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research, 95*(6), 323-332. https://doi.org/10.1080/00220670209596607
16. Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*(1), 20-26.
17. Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.