International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Analysis of a Stochastic Inventory Model with Probabilistic Constraint

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Abstract:
A fixed reorder quantity system with back-order is modeled here. A Multi Objective Stochastic Inventory model [MOSIM] and a Fuzzy Multi Objective Stochastic Inventory model [FMOSIM] with Stochastic constraint are analyzed here and are illustrated numerically considering the uniform demand.

Keywords: Multi Objective Stochastic Inventory model, Fixed Reorder Quantity system, Fuzzy Optimization, Stochastic Model

1. Introduction

In most of the existing inventory models, it is assumed that the inventory parameters, objective goals and constraint goals are deterministic and fixed. But, if we think of their practical meaning, they are uncertain, either random or imprecise. When some or all parameters of an optimization problem are described by random variables, the problem is called stochastic or probabilistic programming problem. In a stochastic programming problem, the uncertainties in the parameters are represented by probability distributions. This distribution is estimated on the basis of the available observed random data.

As classified by Mohon (2000), “There are two main approaches for solving single-objective stochastic programming problem: the ‘wait and see’ (distribution problem) and ‘here and now’ approaches. In 1965, the first publication in fuzzy set theory by Zadeh (1965) showed the intention to accommodate uncertainty in the non-stochastic sense rather than the presence of random variables. Bellman and Zadeh (1970) first introduced fuzzy set theory in decision-making processes. Later, Tanaka, et-al. (1974) considered the objectives as fuzzy goals over the □-cuts of a fuzzy constraint set and Zimmermann (1976) showed that the classical algorithms could be used to solve a fuzzy linear programming problem. Fuzzy mathematical programming has been applied to several fields like project network, reliability optimization, transportation, media selection for advertising; air pollution regulation etc. problems formulated in fuzzy environments. Detail literature on fuzzy linear and non-linear programming with application is available in two well-known books of Lie and Hwang (1992, 1994). Walter (1992) discussed the single period inventory problem with uniform demand. In inventory problem, fuzzy set theory has not been much used. Park (1987) examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with cost data. Alfares and Ghaithan (2016) discussed Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. Faritha, and Henry Amirtharaj (2016) Solved multi objective inventory model of deteriorating items using intuitionistic fuzzy optimization technique. Geetha and Udayakumar (2016) examined Optimal lot sizing policy for noninstantaneous deteriorating items with price and advertisement dependent demand

under partial backlogging. Kar, Roy and Maiti, (2008) studied Multi-objective inventory model of deteriorating items with space constraint in a fuzzy environment. Mahapatra and Maiti (2005) considered Multiobjective inventory models of multi items with quality and stock dependent demand and stochastic deterioration. Mishra and Waliv (2019) investigated Optimizing of multi-objective inventory model by different fuzzy techniques. Palanivel and Uthayakumar (2015) analyzed Finite horizon EOQ model for non-instantaneous deteriorating items with price and advertisement dependent demand and partial backlogging. Shaikh et al. (2017) examined Noninstantaneous deterioration inventory model with price and stock dependent demand for fully backlogged shortages.
In this paper A Multi Objective Stochastic Inventory model[MOSIM] with crisp constraint and Fuzzy Multi Objective Stochastic Inventory model[FMOSIM] with stochastic constraint are analyzed here.

2. Mathematical Model

A Fixed Reorder Quantity System with Back-order

Here the policy is to order a lot size Q when the inventory level drops to a reorder point r and it is supposed that the inventory position of an item is monitored after every transaction. The demand in any given interval of time is a random variable and the expected value of demand in a unit of time, say a year, is D. We let x denote the demand during the lead-time and f (x) denote its probability distribution.
The fixed procurement cost is A and the unit variable procurement cost is C. The cost of carrying a unit of inventory for one unit of time is h. All shortages are backordered at a cost of π per unit short, regardless of the duration of the shortage. Because of the probabilistic nature of demand, the number of cycles per year is a random variable that averages D/Q. The procurement cost per cycle is A+CQ.
The shortages cost per cycle is $\pi\bar{b}(r)$, where $\bar{b}(r)$ is the expected number of shortages per cycle and is a function of reorder point r. The amount of the shortage at the end of a cycle, when the replenishment order is received, is b (x, r) = max [0, x - r], which has the expected value

$$\bar{b}(r) = \int_r^\infty (x-r)f(x)dx$$

μ is the expected demand during a lead time. The quantity Q/2 is often called the cycle stock and r - μ is referred to as the safety stock. Thus safety stock for the system is the amount by which the reorder point exceeds the average usage during a lead-time.
The average annual cost is:

$$K(Q,r) = \frac{AD}{Q} + CD + h(\frac{Q}{2} + r - \mu) + \frac{\pi D\bar{b}(r)}{Q} \qquad \ldots(1)$$

2.1 Model I: Multi Objective Stochastic Inventory Model [MOSIM]

Traditional single objective linear or non-linear programming problem aims at optimization of the performance in terms of combination of resources. In reality, a managerial problem of a responsible organization involves several conflicting objectives to be achieved simultaneously subject to a system of restrictions (constraints) that refers to a situation on which the DM has no control. For this purpose a latest tool is linear or non-linear programming problem with multiple conflicting objectives. So the following model may be considered:
Minimize total annual cost of not only the first objective, but also the second objective but also up to the nth objective. It is a Multi-Objective Stochastic Inventory Model [MOSIM].
To solve the problem (3.1) as a MOSIM, it can be reformulated as:

$$Min\ K_i(Q_i,r_i)=(\frac{A_iD_i}{Q_i}+C_iD_i+h_i(\frac{Q_i}{2}+r_i-\mu_i)+(\frac{\pi_iD_i}{Q_i})\bar{b}_i(r_i)) \quad ….(2.1)$$

subject to the constraints

$p_iQ_i \le B$ (i = 1, 2, …., n).

$Q_i \ge 0$ (i = 1, 2, …., n).

2.2 Model II. Fuzzy Multi Objective Stochastic Model with Stochastic Budget

To achieve several conflicting objectives simultaneously subject to a system of restrictions (constraints), sometimes, the objective goals are not stated clearly i.e. they are imprecise in nature. Then, fuzzification of the MOSIM is needed. Thus, when total annual cost is imprecise in nature, model (2.1) can be reformulated as: $Min\ \widetilde{K}_i(Q_i,r_i)=(\frac{A_iD_i}{Q_i}+C_iD_i+h_i(\frac{Q_i}{2}+r_i-\mu_i)+(\frac{\pi_iD_i}{Q_i})\bar{b}_i(r_i))$ ….(2.2)

subject to the constraints

$\hat{p}_iQ_i \le \hat{B}$ (i = 1, 2, …., n).

$Q_i \ge 0$ (i = 1, 2, …., n).

[Here '^' indicates randomization of the parameter]

3. Fuzzy Non-linear Programming (FNLP) Technique to Solve Multi-Objective Non-Linear Programming Problem (MONLP)

A Multi-Objective Non-Linear Programming (MONLP) problem or Vector Minimization problem (VMP) may be taken in the following form: $\text{Min} f(x)=(f_1(x),f_2(x),........,f_k(x))^T$ ….(3.1)

Subject to $x\in X=\{x\in R^n : g_j(x)\le or = or \ge b_j\ for\ j=1,2,......,m$ and $l_i \le x \le u_i (i=1,2,....,n)\}$.

Zimmermann (1978) showed that fuzzy programming technique can be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used**:**

STEP 1: Solve the MONLP problem of equation (3.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

STEP 2: From the result of step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ .... \\ x^k \end{array} \begin{array}{c} \begin{matrix} f_1(x) & f_2(x) & .... & f_k(x) \end{matrix} \\ \begin{bmatrix} f_1^*(x^1) & f_2(x^1) & .... & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & .... & f_k(x^2) \\ .... & .... & .... & .... \\ f_1(x^k) & f_2(x^k) & .... & f_k^*(x^k) \end{bmatrix} \end{array}$$

Here $x^1,x^2,......,x^k$ are the ideal solutions of the objective functions $f_1(x),f_2(x),.......,f_k(x)$ respectively.

So $U_r = \max\{f_r(x_1), f_r(x_2), \ldots\ldots. f_r(x_k)\}$

and $L_r = \min\{f_r(x_1), f_r(x_2), \ldots\ldots. f_r(x_k)\}$

[L_r and U_r be lower and upper bounds of the r^{th} objective functions $f_r(x)$ $r = 1,2,\ldots.., k$)]

STEP 3: Using aspiration level of each objective of the MONLP problem of equation (3.1) may be written as follows:

Find x so as to satisfy

$f_r(x) \tilde{\leqq} L_r \quad (r = 1,2,\ldots\ldots.., k)$

$x \in X$

Here objective functions of equation (2) are considered as fuzzy constraints. These types of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$\mu_r(f_r(x) = 0$ or $\to 0$ if $f_r(x) \geq U_r$

$= \mu_r(f_r(x)$ if $L_r \leq f_r(x) \leq U_r$ $(r = 1,2,\ldots\ldots.., k)$

$= 1$ if $f_r(x) \leq L_r$ ….(3.2)

Having elicited the membership functions (as in equation (4.8)) $\mu_r(f_r(x)$ for r = 1, 2, …… , k, introduce a general aggregation function

$\mu_{\tilde{D}}(x) = G(\mu_1(f_1(x)), \mu_2(f_2(x)), \ldots\ldots\ldots.., \mu_k(f_k(x))).$

So a fuzzy multi-objective decision making problem can be defined as

Max $\mu_{\tilde{D}}(x)$

Subject to $x \in X$ ….(3.3)

Here we adopt the fuzzy decision as:

Fuzzy decision based on minimum operator (like Zimmermann's approach (1976). In this case equation (3.3) is known as $FNLP_M$.

Then the problem of equation (3.3), using the membership function as in equation (3.2), according to min-operator is reduced to:

Max α ….(3.4)

Subject to $\mu_i(f_i(x) \geq \alpha$ for $i = 1,2,\ldots\ldots.., k$)

$x \in X \quad \alpha \in [0,1]$

STEP 4: Solve the equation (3.4) to get optimal solution.

4. Fuzzy Multi-Objective Non-Linear Programming [FMONLP] Problem

Assuming that the Decision Maker (DM) has fuzzy goals for each of the objective functions in the MONLP (3.1), similar to fuzzy multi objective linear programming problem proposed by Zimmermann (1978), it is possible to soften the rigid requirements of the MONLP problem (3.1), to strictly minimize the k objective functions under the given constraints. In such a situation, the MONLP problem may be softened into the following fuzzy version (called Fuzzy Multi Objective Non-Linear Programming (FMONLP) problem):

$\widetilde{Min} f(x) = [f_1(\mathbf{x}), f_2(x), f_3(x), \ldots, f_k(x)]^T$ ….(4.1)

subject to x ϵ X

Here the symbol $\widetilde{\text{Min}}$ denotes a relaxed or fuzzy version of 'Min' with the interpretation that the k objective function should be minimized as much as possible under the given constraints. So the problem (4.1) is reduced to following fuzzy optimization problem:

Find x so as to satisfy

$f_r(x) \lessapprox f_r^0$ for r = 1, 2, ….. , k

$x \in X$.

Those fuzzy requirements ($f_r(x) \lessapprox f_r^0$ for r = 1, 2, ….. , k) can be quantified by eliciting the membership functions $\mu_r f_r(x)$.

To elicit a membership function $\mu_r f_r(x)$ from the DM for each of the objective function $f_r(x)$ of the FMONLP problem (4.1) one can suggest the following approach:

First calculate the individual minimum L_r and maximum U_r of each $f_r(x)$ under the given constraints. Then by taking account of the calculated individual minimum and maximum of each objective function together with the rate of increase of membership of satisfaction, the DM is used to select a membership function in a subjective manner from among the several types of functions (e.g. linear, exponential, hyperbolic, hyperbolic inverse, piecewise linear etc.). The parameter values are determined through the interaction with the DM.

So membership function $\mu_r f_r(x)$ may be written as follows:

$\mu_r f_r(x) =$ 0 or $\to 0$ if $f_r(x) \geq f_r^1$

$= \nu_r(x)$ if $f_r^0 \leq f_r(x) \leq f_r^1$

$=$ 1 or $\to 1$ if $f_r(x) \leq f_r^0$ (for r = 1, 2, …… , k)

Here $\nu_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

This membership function is determined by asking the DM to specify the two points f_r^0 and f_r^1 within L_r and U_r (i.e $L_r \leq f_r^0 \leq f_r^1 \leq U_r$).

Having determined the membership function for each of the objective functions we propose Bellman and Zadeh's (1970) fuzzy decision and then the FMONLP problem (4.1) may be reduced to the crisp non-linear programming problems:

Fuzzy Non-Linear Programming Problem based on Min operator ($FNLP_M$) Max $\mu_j f_j(x)$ …(4.2)

subject to $\mu_r f_r(x) \geq \mu_j f_j(x)$

$x \in X$, $0 \leq \mu_r f_r(x) \leq 1$, for r, j = 1, 2, …….., k; $r \neq j$.

5. Demand Follows Uniform distribution

We assume that lead time demand for the period for the i^{th} item is a random variable which follows uniform distribution and if the decision maker feels that demand values for item i below a_i or above b_i are highly unlikely and values between a_i and b_i are equally likely, then the probability density function $f_i(x)$ are given by:

$$f_i(x) = \begin{cases} \dfrac{1}{b_i - a_i} & \text{if } a_i \leq x \leq b_i \\ 0 & \text{otherwise} \end{cases} \quad \text{for } i = 1, 2, \ldots, n.$$

So, $\bar{b}_i(s_i) = \dfrac{(b_i - s_i)^2}{2(b_i - a_i)}$ for i = 1, 2, …, n ...(5.1)

Where, $\bar{b}_i(s_i)$ are the expected number of shortages per cycle and all these values of $\bar{b}_i(s_i)$ affects the desired models.

6. Mathematical Analysis

A stochastic non-linear programming problem is considered as:

Min $f_0(X)$

Subject to

$f'_j(X) \le c_j$ (j=1, 2, ……….,m)

$X \ge 0$.

i.e Min $f_0(X)$ ….(6.1)

Subject to

$f_j(X) \le 0$ (j=1, 2, ……….,m)

$X \ge 0$.

Where, $f_j(X) = f'_j(X) - c_j$

Here X is a vector of N random variables $y_1, y_2, \ldots, y_n$ and it includes the decision variables $x_1, x_2, \ldots, x_n$.

Expanding the objective function $f_0(X)$ about the mean value $\bar{y}_i$ of y_i and neglecting the higher order term:

$$f_0(X) = f_0(\bar{X}) + \sum_{i=1}^{N} \left(\frac{\partial f_0}{\partial y_i} \bigg| \bar{X} \right) (y_i - \bar{y}_i) = \xi(X) \text{ (say)} \quad \text{….(6.2)}$$

If y_i (i=1, 2, ……. ,n) follow normal distribution then so does $\xi(X)$. The mean and variance of $\xi(X)$ are given by:

$$\bar{\xi} = \xi(\bar{X}) \quad \text{….(6.3)}$$

$$\sigma_{\xi}^{2} = \sum_{i=1}^{N} \left(\frac{\partial f_0}{\partial y_i} \bigg| \bar{X} \right) \sigma_{y_i}^{2} \quad \text{….(6.4)}$$

When some of the parameters of the constraints are random in nature then the constraints will be probabilistic and thus, the constraints can be written as:

$$P(f_j \le 0) \ge r_j \quad \text{(j=1,2 , ………,m)} \quad \text{….(6.5)}$$

Then in the light of the theoretical convention given above, equivalent deterministic constraints are:

$$\bar{f}_j - \phi_j(r_j) \left[\sum_{i=1}^{N} \left(\frac{\partial f_j}{\partial y_i} \bigg| \bar{X} \right) \sigma_{y_i}^{2} \right]^{1/2} \le 0 \quad \text{(j=1,2 , ………,m)} \quad \text{….(6.6)}$$

where, $\phi_j(r_j)$ is the value of the standard normal variate corresponding to the probability.

7.NUMERICAL

To solve the MOSIM (2.1), where the demand follows uniform distribution, we consider the following data:

A_1 = \$70, A_2 = \$80, h_1 = \$10, h_2 = \$7.5, D_1 = 5000, D_2 = 4000, C_1 = \$50, C_2 = \$37,

π_1 = \$19.2, π_2= \$21, a_1=100 , a_2 =200 , b_1 =400 , b_2 =500 p_1 =\$24,955, p_1=\$24,960, B = \$60,00,000.

To solve the FMOSIM(2.2), where the demand follows uniform distribution, we consider the following additional data:

$\hat{B}$ =(\$60,00,000,\$5,000); $\hat{p}_1$=(\$24,000 m^2, \$250); $\hat{p}_2$=(\$25,000, \$300); f_1^0 =\$85,000, f_1^1 =\$90,000, f_2^0 =\$90,000, f_2^1 =\$95,000.
[Here '^' indicates randomization of the parameter which is actually a Normal variable with respective mean and standard deviation]

MODEL	Q_1	Q_2	r_1	r_2	K_1(\$)	K_2(\$)
MOSIM	187	174	25.3	23.1	88,213.43	93,142.74
FMOSIM	175	164	19.8	17.4	87,225.67	92,612.44

8. CONCLUSION

Firstly, The average annual cost of A Multi Objective Stochastic Inventory model[MOSIM] is illustrated numerically. After that The average annual cost of more imprecise(realistic), Fuzzy Multi Objective Stochastic Inventory model[FMOSIM] with Stochastic constraint is represented here. In both the cases demand follows uniform distribution.

References

1. Alfares, H.K.; Ghaithan, A.M.: Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. Comput. Ind. Eng. 94, 170–177 (2016).
2. Bellman, R. E. and Zadeh, L. A. (1970). Decision-making in a fuzzy environment, Management Science, 17, B141-B164.
3. Faritha, A.; Henry Amirtharaj, E.C.: Solving multi objective inventory model of deteriorating items using intuitionistic fuzzy optimization technique. Int. J. Eng. Sci. Innov. Technol. 5(1), 146–153 (2016).
4. Geetha, K.V.; Udayakumar, R.: Optimal lot sizing policy for noninstantaneous deteriorating items with price and advertisement dependent demand under partial backlogging. Int. J. Appl. Comput. Math. 2(2), 171–193 (2016).
5. Kar, S.; Roy, T.; Maiti, M.: Multi-objective inventory model of deteriorating items with space constraint in a fuzzy environment. Tamsui Oxf. J. Math. Sci. 24(1), 37–60 (2008).
6. Lai, Y. J. and Hwang, C. L. (1992). Fuzzy mathematical programming: Methods and applications, Sprenger-Verlag, Heidelberg.
7. Lai Y.J, Hwang C.L. (1994). Fuzzy Multiple Objective Decision Making: Methods and Application, Springer, New York.
8. Mahapatra, N.; Maiti, M.: Multiobjective inventory models of multi items with quality and stock dependent demand and stochastic deterioration. Adv. Modell. Optim. 7(1), 69–84 (2005).
9. Mahapatra, G.S.; Mitra, M.; Roy, T.K.: Intuitionistic fuzzy multiobjective mathematical programming on reliability optimization model. Int. J. Fuzzy Syst. 12(3), 259–266 (2010).

10. Mishra, U.; Waliv, R.H.; Umap, H.P.: Optimizing of multi-objective inventory model by diferent fuzzy techniques. Int. J. Appl. Comput. Math. 5, 136 (2019). https://doi.org/10.1007/s4081 9-019-0721-0.
11. Mohan, C. (2000). Optimization in fuzzy-stochastic environment and its importance in present day industrial scenario, Proceedings on Mathematics and its Applications in Industry and Business, Copyright © , Narosa Publishing House, India.
12. Palanivel, M.; Uthayakumar, R.: Finite horizon EOQ model for non-instantaneous deteriorating items with price and advertisement dependent demand and partial backlogging under infation. Int. J. Syst. Sci. 46(10), 1762–1773 (2015).
13. Park, K.C. (1987). Fuzzy set theoretic interpretation of economic order quantity, IEEE Transactions on Systems, Man and Cybernetics SMC-17, No.-6, 1082-1084, 167.
14. Shaikh, A.A.; Abu, Mashud; Uddin, H.M.; Khan, M.A.A.: Noninstantaneous deterioration inventory model with price and stock dependent demand for fully backlogged shortages under infation. Int. J. Bus. Forecast. Market. Intell. 3(2), 152–164 (2017).
15. Tanaka, H., Okuda, T. and Asai, K. (1974). On fuzzy mathematical programming, Journal of Cybernetics, 3(4), 37-46.
16. Walter. J. (1992). Single-period Inventory Problem with Uniform Demand, International Journal of Operations and production management, No. 3., pp.79-84.
17. Zadeh, L. A. (1965). Fuzzy sets. Information and Control 8, 338-356.
18. Zimmerman, H. J. (1976). Description and optimization of fuzzy system, International Journal of General Systems, 2, 209-215.