International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Thermoelasticity in Modern Materials: A Review of Linear and Nonlinear Mathematical Models

Sachin B. Lonare

Department of Maths, Mohsinbhai Zaweri Mahavidyalaya, Desaiganj (Wadsa), Dist. - Gadchiroli. Maharashtra, India.

Abstract

Thermoelasticity, the mathematical study of the interaction between mechanical deformations and thermal effects in solids, plays a foundational role in modeling the behavior of advanced engineering systems and materials. This review presents a comparative analysis of linear and nonlinear thermoelastic models, emphasizing their mathematical formulations, physical assumptions, and practical applications. Linear thermoelasticity is explored in the context of classical elasticity theory, with governing equations derived under small strain and constant material property assumptions. Nonlinear thermoelasticity is then examined as a generalization that incorporates large deformations, temperature-dependent parameters, and thermal memory effects. Key developments in theoretical approaches, including micropolar continua, fractal models, and hybrid data-driven frameworks, are discussed with reference to recent literature. Applications in structural engineering, MEMS/NEMS, aerospace systems, and smart materials are used to highlight the advantages and limitations of each modeling regime. The paper concludes by outlining challenges in computational efficiency, experimental validation, and multiphysics integration, offering future directions for research at the intersection of applied mathematics and material science.

Keywords: Thermoelasticity, Nonlinear Continuum Mechanics, Micropolar Media, Thermal-Mechanical Coupling, Mathematical Modeling.

1. Introduction

Thermoelasticity is a fundamental field in continuum mechanics that explores the intricate coupling between mechanical deformations and thermal effects in solid materials. Rooted in the classical theories of elasticity and heat conduction, thermoelasticity plays a pivotal role in modeling and predicting the behavior of materials subjected to both mechanical loads and temperature gradients. Its applications span a wide spectrum—from aerospace structures and civil engineering components to microelectromechanical systems (MEMS) and biomedical devices.

The traditional linear theory of thermoelasticity, built upon assumptions of small deformations, temperature-independent material properties, and linear constitutive relations, has been widely used for its analytical tractability and effectiveness in many engineering applications. However, with the increasing complexity of modern materials and operating environments, these assumptions often fall short. Situations involving large deformations, nonlinear thermal expansion, or temperature-dependent properties necessitate the adoption of nonlinear thermoelastic models that offer a more realistic yet mathematically challenging framework.

In recent years, significant efforts have been devoted to developing and analyzing both linear and nonlinear formulations of thermoelasticity. These models are governed by systems of coupled partial differential equations (PDEs), whose mathematical structure varies significantly with the level of nonlinearity incorporated. While linear models are often solvable using classical methods such as separation of variables and integral transforms, nonlinear systems typically require advanced techniques including perturbation theory, finite element analysis, and numerical time-stepping schemes.
This review aims to provide a comprehensive comparative analysis of linear and nonlinear thermoelasticity from a mathematical modeling standpoint. We begin by establishing the governing equations and assumptions underlying each class of model. Subsequently, we explore recent developments in solution techniques, stability analysis, and applications. By contrasting these two regimes, we highlight their respective strengths, limitations, and domains of applicability. Ultimately, this work serves as a bridge for researchers and students to transition from classical linear theory to the more advanced and versatile nonlinear formulations required in contemporary material modeling.

2. Governing Equations in Thermoelasticity

Thermoelasticity is governed by a coupled set of partial differential equations (PDEs) that describe the balance of momentum and energy in a deformable solid subject to thermal effects. These equations evolve significantly in complexity when transitioning from linear to nonlinear theories. This section outlines the core mathematical framework that underpins both regimes.

2.1 Kinematic and Thermodynamic Preliminaries

Let $u(x,t)$ denote the displacement field and $\theta(x,t)$ the temperature changes relative to a reference configuration. The strain tensor ε and stress tensor σ are related through constitutive equations that differ based on the linearity of the model.
The governing equations are derived from:

- **Conservation of linear momentum** (force balance),
- **Energy balance**, and
- **Constitutive laws** that link stress, strain, and temperature.

2.2 Linear Thermoelasticity

In the classical theory, the assumptions of:

- **small strains**,
- **linear material behavior**, and
- **constant material properties**

lead to a linear PDE system.
Governing Equations:

1. **Momentum Balance:**

$$\rho\ddot{u} = \nabla \cdot \sigma + f$$

2. **Hookean Stress-Strain Relation (Thermoelastic Form):**

$$\sigma = \mathbb{C} : (\varepsilon - \alpha\theta I)$$

Where:

- $\mathbb{C}$ is the fourth-order elasticity tensor,
- α is the thermal expansion coefficient.

3. Heat Conduction Equation (Fourier's Law):

$$\rho c \frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta + \beta \frac{\partial}{\partial t}(\nabla \cdot u)$$

This formulation assumes **instantaneous heat propagation**, which leads to parabolic PDEs. Surana & Mathi (2025) applied this formulation to benchmark problems in 1D and 2D geometries [1]. Ruggeri (2024) provides a pedagogical derivation of these equations in the context of hyperbolic systems [2].

2.3 Nonlinear Thermoelasticity

Nonlinear thermoelasticity relaxes the simplifying assumptions of the linear theory, especially in scenarios involving:

- **Large deformations**,
- **Nonlinear thermal expansion**,
- **Temperature-dependent material properties**, and
- **Geometric nonlinearities**.

Key Modifications:

1. **Strain tensor** replaced by Green–Lagrange strain:

$$E = \frac{1}{2}(\nabla u + \nabla u^T + \nabla u^T \nabla u)$$

2. Stress tensor becomes the **Second Piola–Kirchhoff tensor**.

3. Constitutive laws derived from nonlinear thermodynamic potentials (e.g., Helmholtz free energy):

$$S = \frac{\partial \Psi}{\partial E}, \quad q = -\frac{\partial \Psi}{\partial \theta}$$

4. Generalized heat conduction models, including *non-Fourier laws* and *memory effects*:

$$\tau \frac{\partial q}{\partial t} + q = -\kappa \nabla \theta \quad \text{(Cattaneo-Vernotte law)}$$

Nonlinear models also modify the heat conduction relation. For example, temperature-dependent conductivity $\kappa(\theta)$ or memory effects can be included, as discussed in the nonlinear micropolar continuum theory proposed by **Surana and Mathi (2025) [1]**.

Moreover, **Mustapha and others (2024)** examined nonlinear thermoelastic systems with infinite history and distributed delay terms, emphasizing the role of thermal memory in stability [3].

Additionally, fractal models proposed by **Sur (2025)** introduced thermoelasticity in non-integer dimensional spaces, requiring the modification of classical governing equations to accommodate fractal geometries [4].

2.4 Comparative Summary of Assumptions

Feature	Linear Thermoelasticity	Nonlinear Thermoelasticity
Strain	Infinitesimal strain	Finite strain (Green–Lagrange)
Stress	Cauchy tensor	Second Piola–Kirchhoff tensor
Heat conduction	Fourier's law	Cattaneo–Vernotte or more general
Geometry	Fixed reference	Moving/deforming reference
PDE Type	Parabolic (heat), hyperbolic (wave)	Fully coupled, often hyperbolic or mixed
Example Application	Beam deflection at low temps	Microbeam in MEMS with large deformation

3. Comparative Review of Key Papers

This section provides a structured and critical review of influential works in linear and nonlinear thermoelasticity, highlighting the evolution of mathematical modeling approaches, theoretical frameworks, and practical applications. By examining diverse contributions—ranging from foundational theories to advanced numerical methods—this comparative synthesis offers a comprehensive perspective on the development and refinement of thermoelastic models, underscoring their relevance in both academic research and real-world engineering contexts.

Recent advancements in nonlinear micropolar thermoelasticity have introduced sophisticated models that integrate classical rotational effects within thermoviscoelastic solid frameworks, extending beyond conventional continuum theories by capturing microstructure-driven deformation behaviors. These developments include spatiotemporal nonlocal formulations, field-theoretical approaches, and rotationally invariant constitutive models, all designed to address size-dependent effects, laser-induced interactions, and complex thermo-mechanical dynamics in materials with intrinsic micro-rotations—such as biological tissues, composites, and microscale continua. Additional contributions cover binary mixtures, acoustic wave propagation, and nonlinear finite element implementations, significantly enriching the theoretical and computational understanding of thermoelastic behavior in anisotropic and micropolar media [1, 5-9].

Recent progress in nonlinear thermoelasticity has yielded advanced models that address memory-dependent thermal effects, nonlocal interactions, and size-dependent behaviors in complex materials and structures. Studies on Timoshenko-type beams reveal how thermal history and distributed delays significantly influence mechanical stability, necessitating nonlinear formulations beyond the scope of classical theories. In parallel, developments in micropolar thermoelasticity incorporate rotational degrees of freedom, spatiotemporal nonlocality, and field-theoretical methods to model microscale and anisotropic media such as composites, biological tissues, and binary mixtures, thereby enhancing the mathematical and physical understanding of coupled thermo-mechanical responses at small scales [3,10-15].

Recent research on plane thermoelastic problems highlights key distinctions between linear and nonlinear formulations, particularly in how they address thermal gradients, material heterogeneity, and boundary effects. While many studies have traditionally relied on linear models to analyze isotropic media, layered structures, and wave propagation, newer work demonstrates that these models often fall short under significant thermal or material nonuniformity. In contrast, nonlinear and hyperbolic theories—including those based on Cattaneo's law—offer improved accuracy by capturing stress concentrations, asymptotic behavior, and the influence of voids and advanced constitutive relations, thus providing a more realistic description of two-dimensional thermoelastic responses [16-22].

Recent research has investigated the stabilization of coupled hyperbolic-parabolic PDE systems under nonlinear thermoelastic effects, offering critical insights for long-term simulations, energy decay analysis, and control design in thermo-mechanical systems. These studies demonstrate that nonlinear internal and boundary dissipation—even when localized or excluding material interfaces—can ensure uniform decay rates and well-posedness. Although only a subset of this work focuses explicitly on nonlinear thermoelasticity, related contributions involving p-Laplacian and fractional Laplacian operators, fluid-structure interaction, and traveling wave stability significantly advance the theoretical understanding of stabilization mechanisms and their implications for real-world engineering applications [23-29].

Nonlinear thermoelastic Timoshenko beam models, especially those accounting for singular perturbations, boundary delays, and material gradation, are essential for accurately capturing the long-term dynamic response, vibration behavior, and thermal sensitivity of slender structural components used in aerospace,

civil engineering, and MEMS/NEMS systems, thereby enhancing their performance, reliability, and thermal resilience [30-36].
Ruggeri's comprehensive textbook provides a rigorous foundation for both linear and nonlinear thermoelastic theories, situating them within the broader framework of thermomechanics and emphasizing the hyperbolic nature of governing equations. Building on this foundation, recent research has extended classical theories—particularly those inspired by the Green-Naghdi framework—to incorporate finite-speed propagation of thermal, diffusion, and microtemperature waves, along with nonlinear dissipative mechanisms. These advancements deepen the theoretical understanding of asymptotic behavior, energy dissipation, and wave localization, making nonlinear thermoelastic models increasingly relevant for analyzing complex materials and dynamic structures in both academic and engineering contexts [2,37-41].

4. Applications and Real-World Modeling Examples

Thermoelasticity serves as a crucial foundation for understanding and predicting the behavior of materials and structures subjected to combined thermal and mechanical effects. Depending on the complexity of the problem—whether involving small strains and moderate temperatures or large deformations and intense thermal gradients—either linear or nonlinear thermoelastic models are employed. This section outlines significant applications where thermoelastic theories, both linear and nonlinear, are effectively utilized.

4.1 Structural Applications: Beams and Plates

Linear thermoelasticity has been extensively applied to traditional structures such as beams, frames, and bridges, where the assumptions of small deformation and linear material behavior hold valid. In such cases, analytical methods based on classical elasticity and Fourier's heat conduction law are sufficient to model temperature-induced stresses.
However, when structures are subjected to high thermal loads, significant shear deformations, or material memory effects, nonlinear models become necessary. For instance, Mustapha and others (2024) explored a nonlinear thermodiffusion model for a Timoshenko beam system incorporating infinite history and distributed delay terms. Their work highlights how thermal memory and delay mechanisms substantially affect stability, making linear approximations insufficient for capturing the long-term behavior of such systems [3].

4.2 Micro- and Nano-Scale Systems (MEMS/NEMS)

Microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS) operate in regimes where thermal effects significantly influence mechanical behavior. The miniaturization of components leads to enhanced coupling between thermal expansion and mechanical vibrations, often causing phenomena like thermoelastic damping, which impacts performance and reliability.
Surana and Mathi (2025) developed a nonlinear micropolar continuum theory for thermoviscoelastic solids, particularly suited for microstructured materials. Their formulation incorporates classical rotational effects and nonlinearity in the thermoelastic response, offering a refined tool for modeling microscale devices where traditional linear thermoelastic models fail to capture the intricate thermomechanical interactions [1].

4.3 Advanced Material Systems: Fractal and Smart Materials

The advent of smart materials, metamaterials, and materials with fractal-like microstructures necessitates the use of generalized thermoelastic theories beyond classical frameworks. These materials often display complex thermomechanical responses that cannot be adequately described using integer-dimensional mo dels.

Sur (2025) introduced a fractal theory of thermoelasticity operating in non-integer dimensional spaces. By extending thermoelasticity to fractal continua, Sur's model captures the anomalous diffusion and mechanical behavior seen in certain advanced materials. This approach broadens the application of thermoelastic theory into domains previously inaccessible by conventional models [4].

4.4 Data-Driven Modeling and AI in Thermoelasticity

The integration of artificial intelligence and machine learning techniques into thermoelastic modeling marks a significant advancement. Instead of relying solely on first-principal derivations, data-driven approaches allow for the identification of latent thermoelastic models directly from empirical data, especially for highly complex or nonlinear systems.

Rettberg and others (2024) proposed a data-driven framework based on latent port-Hamiltonian systems to model nonlinear thermoelastic behavior. Their method was successfully applied to high-dimensional systems such as disc brakes, demonstrating that AI-assisted modeling can reveal underlying thermoelastic dynamics more effectively than classical methods in certain complex scenarios [42].

5. Comparative Analysis of Recent Research

A careful examination of recent developments in thermoelasticity reveals a distinct trend: while linear models retain their practicality for conventional problems, nonlinear formulations are becoming increasingly essential to accurately describe complex physical phenomena. This section synthesizes and compares major contributions in linear and nonlinear thermoelastic modeling, stability analysis, and applications.

5.1 Advances in Nonlinear Thermoelastic Modeling

Nonlinear models have expanded significantly beyond classical frameworks.Surana and Mathi (2025) proposed a nonlinear micropolar continuum theory for thermoviscoelastic solids, incorporating classical rotational degrees of freedom and enabling accurate modeling of microstructured materials. Their formulation highlights the limitations of linear assumptions in predicting thermoelastic behavior in systems with internal rotations [1]. Similarly, Mustapha and others (2024) addressed nonlinear thermodiffusion phenomena in a Timoshenko beam system. They demonstrated how the incorporation of infinite memory and distributed delays significantly alters system stability and energy decay, features absent from linear theories [3].

5.2 Innovations in Mathematical Techniques

While classical analytical solutions are effective for linear models, complex nonlinear systems require advanced mathematical tools. Rettberg and others (2024) introduced a novel data-driven identification method based on latent port-Hamiltonian systems, bridging classical energy-based modeling with machine learning. Their study demonstrates that even in highly nonlinear systems, it is possible to construct accurate reduced-order models when sufficient empirical data are available [42]. In a different direction, Sur (2025) incorporated fractal geometry into the thermoelastic framework, suggesting that continuum models based on non-integer dimensions provide more accurate descriptions of certain smart and complex materials. His work opens avenues for more flexible mathematical formulations in nontraditional domains [4].

5.3 Transition from Linear to Nonlinear in Structural Applications

At the structural level, the need for transitioning from linear to nonlinear models becomes evident when dealing with critical phenomena such as buckling, post-buckling behavior, thermal fatigue, and damping.

For instance, traditional linear thermoelastic models have sufficed in earlier studies on thin beams and plates. However, research emphasizes that nonlinear effects—including rotational inertia, thermal diffusion delay, and temperature-dependent properties—must be considered to accurately model real-world structures under thermal and mechanical loads.

5.4 Emerging Challenges and Gaps

Despite the advancements, several challenges persist:

- **Coupled Effects:** Full coupling between mechanical deformation, thermal conduction, and electromagnetic effects (e.g., in piezoelectric materials) is not yet completely understood in nonlinear regimes.
- **Computational Complexity:** Nonlinear thermoelastic simulations demand significant computational resources, especially when memory effects or fractal models are involved.
- **Validation:** Experimental validation of advanced nonlinear models remains sparse, partly because measuring internal variables such as thermal stresses in real-time is difficult.

These gaps suggest fertile ground for future research, particularly at the intersection of theory, computation, and experiment.

6. Discussion and Future Directions

The study of thermoelasticity, particularly through the lens of mathematical modeling, continues to evolve in response to the increasing complexity of real-world systems. As evidenced throughout the literature, both linear and nonlinear theories serve pivotal roles—each suited to distinct physical scales, material behaviors, and computational demands. Nevertheless, challenges remain in achieving a balance among analytical tractability, computational efficiency, and physical accuracy.

6.1 Strengths and Limitations of Current Models

Linear Models:- Linear thermoelasticity provides a structured and analytically solvable framework for modeling small-strain, moderate-temperature applications. Its simplicity facilitates efficient design workflows in fields such as structural engineering, optics, and aerospace. However, the neglect of higher-order couplings, temperature dependencies, and geometric nonlinearities restricts its applicability to advanced materials and systems operating under extreme conditions.

Nonlinear Models:- Nonlinear thermoelastic formulations offer greater fidelity by incorporating large deformations, temperature-dependent material properties, nonlinear constitutive laws, and thermal memory effects. These models are crucial for accurately simulating the behavior of smart materials, fractured media, and systems subjected to extreme environmental conditions. Nevertheless, they introduce significant computational complexity and often preclude closed-form analytical solutions, necessitating the development of sophisticated numerical or hybrid computational methods.

6.2 Observations from the Literature

Emerging Hybrid Approaches : Recent studies, such as that by Rettberg and others (2024), demonstrate the integration of classical mechanics with data-driven methodologies, including latent port-Hamiltonian system frameworks. These hybrid models hint at a promising future where machine learning augments partial differential equation–based modeling, enhancing predictive capability while managing model complexity.

Memory and Delay Effects : The incorporation of memory and distributed delay effects into thermoelastic models, as explored by Mustapha and others (2024), underscores their significance in accurately capturing the dynamic behavior of beams and plates. These factors, largely absent in linear theories, play a critical role in the long-term stability and response of thermomechanical systems.

Generalized Theories : Extensions into generalized theories, such as the fractal thermoelasticity framework proposed by Sur (2025) and micropolar continuum models developed by Surana and Mathi (2025), reveal fundamental limitations of classical continuum assumptions. Such theories are essential for modeling heterogeneous, porous, or microstructured materials where traditional models fail.

Stability and Energy Analysis : Theoretical advances in stabilization techniques for nonlinear partial differential equations, as highlighted by Tebou (2025), are vital for ensuring the long-term control and robustness of thermomechanical systems under perturbations. These methods provide insight into the asymptotic behavior and energy dissipation characteristics of complex systems.

6.3 Open Questions and Future Research Directions

Unified Theories : There is a pressing need for unified modeling frameworks that seamlessly transition between linear and nonlinear behavior based on boundary conditions, loading intensities, or internal state variables. Such adaptive models would provide a more comprehensive understanding of material responses across varying regimes.

Computational Efficiency : While nonlinear thermoelastic models offer superior physical fidelity, they often incur high computational costs. Advances in model order reduction (MOR) techniques and the application of physics-informed neural networks (PINNs) present promising avenues for achieving real-time predictions without significant loss of accuracy.

Experimental Validation : Many advanced nonlinear theories remain under-validated due to the scarcity of experimental data at relevant scales. Greater collaboration between theoreticians and experimentalists is essential to refine theoretical assumptions, calibrate models, and validate predictions through controlled testing and observation.

Multiphysics Coupling : Future investigations should increasingly explore the coupling of thermoelasticity with other physical phenomena such as electromagnetism, phase changes, and fluid flow. Such multiphysics models are particularly relevant for smart materials, biomedical implants, microelectromechanical systems (MEMS), and nanoelectromechanical systems (NEMS).

7. Conclusion

Thermoelasticity remains a vital and evolving area of mathematical physics, bridging thermal and mechanical behaviors through a coupled system of partial differential equations. This review has explored the distinctions and overlaps between linear and nonlinear thermoelastic models, focusing on their governing equations, solution techniques, theoretical developments, and real-world applications.

Linear thermoelasticity, characterized by its analytical tractability and historical development, continues to serve as a reliable tool for modeling small-strain, moderate-temperature problems. Its simplicity supports efficient simulations in engineering domains such as beam theory, optics, and structural analysis. However, as modern technologies push the boundaries of material performance and operating environments, the limitations of linear theory become increasingly apparent.

Nonlinear thermoelasticity offers the mathematical sophistication required to address large deformations, temperature-dependent behaviors, and memory effects. Theories incorporating fractal geometries, micropolar continua, and non-Fourier heat conduction demonstrate the field's responsiveness to challenges arising from biomechanics, aerospace applications, MEMS/NEMS, and smart materials. Yet, the computational intensity and lack of closed-form solutions in nonlinear models remain open challenges for the research community.

Looking ahead, future efforts should focus on bridging the linear-nonlinear gap, developing computationally efficient hybrid models, and enhancing the experimental validation of theoretical frameworks. As the field progresses, interdisciplinary research combining mathematics, computational science, and engineering applications will be essential for the robust modeling of next-generation materials and systems.

In conclusion, both linear and nonlinear thermoelasticity play indispensable roles in modeling heat-mechanical interactions. The choice of model should be guided by the physical context, desired accuracy, and computational feasibility, with a growing trend toward generalized and adaptive models capable of spanning multiple regimes.

References

1. Surana, K. S., & Mathi, S. S. C. (2025). Finite Deformation, Finite Strain Nonlinear Micropolar NCCT for Thermoviscoelastic Solids with Rheology. *Applied Mathematics*, *16*(1), 143-168.
2. Ruggeri, T. (2024). *Introduction to the thermomechanics of continua and hyperbolic systems*. Springer.
3. Mustapha, N. B., et al. (2024). Thermodiffusion with delay in nonlinear beams. Research Gate.
4. Sur, A. (2025). *Fractal Theory of Thermoelasticity in Non-Integer Dimension Space*. ZAMM.
5. Abouelregal, A. E., Marin, M., & Öchsner, A. (2025). A modified spatiotemporal nonlocal thermoelasticity theory with higher-order phase delays for a viscoelastic micropolar medium exposed to short-pulse laser excitation. *Continuum Mechanics and Thermodynamics*, *37*(1), 15.
6. Kovalev, V., Murashkin, E., & Radayev, Y. (2017). On a physical field theory of micropolar thermoelasticity. In *Journal of Physics: Conference Series* (Vol. 788, No. 1, p. 012043). IOP Publishing.
7. Kovalev, V. A., & Radaev, Y. N. (2015). *Objective rotationally invariant forms of thermoelastic Lagrangians*. Bulletin of the Samara State Technical University. Series: Physical and Mathematical Sciences, **19**(2), 325–340.
8. Galeş, C. (2007). A Mixture Theory for Micropolar Thermoelastic Solids. *Mathematical Problems in Engineering*, *2007*, 881–901.
9. Ramezani, S., Naghdabadi, R., & Sohrabpour, S. (2008). Non-linear finite element implementation of micropolar hypo-elastic materials. *Computer Methods in Applied Mechanics and Engineering*, *197*(49), 4149–4159.
10. Aouadi, M., & Castejón, A. (2019). Properties of global and exponential attractors for nonlinear thermo-diffusion Timoshenko system. *Journal of Mathematical Physics*, *60*(8), 081503.
11. Messaoudi, S. A., & Mustafa, M. I. (2009). On the stabilization of the Timoshenko system by a weak nonlinear dissipation. *Mathematical Methods in The Applied Sciences*, *32*(4), 454–469.
12. Warminska, A., Manoach, E., Warminski, J., & Samborski, S. (2015). Regular and chaotic oscillations of a Timoshenko beam subjected to mechanical and thermal loadings. *Continuum Mechanics and Thermodynamics*, *27*(4), 719–737.
13. Elhindi, M., & Arwadi, T. (2021). *Analysis of the thermoviscoelastic Timoshenko system with diffusion effect*. *4*, 100156.

14. Aouadi, M., Campo, M., Copetti, M. I. M., & Fernández, J. R. (2019). Existence, stability and numerical results for a Timoshenko beam with thermodiffusion effects. *Zeitschrift Für Angewandte Mathematik Und Physik*, *70*(4), 1–26.
15. Ramos, A., Aouadi, M., Mahfoudhi, I., & Freitas, M. (2024). Asymptotic behavior and numerical analysis for a Timoshenko beam with viscoelasticity and thermodiffusion effects. *Mathematical Control and Related Fields*, *14*(2), 467-492.
16. Knyazeva, A. G., et al. (2025). Plane Thermoelasticity Problems. Applied Mathematical Modelling.
17. Zhu, Y.-Y., Li, Y., & Cheng, C.-J. (2014). Analysis of Nonlinear Characteristics for Thermoelastic Half Plane with Voids. *Journal of Thermal Stresses*, *37*(7), 794–816.
18. Sumi, S., Matsumoto, E., & Sekiya, T. (1965). Mechanical Analog Procedure for Solution of Plane Thermoelastic Problems. *The Journal of the Japan Society of Aeronautical Engineering*, *13*(135), 101–108.
19. Chao, C. K., & Chen, F. M. (2009). Heterogeneous Problems in Plane Thermoelasticity. *Journal of Thermal Stresses*, *32*, 623–655.
20. Chandrasekharaiah, D. S., & Keshavan, H. R. (1991). Thermoelastic plane waves in a transversely isotropic body. *Acta Mechanica*, *87*(1), 11–22.
21. Racke, R. (2009). *Chapter 4 – Thermoelasticity* (Vol. 5, pp. 3–5).
22. Vatul'yan, A. O., Kiryutenko, A. Yu., & Nasedkin, A. V. (1996). Plane waves and fundamental solutions in linear thermoelectroelasticity. *Journal of Applied Mechanics and Technical Physics*, *37*(5), 727–733.
23. Tebou, L. (2025). Stabilization in nonlinear PDE systems. Springer.
24. Lasiecka, I., & Lebiedzik, C. (2000). Decay Rates of Interactive Hyperbolic-Parabolic PDE Models with Thermal Effects on the Interface. *Applied Mathematics and Optimization*, *42*(2), 127–167.
25. Bucci, F. (2002). *Uniform Stability of a Coupled System of Hyperbolic/Parabolic PDE's with Internal Dissipation* (pp. 57–68). Springer, Boston, MA.
26. Tebou, L. (n.d.). *Stabilization of a Nonlinear Hyperbolic/parabolic System*.
27. Bucci, F. (2003). Uniform decay rates of solutions to a system of coupled PDEs with nonlinear internal dissipation. *Differential and Integral Equations*, *16*(7), 865–896.
28. Rottmann-Matthes, J. (2012). Stability of parabolic-hyperbolic traveling waves. *Dynamics of Partial Differential Equations*, *9*(1), 29–62.
29. Valos, G., & Triggiani, R. (2008). Uniform stabilization of a coupled PDE system arising in fluid-structure interaction with boundary dissipation at the interface. *Discrete and Continuous Dynamical Systems*, *22*(4), 817–833.
30. Aouadi, M., Moulahi, T., & Attia, N. (2025). Thermoelastic Extensible Timoshenko Beam with Symport Term.
31. Awrejcewicz, J., Krysko, V. A., Pavlov, S. P., Zhigalov, M. V., Kalutsky, L. A., Krysko, A. V., & Krysko, A. V. (2020). Thermoelastic vibrations of a Timoshenko microbeam based on the modified couple stress theory. *Nonlinear Dynamics*, *99*(2), 919–943.
32. Awrejcewicz, J., Krysko, A. V., Zhigalov, M. V., & Krysko, V. A. (2021). *Thermoelastic Vibrations of Timoshenko Microbeams (Modified Couple Stress Theory)* (pp. 295–332). Springer, Cham.
33. Ansari, R., Oskouie, M. F., Nesarhosseini, S., & Rouhi, H. (2021). Nonlinear Thermally Induced Vibration Analysis of Porous FGM Timoshenko Beams Embedded in an Elastic Medium. *Transport in Porous Media*, 1–25.

34. Ghiasian, S. E., Kiani, Y., & Eslami, M. R. (2015). Nonlinear thermal dynamic buckling of FGM beams. *European Journal of Mechanics A-Solids*, *54*, 232–242.
35. Azimi, M., Mirjavadi, S. S., Shafiei, N., & Hamouda, A. M. (2017). Thermo-mechanical vibration of rotating axially functionally graded nonlocal Timoshenko beam. *Applied Physics A*, *123*(1), 104.
36. Azimi, M., Mirjavadi, S. S., Shafiei, N., Hamouda, A. M., & Davari, E. (2018). Vibration of rotating functionally graded Timoshenko nano-beams with nonlinear thermal distribution. *Mechanics of Advanced Materials and Structures*, *25*(6), 467–480.
37. Aouadi, M., Ciarletta, M., & Passarella, F. (2021). A thermoelastic theory with microtemperatures of type III. *arXiv: Analysis of PDEs.*
38. Pabst, W. (2005). *The linear theory of thermoelasticity from the viewpoint of rational thermomechanics*. *49*(4), 242–251.
39. Aouadi, M., Lazzari, B., & Nibbi, R. (2014). A theory of thermoelasticity with diffusion under Green-Naghdi models. *Zamm-Zeitschrift Fur Angewandte Mathematik Und Mechanik*, *94*(10), 837–852.
40. Green, A. E., & Naghdi, P. M. (1993). Thermoelasticity without energy dissipation. *Journal of Elasticity*, *31*(3), 189–208.
41. Nourmohammadi, Z., Joshi, S., & Vengallatore, S. (2016). Analysis of Nonlinear Thermoelastic Dissipation in Euler-Bernoulli Beam Resonators. *PLOS ONE*, *11*(10).
42. Rettberg, J., Kneifl, J., Herb, J., Buchfink, P., Fehr, J., & Haasdonk, B. (2024). Data-driven identification of latent port-Hamiltonian systems. *arXiv preprint arXiv:2408.08185*.