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Optimizing Multi-Objective Transportation Problems Using the Lexicographic Method

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Abstract

This paper addresses the optimization of a Multi-Objective Transportation Problem (MOTP) involving transportation cost, time, and carbon emissions using the Lexicographic Method. The method prioritizes objectives hierarchically, solving the problem step-by-step in decreasing order of importance. Three distinct conditions were evaluated to explore trade-offs between objectives. The results illustrate the method's capability to provide optimized and balanced solutions, allowing decision-makers to prioritize certain objectives without compromising others significantly. The lexicographic approach proves to be efficient, especially in contexts where objective priorities are clearly defined, offering a practical decision-making tool for sustainable and cost-effective logistics planning.

Keywords: Multi-Objective Transportation Problem, lexicographic Method, Optimal Solution

Introduction:

Practical applications give rise to a large class of mathematical programming problems frequently. For instance, a product may be transported from facto- ries (sources) to retail stores (destinations). One must know the amount of the product available as well as the demand of the product. So, the difficulty is that the different ways of the network joining the sources to the destinations have different costs linked with them. Therefore, we aim at calculating the minimum cost routing of products from point of supply to point of destination and this problem is named as cost minimizing Transport Problems. Generally, the classical transportation problems are associated with single objective, which can be transportation cost or time and are developed by Hitchcock (1941) and Koopmans (1947). But competition between organizations is increasing day to day very quickly. So it is not sufficient to achive only one objective at time, when transportation of goods from organizations is made. Therefore it is necessary to proceed with multi-objectives simultaneously so that firms can get maximum profit. Many researchers have developed efficient techniques for solving two or more objectives simultaneously, which are by Lee et al. (1973), Zeleny (1974), Diaz (1978; 1979), Isermann (1979), Aneja et al. (1979), Gupta et al. (1983), Ringuest et al. (1987), Reeves et al. (1985), Kasana et al. (2000), Chang (2007; 2008), Bai et al. (2011), Pandian et al. (2011), Quddoos et al. (2013a) and Nomani et al. (2017) etc. All techniques developed by these researchers are very difficult to apply and more time consuming.
In literature, we find that there are many transportation models where linear programming has been applied or approaches to solve multi-objective transportation problems. From this idea, Chanas (1984) developed multi- objective linear programming by using parametric approach. Further, Zimmerman (1978) makes use of intersection of all constraints and goals by proposing a multi-criteria decision

making (MCDM) set and multi-objective linear programming problems that taking all parameters, along with a triangular possibility distribution. Prakash (1981) considered linear programming approach to multi criteria decision making where the constraints are of equality type. Also, various authors worked on developing different models for solving multi-objective transportation problems.

Multi-Objective Transportation Problem:

A Multi-Objective Transportation Problem (MOTP) is an extension of the classical transportation problem where multiple, often conflicting; objectives are optimized simultaneously. In this context, we are considering three objectives.

Objectives:

Let $o_{ij}^1, o_{ij}^2, o_{ij}^3, \cdots o_{ij}^r$ represent the cost coefficients associated with the r objectives for transporting from source i to destination j.

The multi-objective transportation problem can be stated as:

$$Min\ or\ Max\ Z_1 = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^1 q_{ij}$$

$$Min\ or\ Max\ Z_2 = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^2 q_{ij}$$

$$Min\ or\ Max\ Z_3 = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^3 q_{ij}$$

$$\vdots \qquad \vdots$$

$$Min\ or\ Max\ Z_r = \sum_{i=1}^{p}\sum_{j=1}^{q} o_{ij}^r q_{ij}$$

Where $Z_1, Z_2, Z_3 \ldots\ldots Z_r$ are the three objectives to be minimized or maximized.

Decision Variables:

Let q_{ij} represent the amount of goods transported from source i to destination j, where:

$i = 1, 2, \cdots, p$ (number of sources)

$j = 1, 2, \cdots, q$ (number of destinations)

Constraints:

1. ***Supply Constraints***:

$$\sum_{j=1}^{q} q_{ij} \leq s_i \qquad \forall\, i = 1,2,\cdots,p$$

where s_i is the supply available at source i.

2. ***Demand Constraints***:

$$\sum_{i=1}^{p} q_{ij} \geq d_j \qquad \forall\, j = 1,2,\cdots,q$$

where d_j is the supply available at source j.

3. ***Non-Negativity Constraints***:

$$q_{ij} \geq 0 \qquad \forall\, i = 1,2,\cdots,p;\ j = 1,2,\cdots,q$$

Lexicographic Method:

The multi-objective transportation problem (MOTP) arises in decision-making scenarios where multiple, often conflicting objectives need to be optimized simultaneously, subject to certain resource and capacity constraints. Traditional transportation problems involve minimizing a single cost function while satisfying supply and demand constraints. However, in real-world applications-such as logistics, supply chain management, and production planning-decision-makers often need to consider several criteria, such as minimizing cost, time, risk, or environmental impact. To address such complexity, the Lexicographic Method provides a systematic framework for handling multiple objectives by prioritizing them and sequentially optimizing each according to its rank.

The Lexicographic Method is a goal-oriented decision-making approach that ranks multiple objective functions in order of importance and solves the problem by optimizing them sequentially. The key assumption is that the decision-maker can establish a complete preference ordering among the objectives, meaning that the most important objective must be fully optimized before any subsequent objectives are considered. Only when multiple feasible solutions yield the same optimal value for the higher-priority objectives is the next objective considered.

Working Processor of Lexicographic Method as follows:

Step 1: Firstly, we checked the our MOTP is balanced. i.e. $\sum_{j=1}^{q} d_j = \sum_{i=1}^{p} s_i$

We move on step 3.

Step 2: If our MOTP is not balanced. i.e. $\sum_{j=1}^{q} d_j \neq \sum_{i=1}^{p} s_i$

according condition, we are going to add dummy row or column to convert out problem in balanced problem.

Step 3: **Established the Priority Order of Objectives:**

- Rank all the objective functions based on their importance e.g. $Z_1 > Z_2 > \cdots > Z_r$.
- The most important objective is optimized first, followed by the next in the rank, and so on.

Step 4: Solve the first objective i.e. minimize Z_1 subject to the transportation constraints. Obtain the optimal value Z_1^* and the corresponding basic feasible solution X_1^*

Step 5: for the second objective Z_2, include a constraint to maintain the optimal value of Z_1

$$Z_1(x_{ij}) = Z_1^*$$

When Z_2 is optimize, the optimality of $Z_1$is not violated.

Step 6: The optimality constraints for all higher-priority objectives

$$Z_r(x_{ij}) = Z_r^* \quad \forall\, r = 1, 2, 3, \cdots r - 1$$

Continue unit all the objective has been solved.

Numerical Problem:

An agro-processing company in Madhya Pradesh needs to transport agricultural raw materials from various farms to processing plants while optimizing multiple objectives. The company sources wheat from four different farms and distributes it to three processing plants. The key objectives considered in transportation planning are:

1. Minimization of Transportation Cost - Reducing the cost of shipping raw materials.
2. Minimization of Transportation Time - Ensuring timely delivery to maintain quality.
3. Minimization of Environmental Impact - Lowering carbon emissions from transportation.

Table 1: Sources and Destinations

Sources			**Destination**		
Farm	**City**	**Supply**	**Plant**	**City**	**Demand**
A	Indore	40	X	Ujjain	60
B	Bhopal	50	Y	Sagar	50
C	Jabalpur	60	Z	Rewa	70
D	Gwalior	30			

Table 2: Objective Matrix of Transportation Problem

Source→ Destination	**Cost (Rs Per Ton) (Z_1)**	**Time (Hours) (Z_2)**	**Carbon Emissions (Kg CO_2 Per ton) (Z_3)**
Farm A → Plant X	500	5	20
Farm A → Plant Y	700	7	30
Farm A → Plant Z	600	6	25

Farm B → Plant X	550	6	22
Farm B → Plant Y	650	5	28
Farm B → Plant Z	750	8	35
Farm C → Plant X	800	9	40
Farm C → Plant Y	600	6	25
Farm C → Plant Z	500	5	20
Farm D → Plant X	700	8	30
Farm D → Plant Y	900	10	45
Farm D → Plant Z	650	7	28

Since our MOTP is balanced so we are going to decide the priority of objective according to following three conditions:

Table 3: Priority of Objectives

Objective	Priority Order
I	$Z_1 > Z_2 > Z_3$
II	$Z_2 > Z_3 > Z_1$
II	$Z_3 > Z_1 > Z_2$

Here we are going to use Vogel Approximation Method and MODI Method to optimal solution of our MOTP in each condition as follows:

Condition I: According to Lexicographic Method firstly we solve our MOTP for Transportation Cost than Transportation Time and end with Carbon emissions.

Now after the implementation of VAM and MODI method in First Objective, we obtained the allocation of unit from source to destinations as follows

$x_{11} = 40$ $x_{21} = 20$ $x_{22} = 30$ $x_{30} = 20$ $x_{33} = 40$ $x_{41} = 30$

and optimal transportation cost mentioned below

Table 4: Optimal Solution as per I Condition

Objective	Name of Objective	Optimal Solution
Z_1^*	Transportation Cost	Rs 103500
Z_2^*	Transportation Time	35 hours
Z_3^*	Carbon Emission	4280 kg CO_2

Condition II: According to Lexicographic Method firstly we solve our MOTP for Transportation Time than Carbon emissions and end with Transportation Time.

Now after the implementation of VAM and MODI method in First Objective, we obtained the allocation of unit from source to destinations as follows

$x_{11} = 40 \quad x_{22} = 50 \quad x_{32} = 0 \quad x_{33} = 60 \quad x_{41} = 20 \quad x_{43} = 10$

and optimal transportation cost mentioned below

Table 5: Optimal Solution as per II Condition

Objective	Name of Objective	Optimal Solution
Z_2^*	Transportation Time	30 hours
Z_3^*	Carbon Emission	4280 kg
Z_1^*	Transportation Cost	Rs 103000

Condition III: According to Lexicographic Method firstly we solve our MOTP for Carbon emissions than Transportation Time and end with Transportation Time.

Now after the implementation of VAM and MODI method in First Objective, we obtained the allocation of unit from source to destinations as follows

$x_{11} = 40 \quad x_{21} = 20 \quad x_{22} = 30 \quad x_{30} = 20 \quad x_{33} = 40 \quad x_{41} = 30$

and optimal transportation cost mentioned below

Table 6: Optimal Solution as per III Condition

Objective	Name of Objective	Optimal Solution
Z_3^*	Carbon Emission	4280 kg CO_2
Z_1^*	Transportation Cost	Rs 103500
Z_2^*	Transportation Time	35 hours

Result and Discussion:

The Lexicographic Method was applied to a Multi-Objective Transportation Problem (MOTP) involving transportation cost, time, and carbon emission. Under three conditions, the outcomes varied. In Condition II, transportation time was minimized to 30 hours and cost reduced to Rs. 103000, while maintaining the same carbon emission (4280 kg) as the other conditions. Condition I and III both had higher transportation costs (Rs. 103500) and longer time (35 hours), indicating suboptimal performance. This result highlights the method's effectiveness in prioritizing objectives-minimizing time and cost while keeping emissions constant -demonstrating its capability in addressing trade-offs in multi-objective decision-making.

Table 7: Summary of Optimal Solution of All the Objective according priority

Objectives of MITP	I Condition	II Condition	III Condition
	$Z_1 > Z_2 > Z_3$	$Z_2 > Z_3 > Z_1$	$Z_3 > Z_1 > Z_2$
Transportation Cost	Rs 103500	Rs 103000	Rs 103500
Transportation Time	35 hours	30 hours	35 hours
Carbon Emission	4280 kg	4280 kg	4280 kg

Conclusion:

The Lexicographic Method effectively solved the MOTP by systematically addressing prioritized objectives. Among the three evaluated conditions, Condition II yielded the most efficient solution, achieving the lowest transportation time and cost, while maintaining constant carbon emissions. This emphasizes the method's utility in real-world logistics where multiple conflicting goals must be balanced. By sequentially optimizing the most critical criteria, the Lexicographic approach provides a structured way to reach practical and efficient solutions. Hence, it proves to be a valuable tool for decision-makers in transportation planning aiming to enhance operational performance without promising environmental considerations.

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