International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 • Website: www.ijfmr.com • Email: editor@ijfmr.com

Outer-Connected Fair Domination in Graphs

Rene L. Duhilag Jr.[1], Margie L. Baterna[2], Grace M. Estrada[3], Mark Kenneth C. Engcot[4], Enrico L. Enriquez[5]

[1]Master's Student in Mathematics, Department of Computer, Information Sciences, and Mathematics, University of San Carlos
[2,4]MS in Mathematics, Department of Computer, Information Sciences, and Mathematics, University of San Carlos
[3,5]PhD in Mathematics, Department of Computer, Information Sciences, and Mathematics, University of San Carlos

Abstract
Let $G = (V(G), E(G))$ be a nontrivial connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$. A set $S \subseteq V(G)$ is said to be an outer-connected dominating set in G if S is dominating and either $S = V(G)$ or $\langle V(G) \setminus S \rangle$ is connected. The outer-connected domination number of G is the minimum cardinality of an outer-connected dominating set of G, denoted by $\tilde{\gamma}_c(G)$. A fair dominating set in graph G is a dominating set S such that all vertices in $V(G) \setminus S$ are dominated by the equal number of vertices in S. The fair domination number of G is the minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$. A nonempty subset $S \subseteq V(G)$ is an outer-connected fair dominating set of G, if S is a fair dominating set of G and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The outer-connected fair domination number of G is the minimum cardinality of an outer-connected fair dominating set of G, denoted by $\tilde{\gamma}_{cfd}(G)$. In this paper, we initiate the study of the concept and we show the existence of a connected graph G with $|V(G)| = n$ and $\tilde{\gamma}_{cfd}(G) = k$ for all positive integer k. Further, give the outer-connected fair domination number of some special graphs.

Keywords: dominating set, outer-connected dominating set, fair dominating set, outer-connected fair dominating set

1. Introduction

A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \cup_{v \in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \cup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [res. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$]. A u-v walk in G is a sequence of vertices in G, beginning with u and ending at v such that consecutive vertices in the sequence are adjacent. A u-v walk in a graph in which no vertices are repeated is a u-v path. If G contains a u-v path, then u and v are said to be connected and u is connected to v (and v is connected to u). A graph G is connected if every two vertices of G are connected, that is, if G contains a u-v path for every pair u, v of vertices of G. Since every vertex is connected to itself, the trivial graph is connected. For the general terminology in graph theory, readers may refer to [1].

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Following an article [3] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$, that is, $N[S] = V(G)$. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [4-17].

A set S of vertices of a graph G is an outer-connected dominating set if every vertex not in S is adjacent to some vertex in S and the sub-graph induced by $V(G) \setminus S$ is connected. The outer-connected domination number $\tilde{\gamma}_c(G)$ is the minimum cardinality of the outer-connected dominating set S of a graph G. The concept of outer-connected domination in graphs was introduced by Cyman [18]. Some related studies of outer-connected domination in graphs are found in [19-28].

A fair dominating set in graph G is a dominating set S such that all vertices in $V(G) \setminus S$ are dominated by the equal number of vertices in S. The fair domination number of G is the minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$. The concepts of fair domination in graphs were introduced by Caro, Hansberg, and Henning [29]. Some related studies of fair domination in graphs are found in [30-35].

Motivated by the introduction of the outer-connected dominating sets and the fair dominating sets, a new variant of domination in graphs is introduced in this paper. Let $G = (V(G), E(G))$ be a nontrivial connected simple graph. A nonempty subset $S \subseteq V(G)$ is an outer-connected fair dominating set of G, if S is a fair dominating set of G and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The outer-connected fair domination number of G is the minimum cardinality of an outer-connected fair dominating set of G, denoted by $\tilde{\gamma}_{cfd}(G)$. In this paper, we initiate the study of the concept and we show that given positive integers k and n such that $n \geq 2$ and $1 \leq k \leq n-1$, there exists a connected graph G with $|V(G)| = n$ and $\tilde{\gamma}_{cfd}(G) = k$. Further, give the outer-connected fair domination number of some special graphs.

2. Results

Definition 2.1 A graph $G = K_n$ is called a complete graph of order n when xy is an edge in G for every distinct pair $x, y \in V(G)$. The complement of a graph G is a graph $\bar{G}$ on the same vertices such that two distinct vertices of $\bar{G}$ are adjacent if and only if they are not adjacent in G.

From the definitions, the following result is immediate.

Remark 2.2 Let G be a connected graph of order $n \geq 2$. Then $1 \leq \tilde{\gamma}_{cfd}(G) \leq n-1$.

It is worth mentioning that the upper bound in Remark 2.2 is sharp. For example, $\tilde{\gamma}_{cfd}(\overline{K}_n) = n-1$ for all $n \geq 2$. The lower bound is also attainable as the following result shows.

Theorem 2.3 Let G be a connected graph of order $n \geq 2$. Then $\tilde{\gamma}_{cfd}(G) = 1$ if $G = K_n$.

Proof. Suppose now that $G = K_n$ and let $S = \{x\}$. Then S is a dominating set of a complete graph G. Since all vertices in $V(G) \setminus S$ is dominated by the equal number of vertices in S, it follows that S is a fair dominating set of G. Further, every vertex in $V(G) \setminus S$ is adjacent to a vertex x in S and the sub-graph induced by $V(G) \setminus S$ is connected since G is a complete graph. Thus, S is an outer-connected fair dominating set of G. Clearly, $S = \{x\}$ is a minimum outer-connected fair dominating set of G. Hence, $\tilde{\gamma}_{cfd}(G) = 1$. ■

It is easy to see that every connected graph G has an outer-connected fair dominating set. The next result says that the value of the parameter $\tilde{\gamma}_{cfd}(G)$ ranges over all positive integers.

Theorem 2.4 Given positive integers k and n such that $n \geq 2$ and $1 \leq k \leq n-1$, there exists a connected graph G with $|V(G)| = n$ and $\tilde{\gamma}_{cfd}(G) = k$.

Proof. Consider the following cases:

***Case* 1. Suppose $k = 1$.**

Let $G = K_n$. Then, $|V(G)| = n$ and by Theorem 2.3, $\tilde{\gamma}_{cfd}(G) = 1$.

***Case* 2. Suppose $k = 2$.**

Let $V(\overline{K}_2) = \{u_1, u_2\}$, $V(P_m) = \{v_1, v_2, \dots, v_m\}$ such that $G = \overline{K}_2 + P_m$ for $m \geq 4$, $n = 2 + m$, see the graph G in Figure 1.

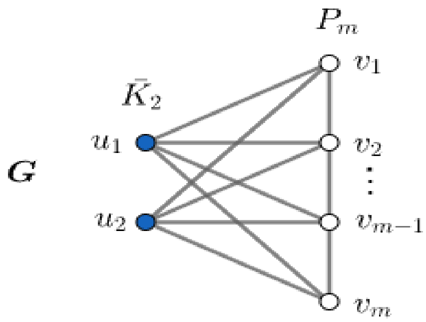

Figure 1

Then $S = V(\overline{K}_2)$ is a dominating set of G and all vertices in $V(G) \setminus S = V(P_m)$ is dominated by the equal number of 2 vertices in S. Thus, S is a fair dominating set of G. Further, every vertex in $V(G) \setminus S$ is adjacent to a vertex in S and the sub-graph induced by $V(G) \setminus S$ is connected. By definition, S is an outer-connected fair dominating set of G. Now, $S = V(\overline{K}_2) = \{u_1, u_2\}$ is a minimum outer-connected fair dominating set of $G = \overline{K}_2 + P_m$ since $\gamma(G) \neq 1$ for $m \geq 4$. Therefore, $|V(G)| = 2 + m = n$ and $\tilde{\gamma}_{cfd}(G) = |S| = 2$.

***Case* 3. Suppose $3 \leq k < n-1$.**

Let $V(P_k) = \{u_1, u_2, \dots, u_{k-1}, u_k\}$, $E(P_k) = \{u_1u_2, u_2u_3, \dots, u_{k-1}u_k\}$, $V(K_m) = \{v_1, v_2, \dots, v_m\}$ with $m \geq 3$ such that $V(G) = V(P_k) \cup V(K_m)$, $E(G) = \{u_1u_2, u_2u_3, \dots, u_{k-1}u_k\} \cup \{u_kv_1\} \cup E(K_m)$, and $n = k + m$, see the graph of G in Figure 2. Consider $S = [V(P_k) \setminus \{u_k\}] \cup \{v_m\}$. Then $S = \{u_1, u_2, \dots u_{k-1}\} \cup \{v_m\}$. Since $\{u_1, u_2, \dots u_{k-1}\}$ is a dominating set in P_k and $\{v_m\}$ is a dominating set in K_m, it follows that S is a dominating set in G. Let $u, v_r \in V(G) \setminus S$. Then $u = u_k$ and $v_r \in V(K_m) \setminus \{v_m\}$. Now, $N_G(u) \cap S = \{u_{k-1}\}$ and $N_G(v_r) \cap S = \{v_m\}$. This implies that $|N_G(u) \cap S| = |N_G(v_r) \cap S| = 1$. Hence, all vertices in $V(G) \setminus S$ are dominated by the equal number of vertices in S, that is, S is a fair dominating set of G. Further, $V(G) \setminus S = \{u_k\} \cup \{v_1, v_2, \dots, v_{m-1}\}$ is connected since $u_kv_1, v_1v_2, \dots, v_{m-2}v_{m-1} \in E(G)$. Thus, every vertex in $V(G) \setminus S$ is adjacent to a vertex in S and the sub-graph induced by $V(G) \setminus S$ is connected. This implies that S is an outer-connected dominating set of G.

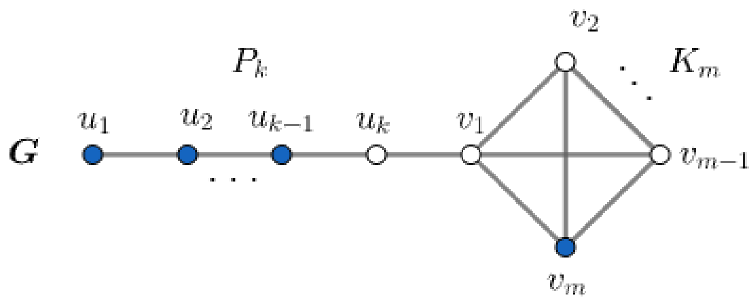

Figure 2

By definition, S is an outer-connected fair dominating set of G. Now, suppose that S is not a minimum outer-connected fair dominating set of G. Then there exists $x \in S$ such that $S \setminus \{x\}$ is an outer-connected dominating set of G. If $x \in \{u_1, u_2, \dots, u_{k-2}\}$, then $V(G) \setminus S$ is not connected. If $x = u_{k-1}$ or $x = v_m$, then S is not a dominating set of G. In either case, $S \setminus \{x\}$ is not an outer-connected fair dominating set of G contrary to our assumption that $S \setminus \{x\}$ is an outer-connected dominating set of G. Thus, S is not a minimum outer-connected fair dominating set of G. Hence, $|V(G)| = k + m = n$ and

$$\tilde{\gamma}_{cfd}(G) = |S| = |\{u_1, u_2, \dots u_{k-1}\} \cup \{v_m\}| = (k-1) + 1 = k.$$

Clearly, $k < (k+m) - 1 = n - 1$, since $n = k + m$ and $m \geq 3$.

***Case* 4. Suppose that $k = n - 1$.**

Let $G = P_1 + \overline{K}_{n-1}$ and let $S = V(\overline{K}_{n-1})$, see the graph of G in Figure 3.

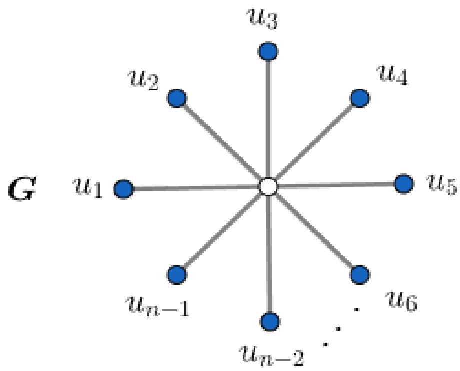

Figure 3

Then S is a dominating set of G. Since the only vertex in P_1 is dominated by all vertices in S, S is a fair dominating set of G. Further, a vertex in P_1 is adjacent to a vertex in S and the sub-graph induced by $V(P_1)$ is connected. This implies that S is an outer-connected dominating set of G. By definition, S is an outer-connected fair dominating set of G. Clearly, $S = V(\overline{K}_{n-1})$ is a minimum outer-connected fair dominating set of G. Hence, $|V(G)| = 1 + (n-1) = n$, and $\tilde{\gamma}_{cfd}(G) = |S| = |V(\overline{K}_{n-1})| = n - 1$.

This proves the assertion. ■

Definition 2.5 A simple graph G is an undirected graph with no loop edges or multiple edges.

Definition 2.6 The path $P_n = \{a_1\, a_2\, a_3 \dots a_n\}$ is the graph with $V(P_n) = \{a_1, a_2, a_3, \dots, a_n\}$ and $E(P_n) = \{a_1 a_2, a_2\, a_3, \dots, a_{n-1} a_n\}$.

Remark 2.7 Let $G = P_n, n \geq 2$. Then $\tilde{\gamma}_{cfd}(G) = \begin{cases} 1, & if\ n = 2 \\ 2, & if\ n = 3\ or\ 4 \\ n-2, & if\ n \geq 5 \end{cases}$

Proof. Suppose that $n = 2$. Let $G = P_2$ such that $V(G) = \{x, y\}$. The set $S = \{x\}$ is a fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{y\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = 1$. Suppose that $n = 3$. Let $G = P_3$ such that $V(G) = \{x, y, z\}$ and $E(G) = \{xy, yz\}$. The set $S = \{x, z\}$ is a fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{y\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = 2$. Suppose that $n = 4$. Let $G = P_4$ such that $V(G) = \{x, y, w, z\}$ and $E(G) = \{xy, yw, wz\}$. The set $S = \{x, z\}$ is a fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{y, w\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = 2$. Suppose that $n = 5$. Let $G = P_5$ such that $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$. The set $S = \{v_1, v_4, v_5\}$ is a fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{v_2, v_3\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = 3 = 5 - 2 = n - 2$. Similarly, if $n \geq 6$, then $\tilde{\gamma}_{cfd}(G) = |S| = n - 2$. Hence, $\tilde{\gamma}_{cfd}(G) = n - 2$ if $n \geq 5$. This proves the assertion. ■

Definition 2.8 The cycle $C_n = \{ a_1a_2a_3 \ldots a_na_1 \}$ is the graph with $V(C_n) = \{a_1, a_2, a_3, \ldots, a_n\}$ and $E(C_n) = \{a_1a_2, a_2a_3, \ldots, a_na_1\}$.

Remark 2.9 Let $G = C_n, n \geq 3$. Then $\tilde{\gamma}_{cfd}(G) = n - 2$.

Proof. Suppose that $n = 3$. Let $G = C_3$ such that $V(G) = \{x, y, z\}$. The set $S = \{x\}$ is a fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{y, z\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = 1 = 3 - 2 = n - 2$. Suppose that $n = 4$. Let $G = C_4$ such that $V(G) = \{x, y, w, z\}$ and $E(G) = \{xy, yw, wz, zx\}$. The set $S = \{x, y\}$ is a fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{wz\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = 2 = 4 - 2 = n - 2$. Similarly, if $n \geq 5$, then $\tilde{\gamma}_{cfd}(G) = |S| = n - 2$. Hence, $\tilde{\gamma}_{cfd}(G) = n - 2$ for all $n \geq 3$. This proves the assertion. ■

Definition 2.10 A graph $K_n = (V(K_n), E(K_n))$ is called a complete graph of order n when xy is an edge in K_n for every distinct pair $x, y \in V(K_n)$.

Definition 2.11 A complete bipartite graph is a graph whose vertex set can be partitioned into V_1 and V_2 such that every edge joins a vertex in V_1 with a vertex in V_2, and every vertex in V_1 is adjacent with every vertex in V_2.

Remark 2.12 Let $G = K_{m,n}$, $m, n \geq 1$. Then $\tilde{\gamma}_{cfd}(G) = \begin{cases} n, & if \quad m = 1 \\ m, & if \quad n = 1. \\ 2, & if\ m, n \geq 2 \end{cases}$

Proof. Suppose that $m = 1$. Let $G = K_{1,n}$ such that $V(G) = V_1 \cup V_m = \{x\} \cup \{v_1, v_2, \ldots, v_m\}$ and $E(G) = \{xv_1, xv_2, \ldots, xv_m\}$. The set $S = V_m = \{v_1, v_2, \ldots, v_m\}$ is fair dominating set of G. Since the subgraph induced by $V(G) \setminus S = \{x\}$ is connected, it follows that S is an outer-connected dominating set of G. Thus, S is an outer-connected fair dominating set of G. Suppose that S is not a minimum outer-connected dominating set of G. Then there exists $v \in S$ such that $S \setminus \{v\}$ is outer-connected dominating set of G. This is a contradiction since for any $v \in S$, $S \setminus \{v\}$ is not a dominating set of G. Thus, S is must be a minimum outer-connected dominating set of G, that is, $\tilde{\gamma}_{cfd}(G) = |S| = |\{v_1, v_2, \ldots, v_n\}| = n$. Similarly, if $n = 1$ for $G = K_{m,1}$, then $\tilde{\gamma}_{cfd}(G) = m$. Suppose that $m, n \geq 2$. Let $G = K_{m,n}$ such that $V(G) = V_m \cup V_n = \{u_1, u_2, \ldots, u_m\} \cup \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{uv: u \in V_m, v \in V_n\}$. Consider the set $S = \{u_1, v_1\}$. Since $u_1 \in V_m$, $u_1 v \in E(G)$ for all $v \in V_n$, that is, $u_1 \in V_m$ dominates V_n. Similarly, $v_1 \in V_n$ dominates V_m. This implies that $S = \{u_1, v_1\}$ is a dominating set of G, see the graph of G in Figure 4.

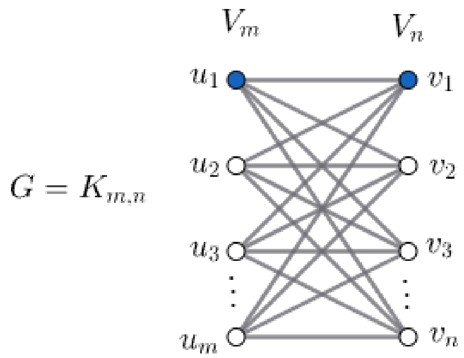

Figure 4.

Let $G = K_{m,n}$ such that $V(G) = V_m \cup V_n = \{u_1, u_2, \ldots, u_m\} \cup \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{uv: u \in V_m, v \in V_n\}$. Consider the set $S = \{u_1, v_1\}$. Since $u_1 \in V_m$, $u_1 v \in E(G)$ for all $v \in V_n$, that is, $u_1 \in V_m$ dominates V_n. Similarly, $v_1 \in V_n$ dominates V_m. This implies that $S = \{u_1, v_1\}$ is a dominating set of G. Since G cannot be dominated by a single vertex, it follows that $S = \{u_1, v_1\}$ is a minimum dominating set of G. Let $x \in V(G) \setminus S$. If $x \in V_m$, then $N_G(x) \cap S = \{v_1\}$, and if $x \in V_n$, then $N_G(x) \cap S = \{u_1\}$. Thus, $|N_G(x) \cap S| = 1$ for all $x \in V(G) \setminus S$. This implies that S is a fair dominating set of G. Further, let $u, v \in V(G) \setminus S$ such that $u \in V_m$ and $v \in V_n$. Then $uv \in E(G)$ by Definition 2.11. Thus, the subgraph induced by $V(G) \setminus S$ is connected. This implies that S is an outer-connected dominating set of G, that is, S is a minimum outer-connected fair dominating set of G. Thus, $\tilde{\gamma}_{cfd}(G)$ if $m, n \geq 2$. This completes the proofs. ■

3. Conclusion and Recommendations

In this work, we introduced a new parameter of domination in graphs - the outer-connected fair domination in graphs. The existence of a graph with outer-connected fair domination number were proven. The outer-connected fair domination number of some special graphs were computed. This study will pave a way to new research such bounds and other binary operations of two graphs. Other parameters involving outer-connected fair domination in graphs may also be explored. Finally, the characterization of an outer-connected fair domination in graphs and its bounds is a promising extension of this study.

References

1. G. Chartrand and P. Zhang, A First Course in Graph Theory. Dover Publication, Inc., New York, 2012.
2. O. Ore. Theory of Graphs. American Mathematical Society, Provedence, R.I., 1962.
3. E.J. Cockayne, and S.T. Hedetniemi Towards a theory of domination in graphs, Networks, (1977) 247-261.
4. JM.S. Leuveras, E.L.Enriquez,, Doubly Restrained Domination in Graphs, International Journal for Multidisciplinary Research (IJFMR), 2025, 7(2), pp 1-6.
5. Y.M. Bandoy, G.M. Estrada, M.L. Baterna, MK.C. Engcot, E.L. Enriquez, Identifying Restrained Domination in Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(6), pp 1-12.
6. R.B. Udtohan, E.L. Enriquez, G.M. Estrada, MC A. Bulay-og, E.M. Kiunisala, Disjoint Perfect Secure Domination in the Cartesian Product and Lexicographic Product of Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(2), pp 1-10.
7. ME.N. Diapo, G.M. Estrada, MC. A. Bulay-og, E.M. Kiunisala, E.L. Enriquez, Disjoint Perfect Domination in the Cartesian Products of Two Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(2), pp 1-7.
8. H.A. Aquiles, E.L. Enriquez, Identifying Secure Domination in the Join and Corona of Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(1), pp 1-10.
9. J.P. Dagodog, E.L. Enriquez, Secure Inverse Domination in the Join of Graphs, International Journal of Latest Engineering Research and Applications (IJLERA), 2024, 9(1), pp 105-112.
10. WR.E. Alabastro, K.B. Fuentes, G.M. Estrada, MC.A. Bulay-og, E.M. Kiunisala, E.L. Enriquez, Restrained Inverse Domination in the Lexicographic and Cartesian Products of Two Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(2), pp 1-11.
11. H.A. Aquiles, MC.A. Bulay-og, G.M. Estrada, C.M. Loquias, E.L. Enriquez, Identifying Secure Domination in the Cartesian and Lexicographic Products of Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(2), pp 1-13.
12. WR.E. Alabastro, E.L. Enriquez, Restrained Inverse Domination in the Join and Corona of Two Graphs, International Journal of Latest Engineering Research and Applications (IJLERA), 2024, 9(1), pp 57-62.
13. ME.N. Diapo, E.L. Enriquez, Disjoint Perfect Domination in Join and Corona of Two Graphs, International Journal of Latest Engineering Research and Applications (IJLERA), 2024, 9(1), pp 43-49.
14. R.B. Udtohan, E.L. Enriquez, Disjoint Perfect Secure Domination in the Join and Corona of Graphs, International Journal of Latest Engineering Research and Applications (IJLERA), 2024, 9(1), pp 63-71.
15. Gohil, HR.A. and Enriquez, E.L., Inverse Perfect Restrained Domination in Graphs, International Journal of Mathematics Trends and Technology, 2020, 66(10), pp. 1-7.

16. K.M. Cruz, E.L. Enriquez, K.B. Fuentes, G.M. Estrada, MC.A. Bulay-og, Inverse Doubly Connected Domination in the Lexicographic Product of Two Graphs, International Journal for Multidisciplinary Research (IJFMR), Vol. 6, 2024, no. 2, pp: 1-6.
17. J.P. Dagodog, E.L. Enriquez, G.M. Estrada, MC.A. Bulay-og, E.M. Kiunisala, Secure Inverse Domination in the Corona and Lexicographic Product of Two Graphs, International Journal for Multidisciplinary Research (IJFMR) Vol. 6, 2024, no. 2, pp: 1-8.
18. J. Cyman, The Outer-connected Domination Number of a graph. Australas. J.Combin., 38(2007), 35-46.
19. AR.E. Montebon, G.M. Estrada, M.L. Barterna, MK.C. Engcot, E.L. Enriquez, Outer-Connected Inverse Domination in the Corona of Two Graphs, International Journal of Latest Engineering Research and Applications (IJLERA), 2024, 9(12), pp 52-56.
20. AR.E. Montebon, E.L. Enriquez, G.M. Estrada, MK. C. Engcot, M.L. Baterna, Outer-Connected Inverse Domination in Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(6), pp 1-8.
21. C.A. Tuble, E.L. Enriquez, Outer-restrained Domination in the Join and Corona of Graphs, International Journal of Latest Engineering Research and Applications, Vol. 09, 2024, no. 01 pp 50-56.
22. C.A. Tuble, E.L. Enriquez, K.B. Fuentes, G.M. Estrada, and E.M. Kiunisala, Outer-restrained Domination in the Lexicographic Product of Two Graphs, International Journal for Multidisciplinary Research, Vol. 6, 2024, no. 2 pp 1-9.
23. E.L. Enriquez, V.V. Fernandez, J.N. Ravina Outer-clique Domination in the Corona and Cartesian Product of Graphs, Journal of Global Research in Mathematical Archives, 5(8), 2018, pp 1-7.
24. J.A. Dayap, E.L. Enriquez Outer-convex Domination in Graphs in the Composition and Cartesian Product of Graphs, Journal of Global Research in Mathematical Archives, 6(3), 2019, pp 34-42.
25. V.V. Fernandez, J.N. Ravina and E.L. Enriquez, Outer-clique domination in the Corona and Cartesian Product of Graphs, Journal of Global Research in Mathematical Archive, Vol. 5, 2018, no. 8 pp 1-7.
26. E.L. Enriquez, V.V. Fernandez, T.J. Punzalan, J.A. Dayap, Perfect outerconnected domination in the join and corona of graphs, Recoletos Multidisciplinary Research Journal, Vol. 4, 2016, no. 2 pp 1-8.
27. J.A. Dayap, E.L. Enriquez, Outer-convex domination in graphs, Discrete Mathematics, Algorithms and Applications, Vol. 12, 2020, no. 01 pp 2030008.
28. J.N. Ravina, V.V. Fernandez, E.L. Enriquez, Outer-clique Domination in Graphs, Journal of Global Research in Mathematical Archives, Vol. 5, 2018, no. 7 pp 102-107.
29. Y. Caro, A. Hansberg, and M.A. Henning, Fair Domination in Graphs. University of Haifa, 1-7, 2011.
30. V.S. Verdad, G.M. Estrada, E.M. Kiunisala, MC.A. Bulay-og, E.L. Enriquez, Inverse Fair Restrained Domination in the Join of Two Graphs, International Journal for Multidisciplinary Research (IJFMR), Vol. 6, 2024, no. 2, pp: 1-11.
31. Enriquez, E.L., Inverse Fair Domination in Join and Corona of graphs, Discrete Mathematics Algorithms and Applications 2023, 16(01), 2350003
32. V.S. Verdad, E.C. Enriquez, M.M. Bulay-og, E.L. Enriquez, Inverse Fair Restrained Domination in Graphs, Journal of Research in Applied Mathematics, Vol. 8, 2022, no. 6, pp: 09-16.
33. JN.C. Serrano, E.L. Enriquez, G.M. Estrada, MC.A. Bulay-og, E.M. Kiunisala, Fair Doubly Connected Domination in the Join of two Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(2), pp 1-11.

34. D.A. Escandallo, M.L. Baterna, MK.C. Engcot, G.M. Estrada, E.L. Enriquez, Fair Inverse Domination in the Corona of Two Connected Graphs, International Journal of Latest Engineering Research and Applications (IJLERA), 2024, 9(12), pp 47-51.
35. D.A. Escandallo, G.M. Estrada, M.L. Baterna, MK. C. Engcot, E.L. Enriquez, Fair Inverse Domination in Graphs, International Journal for Multidisciplinary Research (IJFMR), 2024, 6(6), pp 1-8.