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Identifying Inverse Domination in Graphs

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Abstract

Let $G = (V(G), E(G))$ be a nontrivial connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$. Let D be a minimum dominating set of G. If $S \subseteq V(G) \setminus D$ is a dominating set of G, then S is called an inverse dominating set with respect to D. The inverse domination number of G is the minimum cardinality of an inverse dominating set of G, denoted by $\gamma^{-1}(G)$. An identifying code of a graph G is a dominating set $C \subseteq V(G)$ such that for every $v \in V(G)$, $N_G[v] \cap C$ is distinct. The minimum cardinality of an identifying code of G, denoted by $\gamma^{ID}(G)$, is called the identifying code number of G. An inverse dominating set $S \subseteq V(G) \setminus D$ is an identifying inverse dominating set of G if for every $v \in V(G)$, $N_G[v] \cap C$ is distinct. The minimum cardinality of an identifying inverse dominating set of G, denoted by $\gamma^{ID(-1)}(G)$, is called the identifying inverse domination number of G. In this paper, we initiate the study of the concept and we show the existence of a connected graph G with $|V(G)| = n$ and $\gamma^{ID(-1)}(G) = k$ for all positive integer k. Further, we give the identifying inverse domination number of a path graph.

Keywords: dominating set, identifying code, inverse dominating set, identifying inverse dominating set

1. Introduction

A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \bigcup_{v \in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \bigcup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [res. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$]. A u-v walk in G is a sequence of vertices in G, beginning with u and ending at v such that consecutive vertices in the sequence are adjacent. A u-v walk in a graph in which no vertices are repeated is a u-v path. If G contains a u-v path, then u and v are said to be connected and u is connected to v (and v is connected to u). A graph G is connected if every two vertices of G are connected, that is, if G contains a u-v path for every pair u, v of vertices of G. Since every vertex is connected to itself, the trivial graph is connected. For the general terminology in graph theory, readers may refer to [1].

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Following an article [3] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$, that is, $N[S] = V(G)$. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [4-17].

An identifying code of a graph G is a dominating set $C \subseteq V(G)$ such that for every $v \in V(G)$, $N_G[v] \cap C$ is distinct. The minimum cardinality of an identifying code of G, denoted by $\gamma^{ID}(G)$, is called the identifying code number of G. The identifying code of a graph was studied in 1998 by M.G. Karpovsky, et.al [18] in their paper "On a new class of codes for identifying vertices in graphs". They observed that the concept of identifying codes is that a graph is identifiable if and only if it is twin-free. A vertex x is a twin of another vertex y if $N[x] = N[y]$. A graph G is called twin-free if no vertex has a twin. From a computational point of view, it is shown that given a graph G, finding the exact value of $\gamma^{ID}(G)$ is in the class of NP-hard problems. Some related studies of identifying domination in graphs are found in [19-23].

Let D be a minimum dominating set of G. If $S \subseteq V(G) \setminus D$ is a dominating set of G, then S is called an inverse dominating set with respect to D. The inverse domination number of G is the minimum cardinality of an inverse dominating set of G, denoted by $\gamma^{-1}(G)$. The inverse domination in graph was first found in the paper of Kulli [24]. Some related studies of inverse domination in graphs are found in [25-35].

Motivated by the introduction of the identifying dominating sets and the inverse dominating sets, a new variant of domination in graphs is introduced in this paper. Let $G = (V(G), E(G))$ be a nontrivial connected simple graph. An inverse dominating set $S \subseteq V(G) \setminus D$ is an identifying inverse dominating set of G if for every $v \in V(G)$, $N_G[v] \cap S$ is distinct. The minimum cardinality of an identifying inverse dominating set of G, denoted by $\gamma^{ID(-1)}(G)$, is called the identifying inverse domination number of G. In this paper, we initiate the study of the concept and we show that given positive integers k and n such that $n \geq 2$ and $2 \leq k \leq n-1$, there exists a connected graph G with $|V(G)| = n$ and $\gamma^{ID(-1)}(G) = k$. Further, we give the identifying inverse domination number of a path graph.

2. Results

Definition 2.1 A complete graph K_n is a graph in which each vertex is connected to every other vertex. The complement of a complete graph K_n is a null graph $\overline{K}_n$, a graph that does not have any edges connecting its vertices.

From the definitions, the following result is immediate.

Remark 2.2 Let G be a connected graph of order $n \geq 3$. Then $2 \leq \gamma^{ID(-1)}(G) \leq n-1$.

It is worth mentioning that the upper bound in Remark 2.1 is sharp. For example, $\gamma^{ID(-1)}(P_1 + \overline{K}_n) = n-1$ for all $n \geq 3$. The lower bound is also attainable as $\gamma^{ID(-1)}(P_3) = 2$.

Theorem 2.3 Let G be a connected graph of order $n \geq 3$. Then $\gamma^{ID(-1)}(G) = 2$ if and only if $G = P_3$,

It is easy to see that every connected graph G has an identifying inverse dominating set. The next result says that the value of the parameter $\gamma^{ID(-1)}(G)$ ranges over all positive integers $k \geq 2$.

Theorem 2.4 Given positive integers k and n such that $n \geq 3$ and $2 \leq k \leq n-1$, there exists a connected graph G with $|V(G)| = n$ and $\gamma^{ID(-1)}(G) = k$.

Proof. Consider the following cases:

Case 1. Suppose $k = 2$.

Let $G = P_3$ such that $V(G) = \{v_1, v_2, v_3\}$ and $E(G) = \{v_1v_2, v_2v_3\}$ see the graph G in Figure 1.

Figure 1

Then, $D = \{v_2\}$ is a minimum dominating set of G and $S = V(G) \setminus D = \{v_1, v_3\}$ is an inverse dominating set of G with respect to D. Since $N_G[v_1] \cap S = \{v_1\}$, $N_G[v_2] \cap S = \{v_1, v_3\}$, and $N_G[v_3] \cap S = \{v_3\}$, it follows that $N_G[v] \cap S$ is distinct for every $v \in V(G)$. By definition, S is an identifying inverse dominating set of G. Since S is clearly a minimum inverse dominating set of G with respect to D, it follows that $\gamma^{ID(-1)}(G) = |S| = 2$ and $|V(G)| = 3$.

Case 2. Suppose $k = 3$.

Let $G = P_5$ such that $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$ see the graph G in Figure 2.

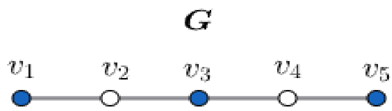

Figure 2

Then, $D = \{v_2, v_4\}$ is a minimum dominating set of G and $S = V(G) \setminus D = \{v_1, v_3, v_5\}$ is an inverse dominating set of G with respect to D. Since $N_G[v_1] \cap S = \{v_1\}$, $N_G[v_2] \cap S = \{v_1, v_3\}$, $N_G[v_3] \cap S = \{v_3\}$, $N_G[v_4] \cap S = \{v_3, v_5\}$, $N_G[v_5] \cap S = \{v_5\}$, it follows that $N_G[v] \cap S$ is distinct for every $v \in V(G)$. By definition, S is an identifying inverse dominating set of G. Since S is a minimum identifying dominating set of G, it follows that $\gamma^{ID(-1)}(G) = |S| = 3$ and $|V(G)| = 5$.

Case 3. Suppose $4 \leq k < n-1$.

Let $G = K_1 + P_{n-1}$ such that $V(K_1) = \{v_0\}$ and $V(P_{n-1}) = \{v_1, v_2, \ldots, v_{n-1}\}$ where $n = 2k$, $k \geq 4$, see the graph G in Figure 3.

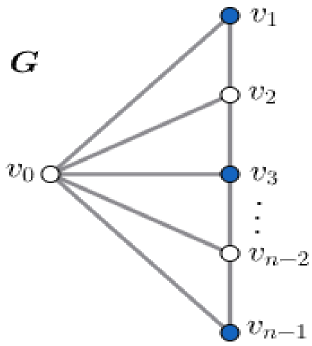

Figure 3

Then, $D = \{v_0\}$ is a minimum dominating set of G and $S = \{v_{2i-1}: i = 1,2, \dots, \frac{n}{2}\}$ is an inverse dominating set of G with respect to D. Since $N_G[v_0] \cap S = \{v_1, v_3, \dots, v_{n-1}\} = S$, $N_G[v_{2i-1}] \cap S = \{v_{2i-1}\}$ for $i = 1,2, \dots, \frac{n}{2}$, and $N_G[v_{2i}] \cap S = \{v_{2i-1}, v_{2i+1}\}$ for $i = 1,2, \dots, \frac{n-2}{2}$, it follows that $N_G[v] \cap S$ is distinct for every $v \in V(G)$. By definition, S is an identifying inverse dominating set of G. Suppose that S is not a minimum identifying inverse dominating set of G. Then there exists $v \in S$ such that $S \setminus \{v\}$ is an identifying inverse dominating set of G. But for any $v \in S$, $S \setminus \{v\}$ is not a dominating set of G contrary to our assumption that $S \setminus \{v\}$ is an identifying inverse dominating set of G. Thus, S must be a minimum identifying inverse dominating set of G, that is, $\gamma^{ID(-1)}(G) = |S| = \frac{n}{2} = \frac{2k}{2} = k$ and $|V(G)| = |V(K_1)| + |V(P_{n-1})| = 1 + (n-1) = n$.

Case 4. Suppose $k = n - 1$.

Let $G = P_1 + \overline{K}_{n-1}$ such that $V(P_1) = \{v_0\}$ and $V(\overline{K}_{n-1}) = \{v_1, v_2, \dots, v_{n-1}\}$ where $n \geq 3$, see the graph G in Figure 4.

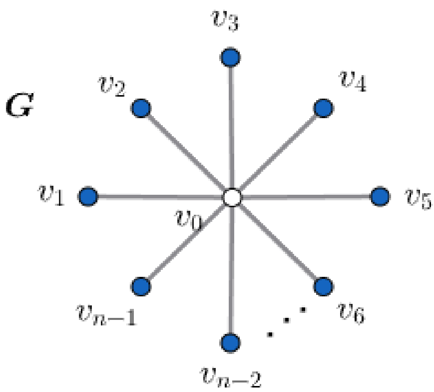

Figure 4

Then, $D = \{v_0\}$ is a minimum dominating set of G and $S = \{v_1, v_2, v_3, \dots, v_{n-1}\}$ is an inverse dominating set of G with respect to D. Since $N_G[v_0] \cap S = S$ and $N_G[v_i] \cap S = \{v_i\}$ for all $i \in \{1,2,3, \dots, n-1\}$, it follows that $N_G[v] \cap S$ is distinct for every $v \in V(G)$. By definition, S is an identifying inverse dominating set of G. Suppose that S is not a minimum identifying inverse dominating set of G. Then there exists $v \in S$ such that $S \setminus \{v\}$ is an identifying inverse dominating set of G. But for any $v \in S$, $S \setminus \{v\}$ is not a dominating set of G contrary to our assumption that $S \setminus \{v\}$ is an identifying inverse dominating set of G. Thus, S must be a minimum identifying inverse dominating set of G, that is, $\gamma^{ID(-1)}(G) = |S| = n - 1 = k$ and $|V(G)| = |V(P_1)| + |V(\overline{K}_{n-1})| = 1 + (n-1) = n$.

This proves the assertion. ■

Corollary 2.5 The difference $\gamma^{ID(-1)} - \gamma$ can be made arbitrarily large.

Proof. Let k be a positive integer. By Theorem 2.4, there exists a connected graph G such that $\gamma^{ID(-1)}(G) = k + 1$ and $\gamma(G) = 1$. Thus, $\gamma^{ID(-1)}(G) - \gamma(G) = k$. Therefore, $\gamma^{ID(-1)} - \gamma$ can be made arbitrarily large. ■

Definition 2.6 A simple graph G is an undirected graph with no loop edges or multiple edges.

Definition 2.7 The path P_n is the graph with $V(P_n) = \{a_1, a_2, a_3, \dots, a_n\}$ and $E(P_n) = \{a_1a_2, a_2a_3, \dots, a_{n-1}a_n\}$.

The following results are needed in the subsequent theorem.

Lemma 2.8 Let $G = P_n$ and let D be a minimum dominating set of G. Then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, if one of the following is satisfied for all integer $k \geq 1$.

(i) $n = 6k - 2$.
(ii) $n = 6k$.
(iii) $n = 6k + 2$.

Proof. Let $G = P_n$ such that $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{v_1v_2, v_2v_3, \dots, v_{n-1}, v_n\}$.

Suppose that statement (i) is satisfied. Then $n = 6k - 2$ for all integer $k \geq 1$. The set $D = \left\{v_{3k-1} : k = 1,2,3, \dots, \frac{n-1}{3}\right\} \cup \{v_n\} = \{v_2, v_5, v_8, \dots, v_{3k-1}, \dots v_{n-2}, v_n\}$ is the minimum dominating set of G. The set $S = V(G) \setminus D = \{v_1, v_3, v_4, v_6, v_7, \dots, v_{3k-2}, v_{3k}, \dots, v_{n-3}, v_{n-1}\}$ is an inverse dominating set of G. But S is not an identifying code of G, say, $N_G[v_3] \cap S = \{v_3, v_4\} = N_G[v_4] \cap S$. Thus, $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, if $n = 6k - 2$ for all $k \geq 1$.

Suppose that statement (ii) is satisfied. Then $n = 6k$ for all integer $k \geq 1$. The set $D = \left\{v_{3k-1} : k = 1,2,3, \dots, \frac{n}{3}\right\} = \{v_2, v_5, v_8, \dots, v_{3k-1}, \dots v_{n-1}\}$ is the minimum dominating set of G. The set $S = V(G) \setminus D = \{v_1, v_3, v_4, v_6, v_7, \dots, v_{3k-2}, v_{3k}, \dots, v_{n-2}\}$ is an inverse dominating set of G. But S is not an identifying code of G by similar arguments in statement (i). Thus, $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, if $n = 6k - 2$ for all $k \geq 1$.

Suppose that statement (iii) is satisfied. Then $n = 6k + 2$ for all integer $k \geq 1$. The set $D = \left\{v_{3k-1} : k = 1,2,3, \dots, \frac{n+1}{3}\right\} = \{v_2, v_5, v_8, \dots, v_{3k-1}, \dots v_n\}$ is the minimum dominating set of G. The set $S = V(G) \setminus D = \{v_1, v_3, v_4, v_6, v_7, \dots, v_{3k-2}, v_{3k}, \dots, v_{n-2}\}$ is an inverse dominating set of G. Similarly, S is not an identifying code of G. Thus, $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set if $n = 6k + 2$ for all $k \geq 1$.

This completes the proof. ■

Lemma 2.9 Let $G = P_n$ and let D be a minimum dominating set of G. Then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, if one of the following is satisfied for all integer $k \geq 1$.

(i) $n = 6k + 3$.
(ii) $n = 6k + 5$
(iii) $n = 6k + 7$.

Proof. Let $G = P_n$ such that $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{v_1v_2, v_2v_3, \dots, v_{n-1}, v_n\}$.

Suppose that statement (i) is satisfied. Then $n = 6k + 3$ for all integer $k \geq 1$. The set $D = \left\{v_{3k-1}: k = 1,2,3, \dots, \frac{n}{3}\right\} = \{v_2, v_5, v_8, \dots, v_{3k-1}, \dots, v_{n-1}\}$ is the minimum dominating set of G. The set $S = V(G) \setminus D = \{v_1, v_3, v_4, v_6, v_7, \dots, v_{3k-2}, v_{3k}, \dots, v_{n-2}, v_n\}$ is an inverse dominating set of G. But S is not an identifying code of G, say $N_G[v_3] \cap S = \{v_3, v_4\} = N_G[v_4] \cap S$. Thus, $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set if $n = 6k + 3$ for all $k \geq 1$.

Suppose that statement (ii) is satisfied. Then $n = 6k + 5$ for all $k \geq 1$. The set $D = \left\{v_{3k-1}: k = 1,2,3, \dots, \frac{n+1}{3}\right\} = \{v_2, v_5, v_8, \dots, v_{3k-1}, \dots, v_n\}$ is the minimum dominating set of G. The set $S = V(G) \setminus D = \{v_1, v_3, v_4, v_6, v_7, \dots, v_{3k-2}, v_{3k}, \dots, v_{n-1}\}$ is an inverse dominating set of G. But S is not an identifying code of G, by similar arguments in statement (i). Thus, $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G if $n = 6k + 5$ for all $k \geq 1$.

Suppose that statement (iii) is satisfied. Then $n = 6k + 7$ for all integer $k \geq 1$. The set $D = \left\{v_{3k-1}: k = 1,2,3, \dots, \frac{n-1}{3}\right\} \cup \{v_n\} = \{v_2, v_5, v_8, \dots, v_{3k-1}, \dots, v_{n-2}, v_n\}$ is the minimum dominating set of G. The $S = V(G) \setminus D = \{v_1, v_3, v_4, v_6, v_7, \dots, v_{3k-2}, v_{3k}, \dots, v_{n-1}\}$ is an inverse dominating set of G. Similarly, S is not an identifying code of G. Thus, $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set if $n = 6k + 5$ for all $k \geq 1$. Therefore, for all odd integer $k \geq 9$, the identifying inverse dominating set does not exist in G.

This completes the proof. ■

Theorem 3.0 Let $G = P_n, n \geq 3$. Then $\gamma^{ID(-1)}(G) = \begin{cases} \frac{n+1}{2}, & if\ n = 3\ or\ n = 5\ or\ n = 7 \\ none, & if\ otherwise \end{cases}$.

Proof. Consider the following cases.

Case 1. Suppose that $n = 3$ or $n = 5$ or $n = 7$.

Let $G = P_3$ such that $V(G) = \{x, y, z\}$ and $E(G) = \{xy, yz\}$. The set $D = \{y\}$ is a minimum dominating set of G and $S = V(G) \setminus D = \{x, z\}$ is an inverse dominating set of G with respect to D. Since $N_G[x] \cap S = \{x\}$, $N_G[y] \cap S = \{x, y\}$, and $N_G[z] \cap S = \{z\}$ it follows that for all $v \in V(G)$, $N_G[v] \cap S$ is distinct. Thus, S is an identifying inverse dominating set of G. Clearly, S is a minimum an identifying inverse dominating set of G. Thus Then $\gamma^{ID(-1)}(G) = |S| = 2 = \frac{3+1}{2} = \frac{n+1}{2}$.

Suppose that $n = 5$. Let $G = P_5$ such that $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$. The set $D = \{v_2, v_4\}$ is a minimum dominating set of G and $S = V(G) \setminus D =$

$\{v_1, v_3, v_5\}$ is an inverse dominating set of G with respect to D. Since $N_G[v_1] \cap S = \{v_1\}$, $N_G[v_2] \cap S = \{v_1, v_3\}$, $N_G[v_3] \cap S = \{v_3\}$, $N_G[v_4] \cap S = \{v_3, v_5\}$, and $N_G[v_5] \cap S = \{v_5\}$, it follows that for all $v \in V(G)$, $N_G[v] \cap S$ is distinct. Thus, S is an identifying inverse dominating set of G. Suppose that S is not a minimum identifying inverse dominating set of G. Then there exists $v \in S$ such that $S \setminus \{v\}$ is an identifying inverse dominating set of G. But for any $v \in S$, $S \setminus \{v\}$ is not a dominating set of G contrary to our assumption that $S \setminus \{v\}$ is an identifying inverse dominating set of G. Thus, S must be a minimum identifying inverse dominating set of G, that is, $\gamma^{ID(-1)}(G) = |S| = 3 = \frac{5+1}{2} = \frac{n+1}{2}$.

Suppose that $n = 7$. Let $G = P_7$ such that $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7\}$. The set $D = \{v_2, v_4, v_6\}$ is a minimum dominating set of G and $S = V(G) \setminus D = \{v_1, v_3, v_5, v_7\}$ is an inverse dominating set of G with respect to D. Since $N_G[v_1] \cap S = \{v_1\}$, $N_G[v_2] \cap S = \{v_1, v_3\}$, $N_G[v_3] \cap S = \{v_3\}$, $N_G[v_4] \cap S = \{v_3, v_5\}$, $N_G[v_5] \cap S = \{v_5\}$, $N_G[v_6] \cap S = \{v_5, v_7\}$, and $N_G[v_7] \cap S = \{v_7\}$, it follows that for all $v \in V(G)$, $N_G[v] \cap S$ is distinct. Thus, S is an identifying inverse dominating set of G. Similarly, S is a minimum identifying inverse dominating set of G, that is, $\gamma^{ID(-1)}(G) = |S| = 4 = \frac{7+1}{2} = \frac{n+1}{2}$.

Case 2. Suppose that $n \neq 3$ and $n \neq 5$ and $n \neq 7$. Let $G = P_n$ such that $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{v_1v_2, v_2v_3, \dots, v_{n-1}, v_n\}$. Let D be a minimum dominating set of G.
Consider an even integer $n \geq 4$. If $n = 6k - 2$ for all $k \geq 1$, then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G by Lemma 2.8(i). If $n = 6k$ for all $k \geq 1$, then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G by Lemma 2.8(ii). If $n = 6k$ for all $k \geq 1$, then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G by Lemma 2.8(iii). Therefore, for all even integer $k \geq 4$, the identifying inverse dominating set does not exist in G.
Consider an odd integer $n \geq 9$. If $n = 6k + 3$ for all $k \geq 1$, then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, by Lemma 2.9(i). If $n = 6k + 5$ for all $k \geq 1$, then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, by Lemma 2.9(ii). If $n = 6k + 7$ for all $k \geq 1$, then $S \subseteq V(G) \setminus D$ is not an identifying inverse dominating set of G, by Lemma 2.9(iii). Therefore, for all odd integer $k \geq 9$, the identifying inverse dominating set does not exist in G.
This completes the proof. ■

3. Conclusion and Recommendations

In this work, we introduced a new parameter of domination in graphs - the identifying inverse domination in graphs. The existence of a graph with identifying inverse domination number was proven. The identifying inverse domination number of a path graph was computed. This study will pave a way to new researches such bounds and other binary operations of two graphs. Other parameters involving identifying inverse domination in graphs may also be explored. Finally, the characterization of an identifying inverse domination in graphs and its bounds is a promising extension of this study.

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