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# Reliability And Availability Analysis of a Four-Component Repairable System with Different Failure And Repair Rates Using Markov Modeling

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**Abstract:**

Markov modelling provides an effective mathematical framework for analysing the reliability and availability of repairable systems in which component failures and repairs occur randomly over time. In the present study, a four-component repairable system with different failure rates and repair rates is considered. The system is represented by a continuous-time Markov chain (CTMC) consisting of sixteen possible states corresponding to the different combinations of good and failed states of the four components. The transition rate matrix is formulated by assigning distinct failure rates $\lambda_i$ and repair rates $\mu_i$, i=1,2,3,4, to the four components. The Kolmogorov forward equations are obtained from the transition matrix and are used to determine the state probabilities. Numerical computations are carried out for different combinations of failure and repair rates. The results demonstrate the influence of component failure nd repair rates on the performance of the repairable system. The proposed model can be useful for reliability assessment and maintenance planning of systems consisting of multiple repairable components.

**Keywords:** Markov model, repairable system, reliability, availability, failure rate, repair rate, MTBF, four-component system, Kolmogorov forward equations.

## 1. Introduction

Reliability analysis plays an important role in the design, operation and maintenance of engineering systems. In practical systems, components may fail during operation and can subsequently be restored to working condition through repair. Therefore, the reliability analysis of repairable systems requires consideration of both failure and repair processes.

Markov analysis provides a mathematical method for representing systems in terms of a finite number of states and transitions between these states. It is particularly useful for systems involving component failures, repairs, redundancy, sequencing and fault tolerance. The research paper by saritha, Thirumala Devi and Sumathi Uma Maheswari applies Markov modelling to both non-repairable and repairable systems and considers different failure and repair rates.

For a non-repairable system, once a component fails, it remains in the failed condition. Hence, measures such as reliability and mean time to failure are important. In a repairable system, however, a failed component can return to the operational condition after repair. Consequently, the system may move

repeatedly between successful and failed states. Measures such as steady-state availability and mean time between failures become important.
Previous Markov reliability studies have considered parallel redundant systems with different failure and repair rates. In the present work, the Markov approach is extended to a four-component repairable system. Since each component can be either good or failed, the system has $2^4 = 16$ possible states. Each component is assigned its own failure rate $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and repair rate $\mu_1, \mu_2, \mu_3, \mu_4$.
The transition rate matrix corresponding to these sixteen states is formulated. The Kolmogorov forward equations are then used to obtain the time-dependent state probabilities. These probabilities re used to obtain the time-dependent state probabilities. These probabilities are used to investigate the reliability, availability and MTBF of the system.
The main purpose of this study is therefore to investigate how different failure and repair rates affect the performance of a four-component repairable system.

## 2. Literature Review

Reliability analysis of repairable systems has been extensively studied using stochastic and Markov modelling techniques. Markov models are particularly useful for repairable systems because they can represent the movement of a system between different operating and failed states through failure and repair transitions.

James Li presented a Markov modelling approach for calculating the reliability of parallel redundant systems having different failure and repair rates. The study derived the mean time to failure (MTTF) for non-repairable systems and mean time between failures (MTBF) for repairable systems. The work demonstrated that Markov modelling can effectively incorporate different failure and repair rates for individual units[1].

Cherian, Jain and Sharma investigated the reliability of a standby system with repair. Their work considered standby units and repair processes and demonstrated the applicability of stochastic modelling to systems in which failed units can be restored to operation [2].

El-Damcese and Temraz developed a mathematical model for the availability nd reliability analysis of a parallel repairable system with different failure modes. Their study considered degradation and common-cause failures and used Markov and supplementary-variable techniques. An example involving four identical repairable components was also analysed[3].

El-Damcese and Temraz also studied the availability and reliability of repairable parallel systems with different failure rates. Their work used Markov models to investigate system availability, reliability and mean time to failure under different failure characteristics[4].

Saritha, Tirumla Devi and Sumathi Uma Maheswari applied Markov modelling to both non-repairable and repairable systems. For the repairable case, they considered a three-component system with different failure rates and repair rates and derived the state probabilities, steady-state availability and mean time between failures. Their study showed that Markov modelling provides a systematic method for analysing repairable systems with different component failure and repair characteristics[5].
The same study reported that, for the repairable system, increasing the failure rate reduce steady-state availability and MTBF, whereas increasing the repair rate improves these measures[5].
Sujatha, Boopathi and Kumar discussed reliability and availability analysis of standby systems with repair using Markov-chain-based differential equations. Their work also considered numerical solution techniques for the resulting reliability equations[6].

## 3. Objectives of the Study

The main objective are:

1. To develop a Markov model for a four-component repairable system.
2. To represent the system using $2^4 = 16$ possible states.
3. To assign different failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ to the four components.
4. To assign different repair rates $\mu_1, \mu_2, \mu_3, \mu_4$ .
5. To formulate the transition rate matrix of the system.
6. To derive the Kolmogorov forward equations.
7. To determine the time-dependent state probabilities.
8. To obtain the system reliability and availability measure according to the specified success and failure states.
9. To calculate the steady-state availability.
10. To calculate the mean time between failures (MTBF).
11. To study the effect of failure rates on system performance.
12. To study the effect of repair rate on system performance.
13. To present numerical results and graphical interpretations.
14.

## 4. Notations

| S.No. | Symbol | Description |
|---|---|---|
| 1 | $\lambda_i$ | Failure rate of component i |
| 2 | $\mu_i$ | Repair rate of component i |
| 3 | i=1,2,3,4 | Component number |
| 4 | $S_i$ | System state |
| 5 | $P_i(t)$ | Probability that the system is in state $S_i$ at time t |
| 6 | Q | Transition rate matrix |
| 7 | $A(t)$ | System availability at time t |
| 8 | MTBF | Mean time between failures |
| 9 | t | Operating time |

## 5. Assumptions

The following assumptions are used:

1. The system consists of four components
2. Each component has two conditions: GOOD or FAILED.
3. Hence, the system has $2^4 = 16$ possible states.
4. The failure time of each component follows an exponential distribution.
5. The repair time of each component follows an exponential distribution.
6. Failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and $\mu_1, \mu_2, \mu_3, \mu_4$ are different.
7. Only one component changes its condition during an elementary transition.
8. The system evolution satisfies the Markov property.
9. The transition rates are independent of the previous history of the system.
10. The transition matrix is constructed according to the specified sixteen-state model.

11. Initially, all four components are assumed to be good: $P_1(0)=1$ and $P_i(0)=0,\ i=2,3,4,.......,16$.

### 6. System Transition diagram

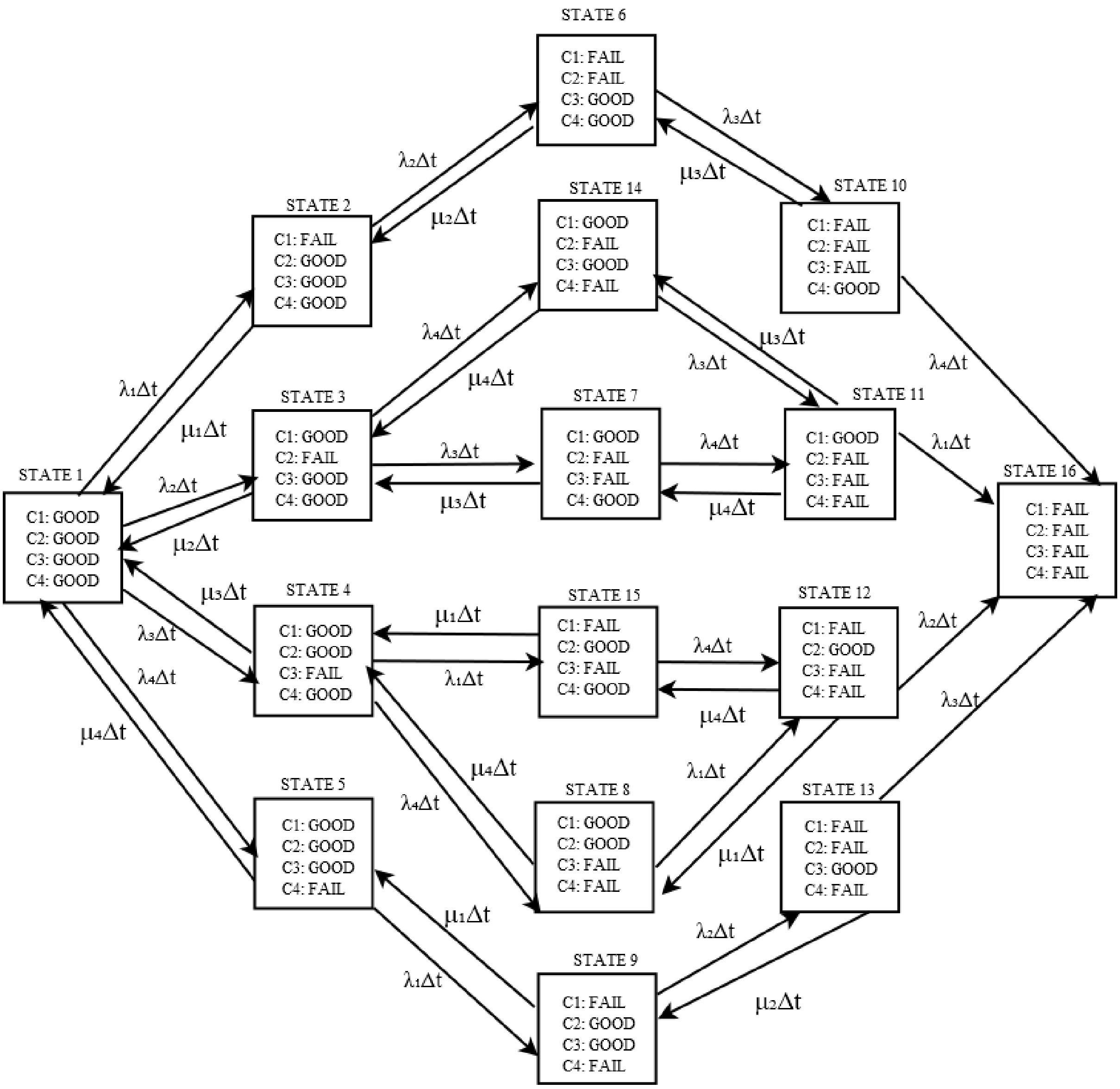


**Fig 1: System Transition Diagram**

### 7. Transition Matrix:

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | $-\begin{pmatrix}\lambda_1+\lambda_2\\+\lambda_3+\lambda_4\end{pmatrix}$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | 0 | 0 | 0 |
| $S_2$ | $\lambda_1$ | $-(\lambda_2+\mu_1)$ | 0 | 0 | 0 | $\mu_2$ | 0 | 0 |
| $S_3$ | $\lambda_2$ | 0 | $-(\lambda_3+\lambda_4+\mu_2)$ | 0 | 0 | 0 | $\mu_3$ | 0 |
| $S_4$ | $\lambda_3$ | 0 | 0 | $-(\lambda_1+\lambda_4+\mu_3)$ | 0 | 0 | 0 | $\mu_4$ |
| $S_5$ | $\lambda_4$ | 0 | 0 | 0 | $-(\lambda_1+\mu_4)$ | 0 | 0 | 0 |
| $S_6$ | 0 | $\lambda_2$ | 0 | 0 | 0 | $-(\lambda_3+\mu_2)$ | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $S_7$ | 0 | 0 | $\lambda_3$ | 0 | 0 | 0 | $-(\lambda_4+\mu_3)$ | 0 |
| $S_8$ | 0 | 0 | 0 | $\lambda_4$ | 0 | 0 | 0 | $-(\lambda_1+\mu_4)$ |
| $S_9$ | 0 | 0 | 0 | 0 | $\lambda_1$ | 0 | 0 | 0 |
| $S_{10}$ | 0 | 0 | 0 | 0 | 0 | $\lambda_3$ | 0 | 0 |
| $S_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\lambda_4$ | 0 |
| $S_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\lambda_1$ |
| $S_{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_{14}$ | 0 | 0 | $\lambda_4$ | 0 | 0 | 0 | 0 | 0 |
| $S_{15}$ | 0 | 0 | 0 | $\lambda_1$ | 0 | 0 | 0 | 0 |
| $S_{16}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $S_9$ | $S_{10}$ | $S_{11}$ | $S_{12}$ | $S_{13}$ | $S_{14}$ | $S_{15}$ |
|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_3$ | 0 | 0 | 0 | 0 | 0 | $\mu_4$ | 0 |
| $S_4$ | 0 | 0 | 0 | 0 | 0 | 0 | $\mu_1$ |
| $S_5$ | $\mu_1$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_6$ | 0 | $\mu_3$ | 0 | 0 | 0 | 0 | 0 |
| $S_7$ | 0 | 0 | $\mu_4$ | 0 | 0 | 0 | 0 |
| $S_8$ | 0 | 0 | 0 | $\mu_1$ | 0 | 0 | 0 |
| $S_9$ | $-(\lambda_2+\mu_1)$ | 0 | 0 | 0 | $\mu_2$ | 0 | 0 |
| $S_{10}$ | 0 | $-(\lambda_4+\mu_3)$ | 0 | 0 | 0 | 0 | 0 |
| $S_{11}$ | 0 | 0 | $-(\lambda_1+\mu_3+\mu_4)$ | 0 | 0 | $\lambda_3$ | 0 |
| $S_{12}$ | 0 | 0 | 0 | $-(\lambda_2+\mu_4+\mu_1)$ | 0 | 0 | $\lambda_4$ |
| $S_{13}$ | $\lambda_2$ | 0 | 0 | 0 | $-(\lambda_3+\mu_2)$ | 0 | 0 |
| $S_{14}$ | 0 | 0 | $\mu_3$ | 0 | 0 | $-(\lambda_3+\mu_4)$ | 0 |
| $S_{15}$ | 0 | 0 | 0 | $\mu_4$ | 0 | 0 | $-(\lambda_4+\mu_1)$ |
| $S_{16}$ | 0 | $\lambda_4$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | 0 | 0 |

## 8. Mathematical Formulation:

From the transition matrix, the probability of the system in state one at $t+\Delta t$ time is

$$P_1(t+\Delta t)=\left[1-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\Delta t\right]P_1(t)+\mu_1\Delta tP_2(t)+\mu_2\Delta tP_3(t)+\mu_3\Delta tP_4(t)+\mu_4\Delta tP_5(t) \qquad (1)$$

The probability of the system in state two at $t+\Delta t$ time is

$$P_2(t+\Delta t)=\lambda_1\Delta tP_1(t)+\left(1-(\lambda_2+\mu_1)\Delta t\right)P_2(t)+\mu_2\Delta tP_6(t)$$

$$=\lambda_1\Delta tP_1(t)+P_2(t)-(\lambda_2+\mu_1)\Delta tP_2(t)+\mu_2\Delta tP_6(t)$$

$$P_2(t+\Delta t)-P_2(t)=\left[\lambda_1P_1(t)-(\lambda_2+\mu_1)P_2(t)+\mu_2P_6(t)\right]\Delta t$$

$$\frac{P_2(t+\Delta t)-P_2(t)}{\Delta t}=\lambda_1P_1(t)-(\lambda_2+\mu_1)P_2(t)+\mu_2P_6(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_2(t+\Delta t)-P_2(t)}{\Delta t}=\lambda_1 P_1(t)-(\lambda_2+\mu_1)P_2(t)+\mu_2 P_6(t)$$

$$\frac{dP_2(t)}{dt}=\lambda_1 P_1(t)-(\lambda_2+\mu_1)P_2(t)+\mu_2 P_6(t)$$

Integrating above equation

$$\int_0^\infty dP_2(t)=\lambda_1\int_0^\infty P_1(t)\,dt-(\lambda_2+\mu_1)\int_0^\infty P_2(t)\,dt+\mu_2\int_0^\infty P_6(t)\,dt$$

The boundary conditions are $P_1(0)=1$ and $P_{16}(\infty)=1$, zero at all other conditions.

$$\lambda_1 T_1-(\lambda_2+\mu_1)T_2+\mu_2 T_6=0$$

Where $T_1, T_2, T_6$ are the expected time in state one, two, six respectively.

$$(\lambda_2+\mu_1)T_2=\lambda_1 T_1+\mu_2 T_6 \Rightarrow T_2=\frac{\mu_2}{\lambda_2+\mu_1}T_6+\frac{\lambda_1}{\lambda_2+\mu_1}T_1 \qquad (2)$$

The probability of the system in state three at $t+\Delta t$ time is

$$P_3(t+\Delta t)=\lambda_2\Delta t P_1(t)+\left(1-(\lambda_3+\lambda_4+\mu_2)\Delta t\right)P_3(t)+\mu_3\Delta t P_7(t)+\mu_4\Delta t P_{14}(t)$$

$$\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4+\mu_2)P_3(t)+\mu_3 P_7(t)+\mu_4 P_{14}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4+\mu_2)P_3(t)+\mu_3 P_7(t)+\mu_4 P_{14}(t)$$

$$\frac{dP_3(t)}{dt}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4+\mu_2)P_3(t)+\mu_3 P_7(t)+\mu_4 P_{14}(t)$$

Integrating above equation

$$\int_0^\infty dP_3(t)=\lambda_2\int_0^\infty P_1(t)\,dt-(\lambda_3+\lambda_4+\mu_2)\int_0^\infty P_3(t)\,dt+\mu_3\int_0^\infty P_7(t)\,dt+\mu_4\int_0^\infty P_{14}(t)\,dt$$

$$\lambda_2 T_1-(\mu_2+\lambda_3+\lambda_4)T_3+\mu_3 T_7+\mu_4 T_{14}=0$$

$$\Rightarrow T_3=\frac{\lambda_2}{\lambda_3+\lambda_4+\mu_2}T_1+\frac{\mu_3}{\lambda_3+\lambda_4+\mu_2}T_7+\frac{\mu_4}{\lambda_3+\lambda_4+\mu_2}T_{14}$$

The probability of the system in state three at $t+\Delta t$ time is

$$P_3(t+\Delta t)=\lambda_2\Delta t P_1(t)+\left(1-(\lambda_3+\lambda_4+\mu_2)\Delta t\right)P_3(t)+\mu_3\Delta t P_7(t)+\mu_4\Delta t P_{14}(t)$$

$$\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4+\mu_2)P_3(t)+\mu_3 P_7(t)+\mu_4 P_{14}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4+\mu_2)P_3(t)+\mu_3 P_7(t)+\mu_4 P_{14}(t)$$

$$\frac{dP_3(t)}{dt}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4+\mu_2)P_3(t)+\mu_3 P_7(t)+\mu_4 P_{14}(t)$$

Integrating above equation

$$\int_0^\infty dP_3(t) = \lambda_2 \int_0^\infty P_1(t)\,dt - (\lambda_3 + \lambda_4 + \mu_2)\int_0^\infty P_3(t)\,dt + \mu_3 \int_0^\infty P_7(t)\,dt + \mu_4 \int_0^\infty P_{14}(t)\,dt$$

$$\lambda_2 T_1 - (\mu_2 + \lambda_3 + \lambda_4) T_3 + \mu_3 T_7 + \mu_4 T_{14} = 0$$

$$\Rightarrow T_3 = \frac{\lambda_2}{\lambda_3 + \lambda_4 + \mu_2} T_1 + \frac{\mu_3}{\lambda_3 + \lambda_4 + \mu_2} T_7 + \frac{\mu_4}{\lambda_3 + \lambda_4 + \mu_2} T_{14} \tag{3}$$

The probability of the system in state four at $t + \Delta t$ time is

$$P_4(t + \Delta t) = \lambda_3 \Delta t P_1(t) + \left(1 - (\lambda_1 + \lambda_4 + \mu_3)\Delta t\right) P_4(t) + \mu_4 \Delta t P_8(t) + \mu\ \Delta t P_{15}(t)$$

$$\frac{P_4(t + \Delta t) - P_4(t)}{\Delta t} = \lambda_3 P_1(t) - (\lambda_1 + \lambda_4 + \mu_3) P_4(t) + \mu_4 P_8(t) + \mu_1 P_{15}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t \to 0} \frac{P_4(t + \Delta t) - P_4(t)}{\Delta t} = \lambda_3 P_1(t) - (\lambda_1 + \lambda_4 + \mu_3) P_4(t) + \mu_4 P_8(t) + \mu_1 P_{15}(t)$$

$$\frac{dP_4(t)}{dt} = \lambda_3 P_1(t) - (\lambda_1 + \lambda_4 + \mu_3) P_4(t) + \mu_4 P_8(t) + \mu_1 P_{15}(t)$$

Integrating above equation

$$\int_0^\infty dP_4(t) = \lambda_3 \int_0^\infty P_1(t)\,dt - (\lambda_1 + \lambda_4 + \mu_3)\int_0^\infty P_4(t)\,dt + \mu_4 \int_0^\infty P_8(t)\,dt + \mu_1 \int_0^\infty P_{15}(t)\,dt$$

$$\lambda_3 T_1 - (\mu_3 + \lambda_1 + \lambda_4) T_4 + \mu_4 T_8 + \mu_1 T_{15} = 0$$

$$\Rightarrow T_4 = \frac{\lambda_3}{\lambda_1 + \lambda_4 + \mu_3} T_1 + \frac{\mu_4}{\lambda_1 + \lambda_4 + \mu_3} T_8 + \frac{\mu_1}{\lambda_1 + \lambda_4 + \mu_3} T_{15} \tag{4}$$

The probability of the system in state five at $t + \Delta t$ time is

$$P_5(t + \Delta t) = \lambda_4 \Delta t P_1(t) + \left(1 - (\lambda_1 + \mu_4)\Delta t\right) P_5(t) + \mu_1 \Delta t P_9(t)$$

$$\frac{P_5(t + \Delta t) - P_5(t)}{\Delta t} = \lambda_4 P_1(t) - (\lambda_1 + \mu_4) P_5(t) + \mu_1 P_9(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t \to 0} \frac{P_5(t + \Delta t) - P_5(t)}{\Delta t} = \lambda_4 P_1(t) - (\lambda_1 + \mu_4) P_5(t) + \mu_1 P_9(t)$$

$$\frac{dP_5(t)}{dt} = \lambda_4 P_1(t) - (\lambda_1 + \mu_4) P_5(t) + \mu_1 P_9(t)$$

Integrating above equation

$$\int_0^\infty dP_5(t) = \lambda_4 \int_0^\infty P_1(t)\,dt - (\lambda_1 + \mu_4)\int_0^\infty P_5(t)\,dt + \mu_1 \int_0^\infty P_9(t)\,dt$$

$$\Rightarrow \lambda_4 T_1 - (\lambda_1 + \mu_4) T_5 + \mu_1 T_9 = 0$$

$$\Rightarrow T_5 = \frac{\lambda_4}{\lambda_1 + \mu_4} T_1 + \frac{\mu_1}{\lambda_1 + \mu_4} T_9 \tag{5}$$

The probability of the system in state six at $t + \Delta t$ time is

$$P_6(t + \Delta t) = \lambda_2 \Delta t P_2(t) + \left(1 - (\lambda_3 + \mu_2)\Delta t\right) P_6(t) + \mu_3 \Delta t P_{10}(t)$$

$$\frac{P_6(t+\Delta t)-P_6(t)}{\Delta t}=\lambda_2 P_2(t)-(\lambda_3+\mu_2)P_6(t)+\mu_3 P_{10}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_6(t+\Delta t)-P_6(t)}{\Delta t}=\lambda_2 P_2(t)-(\lambda_3+\mu_2)P_6(t)+\mu_3 P_{10}(t)$$

$$\frac{dP_6(t)}{dt}=\lambda_2 P_2(t)-(\lambda_3+\mu_2)P_6(t)+\mu_3 P_{10}(t)$$

Integrating above equation

$$\int_0^\infty dP_6(t)=\lambda_2\int_0^\infty P_2(t)\,dt-(\lambda_3+\mu_2)\int_0^\infty P_6(t)\,dt+\mu_3\int_0^\infty P_{10}(t)\,dt$$

$$\Rightarrow\ \lambda_2 T_2-(\lambda_3+\mu_2)T_6+\mu_3 T_{10}=0$$

$$\Rightarrow T_6=\frac{\lambda_2}{\lambda_3+\mu_2}T_2+\frac{\mu_3}{\lambda_3+\mu_2}T_{10} \qquad (6)$$

The probability of the system in state seven at $t+\Delta t$ time is

$$P_7(t+\Delta t)=\lambda_3\Delta t P_3(t)+\left(1-(\lambda_4+\mu_3)\Delta t\right)P_7(t)+\mu_4\Delta t P_{11}(t)$$

$$\frac{P_7(t+\Delta t)-P_7(t)}{\Delta t}=\lambda_3 P_3(t)-(\lambda_4+\mu_3)P_7(t)+\mu_4 P_{11}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_7(t+\Delta t)-P_7(t)}{\Delta t}=\lambda_3 P_3(t)-(\lambda_4+\mu_3)P_7(t)+\mu_4 P_{11}(t)$$

$$\frac{dP_7(t)}{dt}=\lambda_3 P_3(t)-(\lambda_4+\mu_3)P_7(t)+\mu_4 P_{11}(t)$$

Integrating above equation

$$\int_0^\infty dP_7(t)=\lambda_2\int_0^\infty P_3(t)\,dt-(\lambda_4+\mu_3)\int_0^\infty P_7(t)\,dt+\mu_4\int_0^\infty P_{11}(t)\,dt$$

$$\Rightarrow\ \lambda_3 T_3-(\lambda_4+\mu_3)T_7+\mu_4 T_{11}=0$$

$$\Rightarrow T_7=\frac{\lambda_3}{\lambda_4+\mu_3}T_3+\frac{\mu_4}{\lambda_4+\mu_3}T_{11} \qquad (7)$$

The probability of the system in state eight at $t+\Delta t$ time is

$$P_8(t+\Delta t)=\lambda_4\Delta t P_4(t)+\left(1-(\lambda_1+\mu_4)\Delta t\right)P_8(t)+\mu_1\Delta t P_{12}(t)$$

$$\frac{P_8(t+\Delta t)-P_8(t)}{\Delta t}=\lambda_4 P_4(t)-(\lambda_1+\mu_4)P_8(t)+\mu_1 P_{12}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_8(t+\Delta t)-P_8(t)}{\Delta t}=\lambda_4 P_4(t)-(\lambda_1+\mu_4)P_8(t)+\mu_1 P_{12}(t)$$

$$\frac{dP_8(t)}{dt}=\lambda_4 P_4(t)-(\lambda_1+\mu_4)P_8(t)+\mu_1 P_{12}(t)$$

Integrating above equation

$$\int_0^\infty dP_8(t) = \lambda_4 \int_0^\infty P_4(t)\,dt - (\lambda_1+\mu_4)\int_0^\infty P_8(t)\,dt + \mu_1 \int_0^\infty P_{12}(t)\,dt$$

$$\Rightarrow \lambda_4 T_4 - (\lambda_1+\mu_4)T_8 + \mu_1 T_{12} = 0$$

$$\Rightarrow T_8 = \frac{\lambda_4}{\lambda_1+\mu_4}T_4 + \frac{\mu_1}{\lambda_1+\mu_4}T_{12} \tag{8}$$

The probability of the system in state nine at $t+\Delta t$ time is

$$P_9(t+\Delta t) = \lambda_1 \Delta t P_5(t) + \left(1-(\lambda_2+\mu_1)\Delta t\right)P_9(t) + \mu_2 \Delta t P_{13}(t)$$

$$\frac{P_9(t+\Delta t)-P_9(t)}{\Delta t} = \lambda_1 P_5(t) - (\lambda_2+\mu_1)P_9(t) + \mu_2 P_{13}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_9(t+\Delta t)-P_9(t)}{\Delta t} = \lambda_1 P_5(t) - (\lambda_2+\mu_1)P_9(t) + \mu_2 P_{13}(t)$$

$$\frac{dP_9(t)}{dt} = \lambda_1 P_5(t) - (\lambda_2+\mu_1)P_9(t) + \mu_2 P_{13}(t)$$

Integrating above equation

$$\int_0^\infty dP_9(t) = \lambda_1 \int_0^\infty P_5(t)\,dt - (\lambda_2+\mu_1)\int_0^\infty P_9(t)\,dt + \mu_2 \int_0^\infty P_{13}(t)\,dt$$

$$\Rightarrow \lambda_1 T_5 - (\lambda_2+\mu_1)T_9 + \mu_2 T_{13} = 0$$

$$\Rightarrow T_9 = \frac{\lambda_1}{\lambda_2+\mu_1}T_5 + \frac{\mu_2}{\lambda_2+\mu_1}T_{13} \tag{9}$$

The probability of the system in state ten at $t+\Delta t$ time is

$$P_{10}(t+\Delta t) = \lambda_3 \Delta t P_6(t) + \left(1-(\lambda_4+\mu_3)\Delta t\right)P_{10}(t)$$

$$\frac{P_{10}(t+\Delta t)-P_{10}(t)}{\Delta t} = \lambda_3 P_6(t) - (\lambda_4+\mu_3)P_{10}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{10}(t+\Delta t)-P_{10}(t)}{\Delta t} = \lambda_3 P_6(t) - (\lambda_4+\mu_3)P_{10}(t)$$

$$\frac{dP_{10}(t)}{dt} = \lambda_3 P_6(t) - (\lambda_4+\mu_3)P_{10}(t)$$

Integrating above equation

$$\int_0^\infty dP_{10}(t) = \lambda_3 \int_0^\infty P_6(t)\,dt - (\lambda_4+\mu_3)\int_0^\infty P_{10}(t)\,dt$$

$$\Rightarrow \lambda_3 T_6 - (\lambda_4+\mu_3)T_{10} = 0$$

$$\Rightarrow T_{10} = \frac{\lambda_3}{\lambda_4+\mu_3}T_6 \tag{10}$$

The probability of the system in state eleven at $t+\Delta t$ time is

$$P_{11}(t+\Delta t) = \lambda_4 \Delta t P_7(t) + \left(1-(\lambda_1+\mu_3+\mu_4)\Delta t\right)P_{11}(t) + \lambda_3 \Delta t P_{14}(t)$$

$$\frac{P_{11}(t+\Delta t)-P_{11}(t)}{\Delta t}=\lambda_4 P_7(t)-(\lambda_1+\mu_3+\mu_4)P_{11}(t)+\lambda_3 P_{14}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{11}(t+\Delta t)-P_{11}(t)}{\Delta t}=\lambda_4 P_7(t)-(\lambda_1+\mu_3+\mu_4)P_{11}(t)+\lambda_3 P_{14}(t)$$

$$\frac{dP_{11}(t)}{dt}=\lambda_4 P_7(t)-(\lambda_1+\mu_3+\mu_4)P_{11}(t)+\lambda_3 P_{14}(t)$$

Integrating above equation

$$\int_0^\infty dP_{11}(t)=\lambda_4\int_0^\infty P_7(t)\,dt-(\lambda_1+\mu_3+\mu_4)\int_0^\infty P_{11}(t)\,dt+\lambda_3\int_0^\infty P_{14}(t)dt$$

$$\Rightarrow\ \lambda_4 T_7-(\lambda_1+\mu_3+\mu_4)T_{11}+\lambda_3 T_{14}=0$$

$$\Rightarrow T_{11}=\frac{\lambda_4}{\lambda_1+\mu_3+\mu_4}T_7+\frac{\lambda_3}{\lambda_1+\mu_3+\mu_4}T_{14} \qquad (11)$$

The probability of the system in state twelve at $t+\Delta t$ time is

$$P_{12}(t+\Delta t)=\lambda_1\Delta t P_8(t)+\left(1-(\lambda_2+\mu_1+\mu_4)\Delta t\right)P_{12}(t)+\lambda_4\Delta t P_{15}(t)$$

$$\frac{P_{12}(t+\Delta t)-P_{12}(t)}{\Delta t}=\lambda_1 P_8(t)-(\lambda_2+\mu_1+\mu_4)P_{12}(t)+\lambda_4 P_{15}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{12}(t+\Delta t)-P_{12}(t)}{\Delta t}=\lambda_1 P_8(t)-(\lambda_2+\mu_1+\mu_4)P_{12}(t)+\lambda_4 P_{15}(t)$$

$$\frac{dP_{12}(t)}{dt}=\lambda_1 P_8(t)-(\lambda_2+\mu_1+\mu_4)P_{12}(t)+\lambda_4 P_{15}(t)$$

Integrating above equation

$$\int_0^\infty dP_{12}(t)=\lambda_1\int_0^\infty P_8(t)\,dt-(\lambda_2+\mu_1+\mu_4)\int_0^\infty P_{12}(t)\,dt+\lambda_4\int_0^\infty P_{15}(t)dt$$

$$\Rightarrow\ \lambda_1 T_8-(\lambda_2+\mu_1+\mu_4)T_{12}+\lambda_4 T_{15}=0$$

$$\Rightarrow T_{12}=\frac{\lambda_1}{\lambda_2+\mu_1+\mu_4}T_8+\frac{\lambda_4}{\lambda_2+\mu_1+\mu_4}T_{15} \qquad (12)$$

The probability of the system in state thirteen at $t+\Delta t$ time is

$$P_{13}(t+\Delta t)=\lambda_2\Delta t P_9(t)+\left(1-(\lambda_3+\mu_2)\Delta t\right)P_{13}(t)$$

$$\frac{P_{13}(t+\Delta t)-P_{13}(t)}{\Delta t}=\lambda_2 P_9(t)-(\lambda_3+\mu_2)P_{13}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{13}(t+\Delta t)-P_{13}(t)}{\Delta t}=\lambda_2 P_9(t)-(\lambda_3+\mu_2)P_{13}(t)$$

$$\frac{dP_{13}(t)}{dt}=\lambda_2 P_9(t)-(\lambda_3+\mu_2)P_{13}(t)$$

Integrating above equation

$$\int_0^\infty dP_{13}(t)=\lambda_2\int_0^\infty P_9(t)\,dt-(\lambda_3+\mu_2)\int_0^\infty P_{13}(t)\,dt$$

$$\Rightarrow \lambda_2 T_9-(\lambda_3+\mu_2)T_{13}=0$$

$$\Rightarrow T_{13}=\frac{\lambda_2}{\lambda_3+\mu_2}T_9 \tag{13}$$

The probability of the system in state fourteen at $t+\Delta t$ time is

$$P_{14}(t+\Delta t)=\lambda_4\Delta t P_3(t)+\left(1-(\lambda_3+\mu_4)\Delta t\right)P_{14}(t)+\mu_3\Delta t P_{11}(t)$$

$$\frac{P_{14}(t+\Delta t)-P_{14}(t)}{\Delta t}=\lambda_4 P_3(t)-(\lambda_3+\mu_4)P_{14}(t)+\mu_3 P_{11}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{14}(t+\Delta t)-P_{14}(t)}{\Delta t}=\lambda_4 P_3(t)-(\lambda_3+\mu_4)P_{14}(t)+\mu_3 P_{11}(t)$$

$$\frac{dP_{14}(t)}{dt}=\lambda_4 P_3(t)-(\lambda_3+\mu_4)P_{14}(t)+\mu_3 P_{11}(t)$$

Integrating above equation

$$\int_0^\infty dP_{14}(t)=\lambda_4\int_0^\infty P_3(t)\,dt-(\lambda_3+\mu_4)\int_0^\infty P_{14}(t)\,dt+\mu_3\int_0^\infty P_{11}(t)dt$$

$$\Rightarrow \lambda_4 T_3-(\lambda_3+\mu_4)T_{14}+\mu_3 T_{11}=0$$

$$\Rightarrow T_{14}=\frac{\lambda_4}{\lambda_3+\mu_4}T_3+\frac{\mu_3}{\lambda_3+\mu_4}T_{11} \tag{14}$$

The probability of the system in state fifteen at $t+\Delta t$ time is

$$P_{15}(t+\Delta t)=\lambda_1\Delta t P_4(t)+\left(1-(\lambda_4+\mu_1)\Delta t\right)P_{15}(t)+\mu_4\Delta t P_{12}(t)$$

$$\frac{P_{15}(t+\Delta t)-P_{15}(t)}{\Delta t}=\lambda_1 P_4(t)-(\lambda_4+\mu_1)P_{15}(t)+\mu_4 P_{12}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{15}(t+\Delta t)-P_{15}(t)}{\Delta t}=\lambda_1 P_4(t)-(\lambda_4+\mu_1)P_{15}(t)+\mu_4 P_{12}(t)$$

$$\frac{dP_{15}(t)}{dt}=\lambda_1 P_4(t)-(\lambda_4+\mu_1)P_{15}(t)+\mu_4 P_{12}(t)$$

Integrating above equation

$$\int_0^\infty dP_{15}(t)=\lambda_1\int_0^\infty P_4(t)\,dt-(\lambda_4+\mu_1)\int_0^\infty P_{15}(t)\,dt+\mu_4\int_0^\infty P_{12}(t)dt$$

$$\Rightarrow \lambda_1 T_4-(\lambda_4+\mu_1)T_{15}+\mu_4 T_{12}=0$$

$$\Rightarrow T_{15}=\frac{\lambda_1}{\lambda_4+\mu_1}T_4+\frac{\mu_4}{\lambda_4+\mu_1}T_{12} \tag{15}$$

The expressions for $T_2,T_3,T_4,T_5,T_6,T_7,T_8,T_9,T_{10},T_{11},T_{12},T_{13},T_{14},T_{15}$ in terms of $T_1$ are

$$T_2=\frac{\lambda_1(\lambda_3+\lambda_4)+\mu_2(\lambda_4+\mu_3)}{\mu_1\mu_2(\mu_3+\lambda_4)+\lambda_3\lambda_4\mu_1+\lambda_2\lambda_3\lambda_4}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then $T_2=\dfrac{\lambda\left(2\lambda+\mu(\lambda+\mu)\right)}{\mu^2(\mu+\lambda)+\lambda^2\mu+\lambda^3}T_1$ (16)

$$T_3=\frac{\lambda_2\left(\lambda_1(\lambda_4+\mu_3)(\lambda_3+\mu_4)+\mu_3\mu_4(\lambda_3+\lambda_4+\mu_3+\mu_4)\right)}{\left(\begin{array}{l}\lambda_1\lambda_3\left(\lambda_3\lambda_4+(\lambda_4+\mu_2)(\lambda_4+\mu_3)\right)+\lambda_1\left(\lambda_3\lambda_4+\mu_2(\lambda_4+\mu_3)\right)\mu_4\\+\mu_2\mu_3\mu_4(\lambda_3+\lambda_4+\mu_3+\mu_4)\end{array}\right)}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then

$$T_3=\frac{\lambda\left(\lambda(\lambda+\mu)^2+\mu^2(2\lambda+2\mu)\right)}{\left(\lambda^2\left(\lambda^2+(\lambda+\mu)^2\right)+\lambda\left(\lambda^2+\mu(\lambda+\mu)\right)\mu+\mu^3(2\lambda+2\mu)\right)}T_1 \tag{17}$$

$$T_4=\frac{\lambda_3\left(\lambda_1\lambda_2(\lambda_4+\mu_1)+\lambda_1\mu_1\mu_4+\mu_4\left((\lambda_2+\mu_1)(\lambda_4+\mu_1)+\mu_1\mu_4\right)\right)}{\begin{array}{l}\lambda_1^2\lambda_2\lambda_4+\lambda_1\lambda_2(\lambda_4+\mu_1)(\lambda_4+\mu_3)+\lambda_1(\lambda_2\lambda_4+\mu_1\mu_3)\mu_4\\+\mu_3\mu_4\left((\lambda_2+\mu_1)(\lambda_4+\mu_1)+\mu_1\mu_4\right)\end{array}}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then

$$T_4=\frac{\lambda\left(\lambda^2(\lambda+\mu)+\lambda\mu^2+\mu\left((\lambda+\mu)^2+\mu^2\right)\right)}{\lambda^4+\lambda^2(\lambda+\mu)^2+\lambda\left(\lambda^2+\mu^2\right)\mu+\mu^2\left((\lambda+\mu)^2+\mu^2\right)}T_1 \tag{18}$$

$$T_5=\frac{\lambda_4\left(\lambda_2\lambda_3+\mu_1(\lambda_3+\mu_2)\right)}{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_4+\mu_1(\lambda_3+\mu_2)\mu_4}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then $T_5=\dfrac{\lambda\left(\lambda^2+\mu(\lambda+\mu)\right)}{\lambda^3+\lambda^2\mu+\mu(\lambda+\mu)\mu}T_1$ (19)

$$T_6=\frac{\lambda_1\lambda_2(\lambda_4+\mu_3)}{\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\mu_1+\mu_1\mu_2(\lambda_4+\mu_3)}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then $T_6=\dfrac{\lambda^2(\lambda+\mu)}{\lambda^3+\lambda^2\mu+\mu^2(\lambda+\mu)}T_1$ (20)

$$T_7=\frac{\lambda_2\lambda_3\left(\lambda_1(\lambda_3+\mu_4)+\mu_4(\lambda_3+\lambda_4+\mu_3+\mu_4)\right)}{\begin{array}{l}\lambda_1\lambda_3\left(\lambda_3\lambda_4+(\lambda_4+\mu_2)(\lambda_4+\mu_3)\right)+\lambda_1\left(\lambda_3\lambda_4+\mu_2(\lambda_4+\mu_3)\right)\mu_4\\+\mu_2\mu_3\mu_4(\lambda_3+\lambda_4+\mu_3+\mu_4)\end{array}}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then

$$T_7=\frac{\lambda^2\left(\lambda(\lambda+\mu)+\mu(2\lambda+2\mu)\right)}{\lambda^2\left(\lambda^2+(\lambda+\mu)^2\right)+\lambda\left(\lambda^2+\mu(\lambda+\mu)\right)\mu+\mu^3(2\lambda+2\mu)}T_1 \tag{21}$$

$$T_8=\frac{\lambda_3\lambda_4\left(\lambda_2(\lambda_4+\mu_1)+\mu_1(\lambda_1+\lambda_4+\mu_1+\mu_4)\right)}{\begin{array}{l}\lambda_1^2\lambda_2\lambda_4+\lambda_1\lambda_2(\lambda_4+\mu_1)(\lambda_4+\mu_3)+\lambda_1(\lambda_2\lambda_4+\mu_1\mu_3)\mu_4\\+\mu_3\mu_4(\lambda_2+\mu_1)(\lambda_4+\mu_1)+\mu_1\mu_4\end{array}}T_1$$

If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ then

$$T_8 = \frac{\lambda^2\left(\lambda(\lambda+\mu)+\mu(2\lambda+2\mu)\right)}{\lambda^4+\lambda^2(\lambda+\mu)^2+\lambda\left(\lambda^2+\mu^2\right)\mu+\mu^2(\lambda+\mu)^2+\mu^2}T_1 \quad (22)$$

$$T_9 = \frac{\lambda_1\lambda_4(\lambda_3+\mu_2)}{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_4+\mu_1(\lambda_3+\mu_2)\mu_4}T_1$$

If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ then $T_9 = \dfrac{\lambda^2(\lambda+\mu)}{\lambda^3+\lambda^2\mu+\mu(\lambda+\mu)\mu}T_1$ (23)

$$T_{10} = \frac{\lambda_1\lambda_2\lambda_3}{\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\mu_1+\mu_1\mu_2(\lambda_4+\mu_3)}T_1$$

If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ then $T_{10} = \dfrac{\lambda^3}{\lambda^3+\lambda^2\mu+\mu^2(\lambda+\mu)}T_1$ (24)

$$T_{11} = \frac{\lambda_2\lambda_3\lambda_4(\lambda_3+\lambda_4+\mu_3+\mu_4)}{\begin{array}{l}\lambda_1\lambda_3\left(\lambda_3\lambda_4+(\lambda_4+\mu_2)(\lambda_4+\mu_3)\right)+\lambda_1\left(\lambda_3\lambda_4+\mu_2(\lambda_4+\mu_3)\right)\mu_4\\ +\mu_2\mu_3\mu_4(\lambda_3+\lambda_4+\mu_3+\mu_4)\end{array}}T_1$$

If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ then

$$T_{11} = \frac{\lambda^3(2\lambda+2\mu)}{\lambda^2\left(\lambda^2+(\lambda+\mu)^2\right)+\lambda\left(\lambda^2+\mu(\lambda+\mu)\right)\mu+\mu^3(2\lambda+2\mu)}T_1 \quad (25)$$

$$T_{12} = \frac{\lambda_1\lambda_3\lambda_4(\lambda_1+\lambda_4+\mu_1+\mu_4)}{\begin{array}{l}\lambda_1^2\lambda_2\lambda_4+\lambda_1\lambda_2(\lambda_4+\mu_1)(\lambda_4+\mu_3)+\lambda_1(\lambda_2\lambda_4+\mu_1\mu_3)\mu_4\\ +\mu_3\mu_4\left((\lambda_2+\mu_1)(\lambda_4+\mu_1)+\mu_1\mu_4\right)\end{array}}T_1$$

If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ then

$$T_{12} = \frac{\lambda^3(2\lambda+2\mu)}{\lambda^4+\lambda^2(\lambda+\mu)^2+\lambda\left(\lambda^2+\mu^2\right)\mu+\mu^2\left((\lambda+\mu)^2+\mu^2\right)}T_1 \quad (26)$$

$$T_{13} = \frac{\lambda_1\lambda_2\lambda_4}{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_4+\mu_1(\lambda_3+\mu_2)\mu_4}T_1$$

If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ then $T_{13} = \dfrac{\lambda^3}{\lambda^3+\lambda^2\mu+\mu^2(\lambda+\mu)}T_1$ (27)

$$T_{14}=\frac{\lambda_2\lambda_4\left(\lambda_1\left(\lambda_4+\mu_3\right)+\mu_3\left(\lambda_3+\lambda_4+\mu_3+\mu_4\right)\right)}{\lambda_1\lambda_3\left(\lambda_3\lambda_4+\left(\lambda_4+\mu_2\right)\left(\lambda_4+\mu_3\right)\right)+\lambda_1\left(\lambda_3\lambda_4+\mu_2\left(\lambda_4+\mu_3\right)\right)\mu_4+\mu_2\mu_3\mu_4\left(\lambda_3+\lambda_4+\mu_3+\mu_4\right)}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then

$$T_{14}=\frac{\lambda^2\left(\lambda\left(\lambda+\mu\right)+\mu\left(2\lambda+2\mu\right)\right)}{\lambda^2\left(\lambda^2+\left(\lambda+\mu\right)^2\right)+\lambda\left(\lambda^2+\mu\left(\lambda+\mu\right)\right)\mu+\mu^3\left(2\lambda+2\mu\right)}T_1 \tag{28}$$

$$T_{15}=\frac{\lambda_1\lambda_3\left(\lambda_1\left(\lambda_2+\mu_4\right)+\mu_4\left(\lambda_2+\lambda_4+\mu_1+\mu_4\right)\right)}{\lambda_1^2\lambda_2\lambda_4+\lambda_1\lambda_2\left(\lambda_4+\mu_1\right)\left(\lambda_4+\mu_3\right)+\lambda_1\left(\lambda_2\lambda_4+\mu_1\mu_3\right)\mu_4+\mu_3\mu_4\left(\lambda_2+\mu_1\right)\left(\lambda_4+\mu_1\right)+\mu_1\mu_4}T_1$$

If $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda$ and $\mu_1=\mu_2=\mu_3=\mu_4=\mu$ then

$$T_{15}=\frac{\lambda^2\left(\lambda\left(\lambda+\mu\right)+\mu\left(2\lambda+2\mu\right)\right)}{\lambda^4+\lambda^2\left(\lambda+\mu\right)^2+\lambda\left(\lambda^2+\mu^2\right)\mu+\mu^2\left(\lambda+\mu\right)^2+\mu^2}T_1 \tag{29}$$

The probability of the system in state sixteen at $t+\Delta t$ time is

$$P_{16}\left(t+\Delta t\right)=\lambda_4\Delta tP_{10}\left(t\right)+\lambda_1\Delta tP_{11}\left(t\right)+\lambda_2\Delta tP_{12}\left(t\right)+\lambda_3\Delta tP_{13}\left(t\right)$$

$$\frac{P_{16}\left(t+\Delta t\right)-P_{16}\left(t\right)}{\Delta t}=\lambda_4P_{10}\left(t\right)+\lambda_1P_{11}\left(t\right)+\lambda_2P_{12}\left(t\right)+\lambda_3P_{13}\left(t\right)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_{16}\left(t+\Delta t\right)-P_{16}\left(t\right)}{\Delta t}=\lambda_4P_{10}\left(t\right)+\lambda_1P_{11}\left(t\right)+\lambda_2P_{12}\left(t\right)+\lambda_3P_{13}\left(t\right)$$

$$\frac{dP_{16}\left(t\right)}{dt}=\lambda_4P_{10}\left(t\right)+\lambda_1P_{11}\left(t\right)+\lambda_2P_{12}\left(t\right)+\lambda_3P_{13}\left(t\right)$$

Integrating above equation

$$\int_0^\infty dP_{16}\left(t\right)=\lambda_4\int_0^\infty P_{10}\left(t\right)dt+\lambda_1\int_0^\infty P_{11}\left(t\right)+\lambda_2\int_0^\infty P_{12}\left(t\right)+\lambda_3\int_0^\infty P_{13}\left(t\right)$$

$$\Rightarrow 1=\lambda_4T_{10}+\lambda_1T_{11}+\lambda_2T_{12}+\lambda_3T_{13}$$

Substituting $T_{10},T_{11},T_{12},T_{13}$ values in the above equation then we get

$$T_1=\frac{\left(\lambda+\mu\right)\left(\lambda^2+\mu^2\right)\left(2\lambda^2-\lambda\mu+2\mu^2\right)}{2\lambda^4\left(4\lambda^2-\lambda\mu+4\mu^2\right)} \tag{30}$$

The MTBF is the sum of the expected time in state 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15, if all the 4-components have the same failure rate and repair rate, we get

$$MTBF=T_1+T_2+T_3+T_4+T_5+T_6+T_7+T_8+T_9+T_{10}+T_{11}+T_{12}+T_{13}+T_{14}+T_{15}$$

$$=\frac{22\lambda^5+9\lambda^4\mu+27\lambda^3\mu^2+15\lambda^2\mu^3+9\lambda\mu^4+2\mu^5}{2\lambda^4\left(4\lambda^2-\lambda\mu+4\mu^2\right)} \tag{31}$$

**9. Mean time to repair (MTTR):**

The MTTR is the basic measure of the maintainability of repairable items. It represents the average time required to repair a failed component or device.

$$MTTR = \int_0^\infty \left[1 - M(t)\right] dt$$

Where $M(T) = 1 - e^{-\int_0^t \mu(t)dt}$

If the repair rate $\mu(t)$ is constant and is equal to $\mu$ then $MTTR = \frac{1}{\mu}$

In this case the repair rates $\mu_1, \mu_2, \mu_3, \mu_4$ are constants and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$, then

$$MTTR = \frac{1}{4\mu} \tag{32}$$

The steady state availability is

$$A(t \to \infty) = \frac{MTBF}{MTBF + MTTR}$$

On simplification, we get

$$A = \frac{2\mu\left(22\lambda^5 + 9\lambda^4\mu + 27\lambda^3\mu^2 + 15\lambda^2\mu^3 + 9\lambda\mu^4 + 2\mu^5\right)}{4\lambda^6 + 43\lambda^5\mu + 22\lambda^4\mu^2 + 54\lambda^3\mu^3 + 30\lambda^2\mu^4 + 18\lambda\mu^5 + 4\mu^6} \tag{33}$$

**10. Numerical Results:**4-component repairable system with same failure rate and repair rates

| S.No. | $\lambda$ | $\mu$ | $A(t \to \infty)$ | MTBF |
|---|---|---|---|---|
| 1. | 0.02 | 0.1 | 0.999317 | 3659.6 |
| 2. | 0.04 | 0.1 | 0.994699 | 469.104 |
| 3. | 0.06 | 0.1 | 0.985628 | 171.452 |
| 4. | 0.08 | 0.1 | 0.973609 | 92.2295 |
| 5. | 0.1 | 0.1 | 0.96 | 60.0 |
| 6. | 0.12 | 0.1 | 0.945664 | 43.5102 |
| 7. | 0.14 | 0.1 | 0.931098 | 33.7832 |
| 8. | 0.16 | 0.1 | 0.916575 | 27.4671 |
| 9. | 0.18 | 0.1 | 0.902248 | 23.075 |

**Table-1**

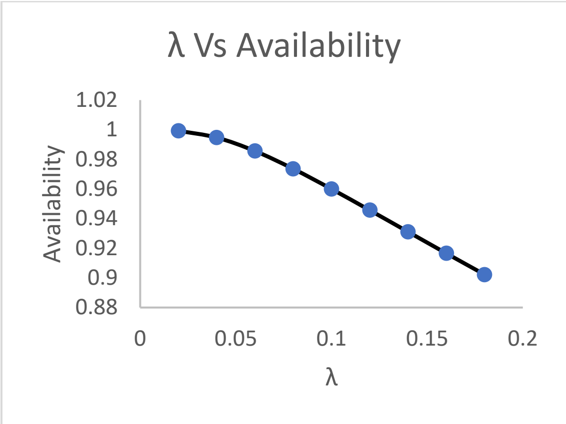


Fig.2

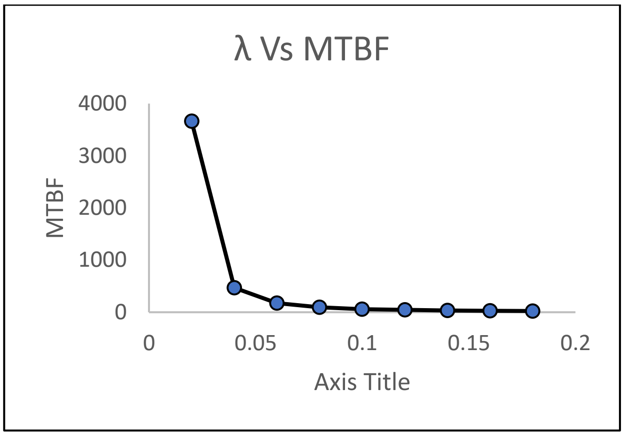


Fig.3

| S.No. | $\lambda$ | $\mu$ | $A(t \to \infty)$ | MTBF |
|---|---|---|---|---|
| 1. | 0.05 | 0.1 | 0.990635 | 264.444 |
| 2. | 0.05 | 0.12 | 0.994096 | 350.784 |
| 3. | 0.05 | 0.14 | 0.996101 | 456.252 |
| 4. | 0.05 | 0.16 | 0.997326 | 582.76 |
| 5. | 0.05 | 0.18 | 0.998107 | 732.224 |
| 6. | 0.05 | 0.2 | 0.998623 | 906.563 |
| 7. | 0.05 | 0.22 | 0.998975 | 1107.7 |
| 8. | 0.05 | 0.24 | 0.999222 | 1337.55 |
| 9. | 0.05 | 0.26 | 0.999399 | 1598.03 |

**Table-2**

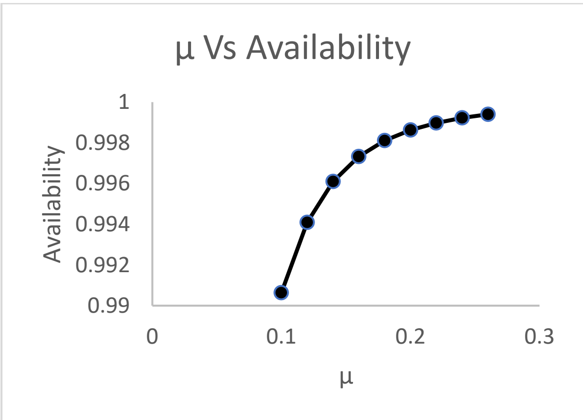


Fig.4

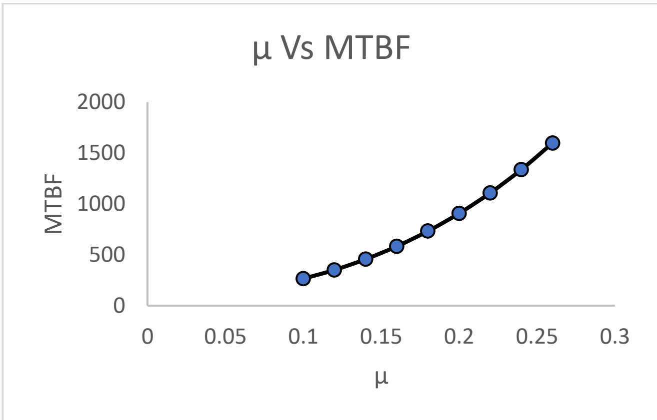


Fig.5

**11. Conclusion:**

The numerical results for the 4-component repairable system with identical failure rate and repair rate show the expected influence of the failure and repair rates on the system performance.

When the repair rate $\mu$ is fixed at 0.1 and the failure rate $\lambda$ is increased from 0.02 to0.18, the steady-state availability $A(t \to \infty)$ decreases from 0.999317 to 0.902248. At the same time, the MTBF decreases significantly from 3659.6 to23.075. The indicates that an increase in the component failure rate adversely affects both system availability and mean time between failures.

On the other hand, when the failure rate $\lambda$ is fixed at 0.05 and the repair rate $\mu$ 0.10 to 0.26, the availability increases from 0.990635 to 0.999399. Th MTBF also increases from 264.444 to 1598.03. Thus, an increase in the repair rate considerably improves the system performance by reducing the time for which failed components remain unavailable.

Therefore, the numerical results obtained from the newly derived expressions for $A(t \to \infty)$ and MTBF demonstrate that higher repair rates improve system reliability and availability, where as higher failure rates reduce them. The results also show that the effect of the repair rate becomes particularly significant as the repair capability of the system is increased.

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