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Numerical Study of Non-Linear Three-Phase-Lag Model for Heat Transfer in Tissue During Thermal Therapy

Neha Sharma[1], Surjan Singh[2], Dinesh Kumar[3]

[1,2]Department of Mathematics, Eternal University, Baru Sahib (H.P.), India
[3]Department of Mathematics, Government Polytechnic College, Nawada, India

Abstract
In this article, the simulation based mathematical modelling on bioheat transfer in living tissue is studied by using non-linear three-phase-lag bioheat transfer model (TPLBHT). The TPLBHT model includes phase lag time due to heat flux, temperature gradient and temperature displacement with energy balance equation. A hybrid numerical method is used for solution which is based on Finite-Difference method with central difference method and Runge-Kutta (4, 5) method to find the solution. The effect of dimensionless temperature with dimensionless time for different parameters like blood perfusion coefficient, metabolic heat source coefficient, phase-lag time parameters and parameters of TPL model are discussed. The full article is studied and presented in the dimensionless form.

Keywords: Bioheat, Blood, Metabolism, Method, Perfusion, Finite, Phase, Tissue

1. Introduction

The mathematical modelling of bioheat transfer which is given by Pennes bioheat transfer model [1] with infinite speed of thermal signal is based on Fourier law of heat conduction is given by

$$\boldsymbol{q}(\boldsymbol{x},\boldsymbol{t}) = -\boldsymbol{K}\boldsymbol{\nabla}\boldsymbol{T}(\boldsymbol{x},\boldsymbol{t}) \quad (1)$$

where $q(x,t)$ is heat flux; K is thermal conductivity; temperature of tissue is T. When the heat moves between blood and tissue, then it takes a finite gap therefore the lagging behavior exist. Cattaneo [2] and Vernotte [3] independently offered a relaxation time (τ_q) which is due to heat flux to control the inconsistency which takes place due to infinite speed gap of thermal signal which is titled as Single-Phase-Lag (SPL) constitutive relation and stated as:

$$q(x,t+\tau_q) = -K\nabla T(x,t) \quad (2)$$

The combination of SPL relation with energy balance equation gives bioheat model of thermal wave. The SPL model was one more time studied by Tzou [4] and gives another phase lag time due to temperature gradient (τ_T) and known as Dual-Phase-Lag (DPL) constitutive relation is as follows:

$$q(x,t+\tau_q) = -K\nabla T(x,t+\tau_T) \quad (3)$$

When DPL relation combined with energy balance equation then it becomes DPLBHT model. Several researcher [5–12] explained the third phase lag τ_v and added in DPL constitutive relation which is due temperature displacement gradient known as TPL constitutive relation i.e.

$$q(x,t+\tau_q) = -[K\nabla T(x,t+\tau_T) + K^*\nabla v(x,t+\tau_v)] \quad (4)$$

where $v(x,t)$ is thermal displacement and $\frac{\partial v(x,t)}{\partial t} = T(x,t)$, K^* is the rate of thermal conductivity of living tissue. We used the expansion of Taylor's series of TPL model (4) upto first order at time t in the problem which is as:

$$\left(1+\tau_q\frac{\partial}{\partial t}\right)q(x,t) = -\left[(\mathrm{K}+\mathrm{K}^*)\nabla T(x,t) + K\tau_T\frac{\partial}{\partial t}\nabla T(x,t) + K^*\nabla v(x,t)\right], \quad (5)$$

Pal (1993) [13] presented the mathematical modelling of the results of metabolic heat production in a two dimensional model of human skin and subcutaneous tissues and flow of blood. For the solution of transient and non-linear 2D bioheat transfer, the dual reciprocity boundary element method was used in heat flux on the skin surface is studied by Deng (2000) [14].Coupled thermo elasticity was developed by using TPL model containing three phase lag time because of heat flux, temperature gradient and thermal displacement gradient by Choudhuri (2007) [15]. Quintanilla [16] discussed the stability in the TPL thermal conduction equation and also shows the effects of different parameters values. Zhang (2009)[17] modified the classical PBHT equation and developed the new DPL model in which the phase-lag times are indicated in the properties of blood and tissue. Ferreira (2009)[8] established the revised model of thermal system of human. To solve real- life problem, the features combined which are 3D heat conductions, the applications of elliptical cylinders to adequately approximate body geometry, the representation of tissues and organs, and the flexibility of the computational applications. Afrin(2011)[18] presented the transfer of heat amidst tissue, blood of venous and arterial tissue by using DPLBHT model for living tissues. They found that when the tissue and blood flow have unique properties then the phase lag because of heat flux and temperature gradient are equivalent. Ahmadikia (2012)[19] compared the hyperbolic and parabolic bioheat transfer models with different boundary conditions and solved by Laplace transform method. A generalized DPL bioheat model is used to examine thermal damage to show the effect of laser irradiation is studied by Afrin (2012) [20]. Hosseininia (2012) [21] focused the mathematical modelling of hyperthermia therapy by considering the bioheat transfer model in the living tissue. They also introduced the 2D-transient, DPL model, variable-order fractional energy equation and used the 2D Legendre wavelets for the solution. The transfer of bioheat process of bioheat transfer with different conditions of blood perfusion rate under the coordinate system and boundary conditions by using radiation of electromagnetic and solved by FDM and Adomian de- composition method is described by Gupta (2013)[22]. Zhang (2014)[23] developed the procedure of fundamental solution coupling with the dual reciprocity method for solution of non-linear steady state bioheat transfer problem. The TPL model is considered for the problem of reflection and refraction because of longitudinal and transverse wave in between uniform elastic solid half-space and thermo elastic solid by Kumar (2013) [24]. Askarizadeh (2014) [25] solved the DPLBHT equation of heat transfer problems inside skin tissue under the pulse train and heat flux periodicity. The differential equations are solved by using Laplace transform and inverse Laplace Transform for results in time domain. The DPLBHT model has been studied by Kumar (2015)[26] by taking Gaussian distribution source underneath the boundary condition for the therapy of hyperthermia. The Finite element wavelet Galerkin method taking the Legendre wavelet as a basis function was used for the solution of the problem. Ghazanfarian (2015)[27] discretized the non-linear PBHT equation and DPL model by taking a mesh-free SPH procedure. They also investigated the temperature distribution in living tissues and

observed the effect of non-linearity in the PDE. Kumar (2016) [28] theoretical study of the DPLBHT model during thermal therapy in tissues underneath the different non-Fourier boundary condition in various coordinate system. The freezing of biological living tissue is discussed by Mochnacki (2017) [29]. They used the DPL model for explaining the thermal interaction in between the soft tissues and tip of cryoprobe. Afrin (2017) [30] reviewed the thermal damage in biological tissue with laser irradiation and showed the effect of uncertainty of phase lag times, scattering coefficients, blood perfusion coefficient etc. For the treatment of infected cells, the simulation based modelling of bioheat transfer in living tissue by using DPLBHT model underneath Dirichlet boundary conditions is studied by Kumar et al.(2018) [31]. They used the FDM and RK (4, 5) schemes for the solution of non-linear problem. Dutta (2018) [32] studied about an analytical solution for 2D thermal field of single layer living biological tissues of Fourier and non-Fourier heat transfer. In conjunction, Laplace Transform Method with the Inversion Theorem is used for the analytical solution. Sharma (2020)[33] studied the simulation based mathematical modelling of non-linear DPLBHT model for examine the temperature in tissue during hyperthermia therapy. They used the hybrid method to solve the problem and also observed the thermal damage of normal tissue. Hobiny (2020) [34] proposed the analytical method solved with Laplace transforms and estimation of experimental data of thermal damages and temperature because of laser irradiation by utilizing measurement in formation of skin surface. Saeed (2020) [35] studied the effect of laser heat source in the spherical tissue which is based on DPLBHT model. They adopted the finite difference method for the solution of bioheat model in the spherical biological tissues. Shah et.al. [36] solved the DPLBHT model for the case of hyperthermia treatment. They used the algorithm of Haar wavelet operational matrix under the different types of boundary conditions. Sharma (2021)[37] described the simulation based on mathematical modelling of bioheat transfer in tissue underneath periodic boundary condition in DPLBHT. Kumar (2020)[5] introduced the TPLBHT model by considering the TPL time because of heat flux, temperature gradient and thermal displacement. They applied the Finite element Legendre Wavelet Galerkin method for the solution of TPLBHT model and compared with experimental data. Akbarzadeh (2014) [38] studied the heat conduction in hollow cylinder which is based on the TPL model. Hobiny et al [39] presented the TPL model of thermo-elastic interactions in the 2D porous medium due to pulse heat flux. In this research paper, the simulation based on mathematical modelling of bioheat transfer in tissue is dealt with by taking non-linear TPLBHT model. The TPLBHT model includes three phase lag times which are due to heat flux, temperature gradient and temperature displacement. We considered the metabolic heat source, blood perfusion heat source which are experimentally validated temperature dependent parameters. The whole problem is converted into dimensionless form. For the solution of the problem, a hybrid numerical method is used which is based on Finite- Difference method and Runge-Kutta (4, 5) method. The effect of dimensionless temperature for different parameters like blood perfusion coefficient, metabolic heat source coefficient, phase-lag time parameters and other parameters of TPL model are reviewed.

Formulation of the problem

In this research paper, we considered a one dimensional inner structure of living skin tissue of length L with initial temperature T_o. The outer surface of skin tissue (x=L) and inner boundary of skin tissue is insulated ($x = 0$) is shown in Fig.1.

The one-dimensional energy balance equation is generally conducted by PBHT equation [1] is

$$\rho c \frac{\partial T(x,t)}{\partial t} = -\nabla q(x,t) + q_m + q_b, \quad (6)$$

Where ρ is the density of tissues; c is the specific heat of local tissues; t is time; $q(x,t)$ is the heat flux; q_b and q_m are the blood perfusion heat source and metabolic heat source respectively. Metabolic heat source is the temperature depended which is the source of local tissue temperature given as [5,40]:

$$q_m = q_{m0} \times \left[1 + \left(\frac{T-T_0}{10}\right)\right], \quad (7)$$

where q_{m0} is the reference heat source term. The blood perfusion source can be given as [23,31]:

$$q_b = w_b \rho_b c_b (T_b - T), \quad (8)$$

where ρ_b, c_b, w_b are the density of blood, specific heat of blood and blood perfusion rate coefficient respectively and T_b is temperature of blood. By using Eqn (5) and Eqn (6), terminating $q(x,t)$ which gives:

$$\tau_q \rho c \frac{\partial^3 T(x,t)}{\partial t^3} + \rho c \frac{\partial^2 T(x,t)}{\partial t^2} - \tau_q \frac{\partial^2 q_b}{\partial t^2} - \tau_q \frac{\partial^2 q_m}{\partial t^2} - \frac{\partial q_b}{\partial t} - \frac{\partial q_m}{\partial t}$$
$$= \left[K^* + (K + K^* \tau_v)\frac{\partial}{\partial t} + K\tau_T\right] \nabla^2 T(x,t), \quad (9)$$

Subject to initial conditions

$$T(x,0) = T_0, \quad \frac{\partial T(x,t)}{\partial t} = 0 \quad and \quad \frac{\partial^2 T(x,t)}{\partial t^2} = 0 \quad (10)$$

Boundary condition

$$T(0,t) = T_w \quad (11)$$

Inner boundary is insulated, therefore the heat flux at boundary is zero, i.e.,

$$\frac{\partial T(L,t)}{\partial t} = 0 \quad (12)$$

Solution of Problem

To re-write the equation into dimensionless form which reduced some of parameters in the equation and make qualitative studies easier, we define the dimensionless variables which are as follows:

$$y = \frac{x}{L}, F_o = \frac{Kt}{\rho c L^2}, F_{oq} = \frac{K\tau_q}{\rho c L^2}, F_{oT} = \frac{K\tau_T}{\rho c L^2}, F_{ov} = \frac{K\tau_v}{\rho c L^2},$$

$$\Theta = \frac{T - T_0}{T_0}, \Theta_w = \frac{T_w - T_0}{T_0}, P_f^2 = \frac{w_{bo} c_b \rho_b}{K} L^2, P_m = \frac{q_{m0} L^2}{K T_0}, C_T = \sqrt{\frac{K^* \rho c}{K}} L, \alpha$$
$$= 0.1 \times T_0 \quad (13)$$

By using dimensionless parameters from eqn(13) in eqns (9) - (12), it become

$$F_{oq} \frac{\partial^3 \Theta(y,F_o)}{\partial F_o^3} = -\left[1 - F_{oq}(P_m \alpha - P_f^2)\right] \frac{\partial^2 \Theta(y,F_o)}{\partial F_o^2} + \left(P_m \alpha - P_f^2\right) \frac{\partial \Theta(y,F_o)}{\partial F_o} + C_T^2 \frac{\partial^2 \Theta(y,F_o)}{\partial y^2} + (1 +$$
$$C_T^2 F_{ov}) \frac{\partial^3 \Theta(y,F_o)}{\partial F_o \partial y^2} + F_{oT} \frac{\partial^4 \Theta(y,F_o)}{\partial F_o^2 \partial y^2}, \quad (14)$$

subjected to initial conditions

$$\Theta(y,0) = 0, \frac{\partial \Theta(y,0)}{\partial F_o} = 0, \frac{\partial^2 \Theta(y,0)}{\partial F_o^2} = 0 \quad (15)$$

boundary condition

$$\Theta(0, F_o) = \Theta_w \quad (16)$$

and symmetric condition

$$\frac{\partial\Theta(1,F_o)}{\partial F_o} = 0, \quad (17)$$

Hybrid Numerical method

The hybrid method is applied to determine the problem. This method is a combination of two different methods. First method in which Eqn (14) is discretized by finite difference scheme discussed by many researchers [31,41,43] is used. After discretization, our problem is turned into system of third order non-linear ordinary differential equations (ODEs) with initial conditions. Again we convert third order ODEs into first order non-linear ordinary differential equations [47]. For the solution of the problem, second method which is Runge-Kutta (4,5) scheme [45,46,48] is adopted. The whole procedure of hybrid method is explained in following subsections.

Spatial discretization scheme

The domain of space coordinate [0,1] is discretized into l+1 subintervals of equivalent length (h) by taking $y_{i+1} = y_i + h$, h is the step length i.e, $0 = y_0 < y_1 < y_2 < y_3 < \cdots < y_i < \cdots < y_l < y_{l+1} = 1$. By using central finite difference formula, the second order derivative is written as,

$$\frac{\partial^2\Theta(y,F_o)}{\partial y^2} = \frac{\Theta_{i+1}(F_o) - 2\Theta_i(F_o) + \Theta_{i-1}(F_o)}{h^2}, \qquad 1 \le i \le l \quad (18)$$

By applying Eqn (18), then Eqns (14−17) are converted into

$$F_{oq}\frac{d^3\Theta_1}{dF_o^3} = -\left[1 - F_{oq}(P_m\alpha - P_f^2)\right]\frac{d^2\Theta_1}{dF_o^2} + (P_m\alpha - P_f^2)\frac{d\Theta_1}{dF_o} + \frac{C_T^2}{21h^2}(-29\Theta_1 + 38\Theta_2 - 9\Theta_3) + \frac{(1+C_T^2F_{ov})}{21h^2}\frac{d}{dF_o}(-29\Theta_1 + 38\Theta_2 - 9\Theta_3) + \frac{F_{oT}}{21h^2}\frac{d^2}{dF_o^2}(-29\Theta_1 + 38\Theta_2 - 9\Theta_3), \quad (19)$$

$$F_{oq}\frac{d^3\Theta_i}{dF_o^3} = -\left[1 - F_{oq}(P_m\alpha - P_f^2)\right]\frac{d^2\Theta_i}{dF_o^2} + \left(P_m\alpha - P_f^2\right)\frac{d\Theta_i}{dF_o} + \frac{C_T^2}{21h^2}\left(\Theta_{i+1}(F_o) - 2\Theta_i(F_o) + \Theta_{i-1}(F_o)\right) + \frac{(1+C_T^2F_{ov})}{21h^2}\frac{d}{dF_o}\left(\Theta_{i+1}(F_o) - 2\Theta_i(F_o) + \Theta_{i-1}(F_o)\right) + \frac{F_{oT}}{21h^2}\frac{d^2}{dF_o^2}\left(\Theta_{i+1}(F_o) - 2\Theta_i(F_o) + \Theta_{i-1}(F_o)\right), \qquad 1 \le i \le l. \quad (20)$$

$$F_{oq}\frac{d^3\Theta_n}{dF_o^3} = -\left[1 - F_{oq}\left(P_m\alpha - P_f^2\right)\right]\frac{d^2\Theta_n}{dF_o^2} + \left(P_m\alpha - P_f^2\right)\frac{d\Theta_n}{dF_o} + \frac{C_T^2}{h^2}(\Theta_w - 2\Theta_n + \Theta_{n-1}) + \frac{(1+C_T^2F_{ov})}{21h^2}\frac{d}{dF_o}(\Theta_w - 2\Theta_n + \Theta_{n-1}) + \frac{F_{oT}}{21h^2}\frac{d^2}{dF_o^2}(\Theta_w - 2\Theta_n + \Theta_{n-1}), \quad (21)$$

Subjected to initial conditions

$$\Theta(y, 0) = 0, \ \frac{d\Theta(y,0)}{dF_o} = 0, \frac{d^2\Theta(y,0)}{dF_o^2} = 0, \quad (22)$$

Runge-Kutta (4, 5) Scheme

Suppose that

$$\frac{d}{dF_o}\left(F_{oq}\frac{d^2\Theta}{dF_o^2}\right) = \frac{d\Phi}{dF_o}, \ \frac{d^2\Theta}{dF_o^2} = \frac{\Phi}{F_{oq}}, \qquad \frac{d}{dF_o}\left(\frac{d\Theta}{dF_o}\right) = \frac{\Phi}{F_{oq}},$$

$$\frac{d\Theta}{dF_o} = \frac{\varphi}{F_{oq}} \qquad 1 \le i \le n \quad (23)$$

By using Eqn (23) in Eqns (19-22) , then the equations becomes

$$\frac{d\Phi_1}{dF_o} = -\left[1 - F_{oq}(P_m\alpha - P_f^2)\right]\frac{\Phi_1}{F_{oq}} + (P_m\alpha - P_f^2)\frac{\varphi_1}{F_{oq}} + \frac{C_T^2}{21F_{oq}h^2}(-29\Theta_1 + 38\Theta_2 - 9\Theta_3) +$$
$$\frac{(1+C_T^2F_{ov})}{21F_{oq}h^2}(-29\varphi_1 + 38\varphi_2 - 9\varphi_3) + \frac{F_{oT}}{21F_{oq}h^2}(-29\Phi_1 + 38\Phi_2 - 9\Phi_3), \quad (24)$$

$$\frac{d\Phi_i}{dF_o} = -\left[1 - F_{oq}(P_m\alpha - P_f^2)\right]\frac{\Phi_i}{F_{oq}} + \left(P_m\alpha - P_f^2\right)\frac{\varphi_i}{F_{oq}} + \frac{C_T^2}{21h^2}(\Theta_{i+1} - 2\Theta_i + \Theta_{i-1}) + \frac{(1+C_T^2F_{ov})}{F_{oq}h^2}(\varphi_{i+1} -$$
$$2\varphi_i + \varphi_{i-1}) + \frac{F_{oT}}{F_{oq}h^2}(\Phi_{i+1} - 2\Phi_i + \Phi_{i-1}), \quad (25)$$

$$\frac{d\Phi_n}{dF_o} = -\left[1 - F_{oq}(P_m\alpha - P_f^2)\right]\frac{\Phi_n}{F_{oq}} + \left(P_m\alpha - P_f^2\right)\frac{\varphi_n}{F_{oq}} + \frac{C_T^2}{F_{oq}h^2}(\Theta_w - 2\Theta_n + \Theta_{n-1}) + \frac{(1+C_T^2F_{ov})}{F_{oq}h^2}(-2\varphi_n +$$
$$\varphi_{n-1}) + \frac{F_{oT}}{F_{oq}h^2}(-2\Phi_n + \Phi_{n-1}) \quad (26)$$

Subjected to initial conditions

$$\Theta(y,0) = 0,\ \varphi(y,0) = 0,\ \Phi(y,0) = 0 \quad (27)$$

Results and Discussion

In this paper, we consider the temperature distribution in biological skin tissue is derived from non-linear TPLBHT model under Dirichlet boundary condition. In non-linear TPLBHT model, the temperature dependent blood perfusion and also metabolic heat source, which are experimentally validated function of temperature is considered. For validation of TPLBHT model, the comparison of TPLBHT model with experimental data is studied by Kumar (2020)[5] with experimental statistics which are obtained by Afrin (2011) [18].

Fig 2 explain the effect of dimensionless phase lag time because of heat flux with respect to dimensionless temperature and dimensionless time. We noticed that as increasing the value of F_{oq} there is decrease in temperature but after F_o=0.1, the temperature increases as increasing the values of F_{oq}. The variation of temperature distribution for different values of dimensionless phase lag time because of temperature gradient is shown in Fig3. It is observed that as increasing the value of F_{oT} with time, the temperature distribution decreases. In Fig4, we shows the effect of dimensionless phase lag time due to thermal displacement F_{ov}. As decreasing the value of thermal displacement, the temperature distribution increases. The effect of dimensionless blood perfusion coefficient is explained in Fig5, which shows that the temperature distribution increases as increasing the values of P_f. The effect of dimensionless metabolic heat source coefficient P_m with respect to dimensionless temperature and dimensionless time is shown in Fig 6. In this figure the temperature is above when the value of P_m is $2.5973e{-}00$ and approximately same for the values of P_m=$2.5973e{-}01$ and P_m= $2.5973e{-}05$. In Fig7, the effect of C_T is shown. From this figure we observed that as increasing the value of C_T, temperature distribution with respect to dimensionless time increases.

In Fig 8, we observed the effect of α, which shows that the temperature increases as the increasing the value of α. From these observations, the TPLBHT model is very beneficial for therapeutically application.

Conclusion

The behavior of non-linear TPLBHT model in living skin tissue is studied. For solution of present problem, the combinational numerical scheme is applied which is based on two different methods. Problem is discretized by central difference technique and Runge-Kutta (4,5) method underneath

Dirichlet boundary condition for skin tissue. The hybrid numerical scheme is applied which is based on two different methods for solution of present problem. Problem is discretized by central difference technique and Runge-Kutta (4,5) method under Dirichlet boundary condition for living tissue is used. The obtained problem is changed into system of non-linear third order ordinary differential equation with initial condition which is then solved by using Runge-Kutta (4, 5) method. After using hybrid method, we get some conclusions which are specified as:

Fig.2 indicate that as increasing the value of F_{oq} the temperature decreases but after F_o=0.1, the temperature increases as increasing the values of F_{oq}. In Fig.3, we observed the effect of dimensionless phase lag time due to heat flux F_{oT} and dimensionless phase lag time due to thermal displacement F_{ov} respectively, from which we concluded as increasing the value of F_{oT} and F_{ov}, the temperature distribution decreases. The effects of dimensionless blood perfusion coefficient P_f, C_T and α are shown in Fig 5,7,8 which shows that the temperature distribution increases as increasing the values of P_f, C_T and α. The results of dimensionless metabolic heat source coefficient P_m with respect to dimensionless temperature and dimensionless time is obtained from Fig6. In this figure, the temperature is exceeding when the value of P_m is $2.5973e-00$ and almost same for the values of P_m= $2.5973e-01$ and P_m= $2.5973e-05$.

Nomenclature

q	heat flux, W/m^2
x	space coordinate, m
t	time, s
κ	Thermal conductivity of tissue, $W/m^\circ C$
T	temperature of tissue, $^\circ C$
τ_q	phase lag due to heat flux, s
τ_T	phase lag due to temperature gradient, s
τ_v	phase lag due to thermal displacement, s
ρ	density of skin tissue, kg/m^3
c	specific heat of tissue, $J/kg^\circ C$
w_b	blood perfusion rate, s^{-1}
ρ_b	density of blood, kg/m^{-3}
c_b	specific heat of blood, $J/kg^\circ C$
T_b	arterial blood temperture, $^\circ C$
q_{mo}	reference metabolic heat generation, W/m^3
K^*	rate of thermal conductivity, $W/m^\circ C/s$
L	Length of tissue, m
T_w	wall temperature of outer boundary, $^\circ C$

Dimensionless variables

y	dimensionless space coordinate
F_o	dimensionless time
F_{oq}	phase lag due to heat flux
F_{oT}	phase lag due to temperature gradient
F_{ov}	phase lag due to thermal displacement local tissue temperature
Θ_b	arterial blood temperature

α	associated metabolism constant
Θ_w	wall temperature at boundary
P_f	blood perfusion coefficient
P_m	metabolic heat source coefficient

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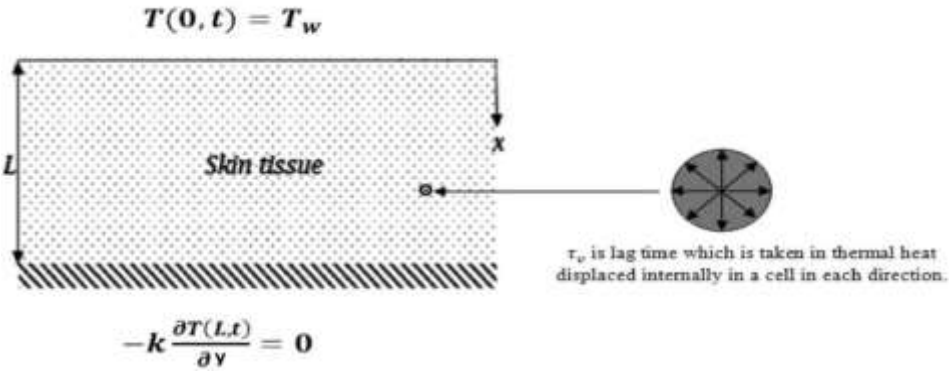

Figure1: One dimensional schematic of skin.

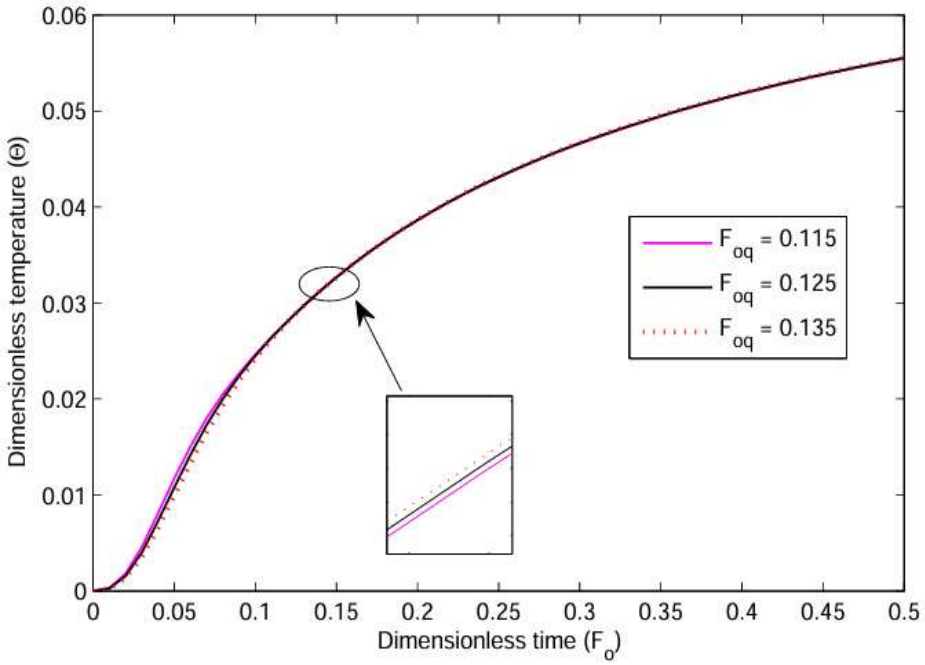

Figure 2: Effect of lagging time due to heat flux F_{oq} on dimensionless temperature in tissue with respect to dimensionless time.

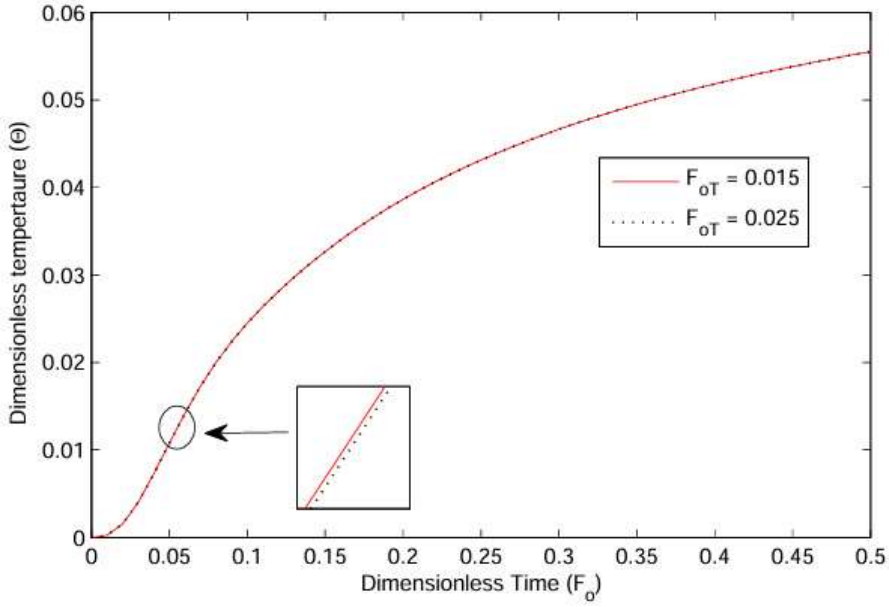

Figure 3: Effect of lagging time due to temperature gradient F_{oT} on dimensionless temperature in tissue with respect to dimensionless time

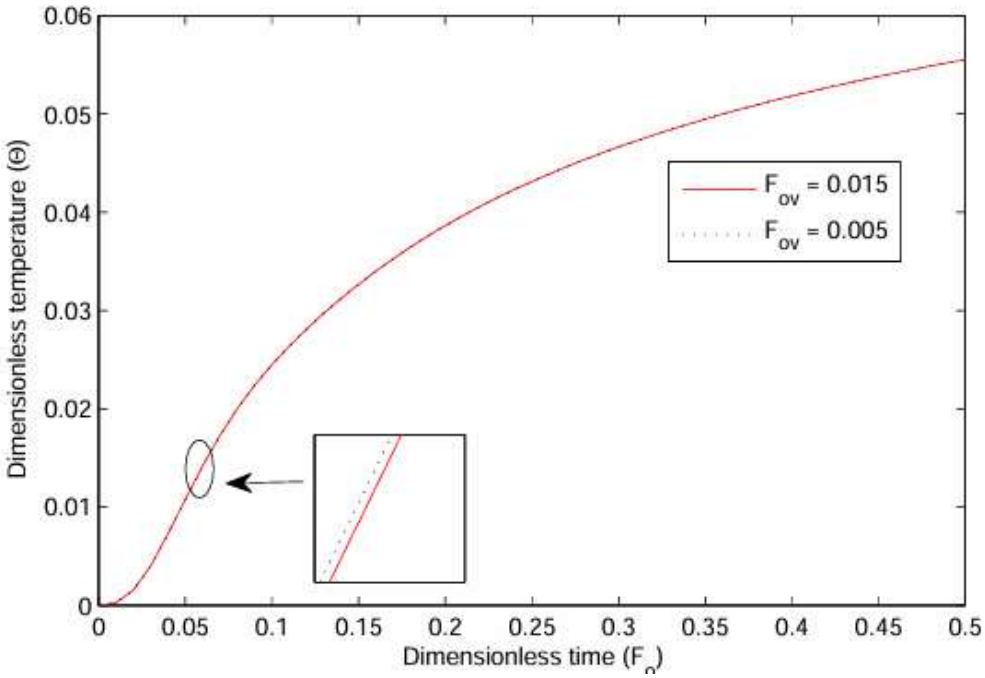

Figure 4: Effect of lagging time due to thermal displacement on dimensionless temperature in tissue with respect to dimensionless time

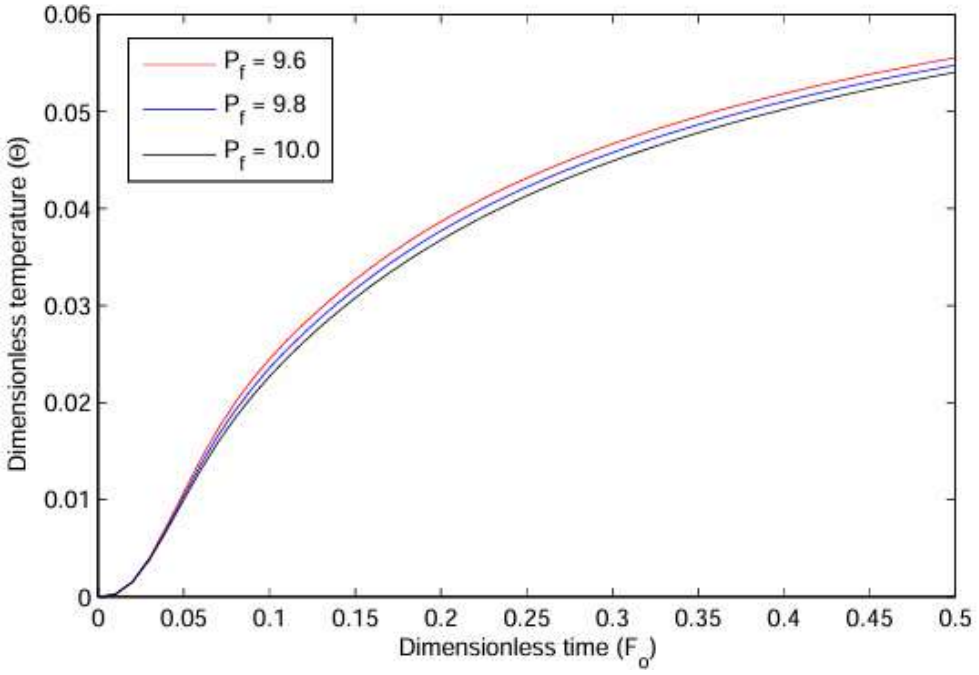

Figure 5: Effect of dimensionless blood perfusion coefficient P_f on dimensionless temperature in tissue with respect to dimensionless time

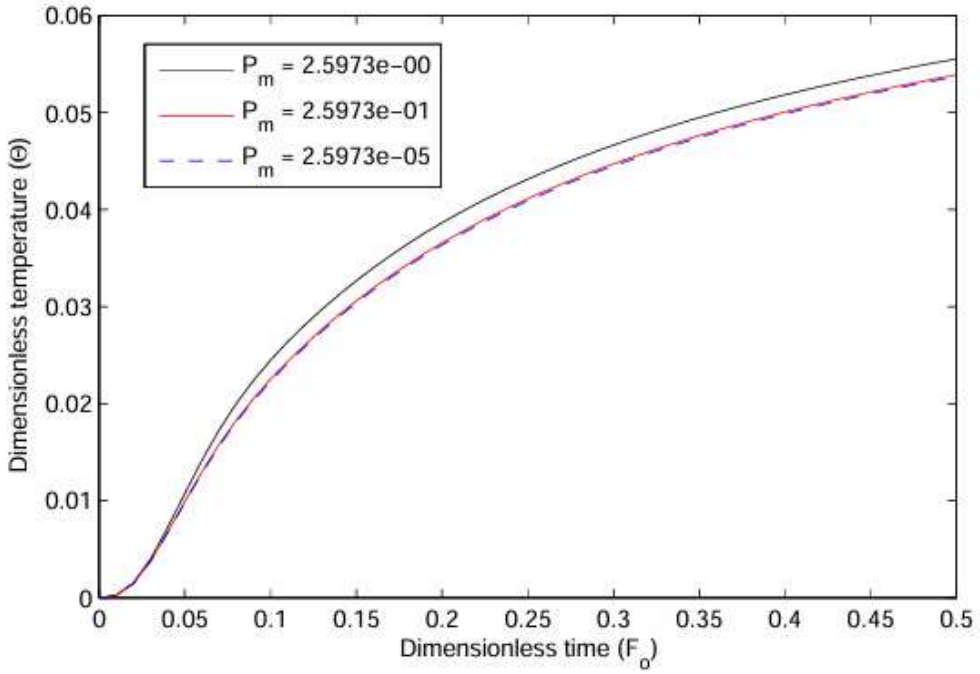

Figure 6: Effect of dimensionless metabolic heat source coefficient P_m on dimensionless temperature in tissue with respect to dimensionless time.

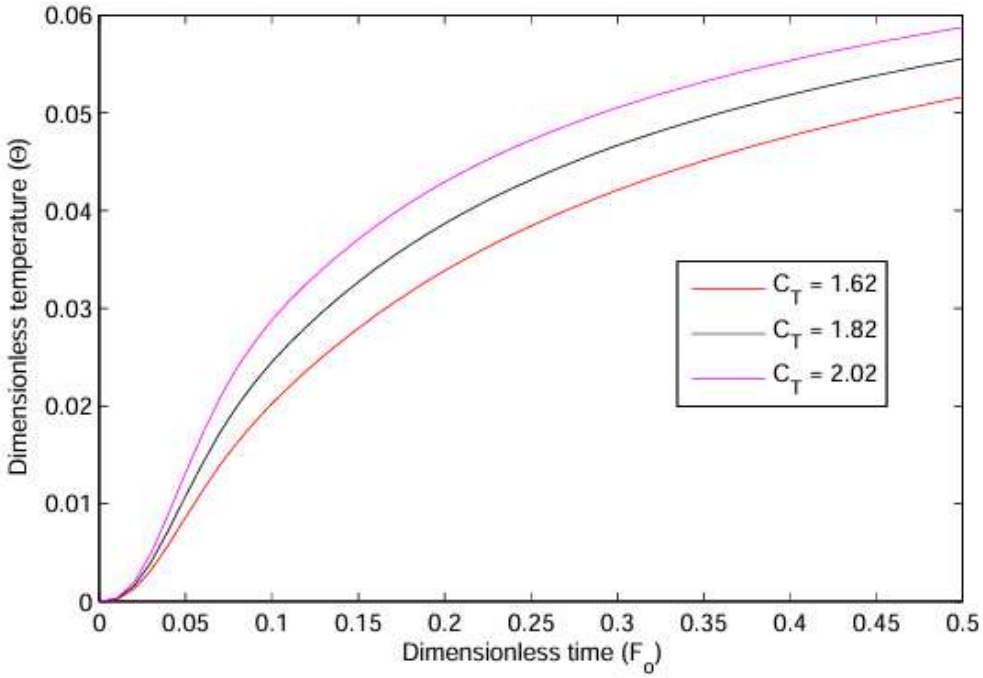

Figure 7: Effect of C_T on dimensionless temperature in tissue with respect to dimensionless time

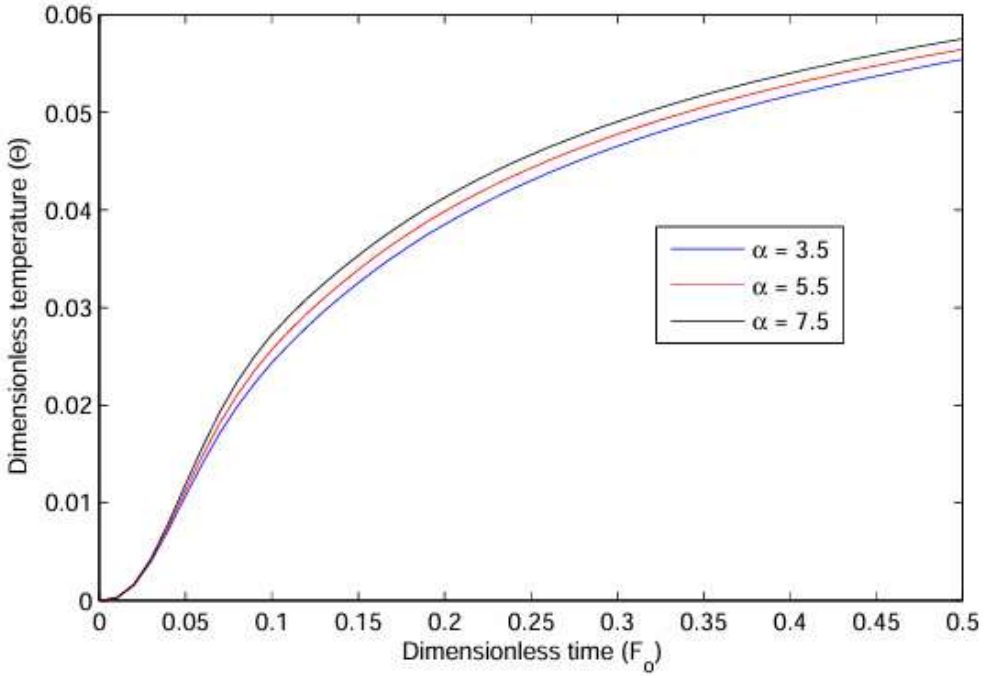

Figure 8: Effect of metabolic constant α on dimensionless temperature in tissue with respect to dimensionless time