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Simplification of Large-Scale Linear Systems Using a New Model Reduction Technique for Design of Controllers

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ABSTRACT

A new model diminution technique is proposed for the reduction of complexity of higher order linear dynamical systems. In this proposed method, a generalized pole clustering technique is used for obtaining the denominator polynomial of the lower order plant and the numerator polynomial is evaluated by applying the Time moment matching technique. The generalized pole clustering algorithm promises the preservation of stability and dominant poles of the actual system in the reduced order plant. The performance error indices such as integral square error (ISE), integral absolute error (IAE), integral time weighted absolute error (ITAE) and relative integral square error (RISE) are used to validate the proposed technique. By using the transfer function of the simplified order plant, the PID and lead/lag compensators are designed by using a moment matching algorithm. This controller is applied to the original large-scale system and the response of the closed loop system is matching with the response of the desired reference model.

KEYWORDS: Large scale systems, Model order reduction, Multidimensional systems, Pole clustering method, Transfer function, Controller design, Integral square error (ISE), Integral absolute error (IAE), Integral time weighted absolute error (ITAE) and relative integral square error (RISE).

1. INTRODUCTION

With the increasing features of dynamical systems, the complexity and order of the systems are increased simultaneously. Analysis and simulation of these systems by a digital system creates several limitations such as storage memory, computational time and computation astringency to the higher order systems [1]–[3]. Model order reduction (MOR) plays an important role in the analysis and synthesis of large scale systems [4]–[6]. MOR is illustrated as the method of approximating a complex system into a simple plant in such a way that the estimated system must preserve the fundamental behavior of the original system. The exploration and controller design for the simple system is easier as compared to the complex system [3], [7]–[11]. From the past few decades, model diminution is a commonly used technique for the reduction of complexity of higher order dynamical plants.

Nowadays, the application of model reduction is extended in the numerous areas of sciences and engineering. The design of an electric motor by finite element analysis is a high-dimensional system and this process is time consuming and computationally inefficient. The MOR based on proper orthogonal decomposition is used to compensate for these problems for the design of electric motors [12]. In the

field of power system stability models, the model diminution schemes are focused on the reduction of complexity of large scale power system models [13]–[17]. The controller design for large scale systems is a difficult task and the model diminution technique plays an imperative role in the field of control systems [18]–[21]. The model reduction is also extended for the neutral type control systems (a time delay system that holds time delays mutually in its state and derivatives of state) [22], [23]. In [24]–[26], the model reduction for decreasing the mathematical complexity of nonlinear analog circuits has been discussed. From the last two decades, the model reduction got great attention in the reduction of higher dimensional electromagnetic systems [27], [28]. In the chemical industry, the model reduction methods are used for the analysis and control of large scale distillation column linear systems [29]. In [30], the authors discussed the applications of model diminution for the simplification of complex stochastic plants. Binion and Chen explained the usage of model reduction for the analysis of complex microelectromechanical models [31]. The computational efficiency of the gear contact simulation is enhanced by using the MOR technique based on Craig–Bampton method [32].

For the simplification of large scale systems, various model reduction techniques are proposed in the frequency domain [33]–[40] and in the time domain [41]–[45]. These methods have numerous advantages but also have limitations. To overcome their limitations, several model reduction methods have been proposed based on the combination of these methods [46]–[55]. Among these methods, the pole clustering method is one of the widely used model reduction methods for the higher dimensional linear dynamic systems [37]. One of the most popular methods for model reduction is pole clustering because it can be used for the simplification of minimum as well as non-minimum phase systems. It can be also used for the simplification of systems having non-strictly and strictly proper transfer functions. In this method, the poles of the actual model are grouped into various clusters and their cluster centers are used for obtaining the reduced model. For the proper matching of the time responses of the original and reduced systems, the extra mathematical calculations are required for the determination of the gain adjustment factor and tuning factor. The process of finding these factors enhances the simulation time and storage memory. To circumvent these drawbacks, various system diminution techniques based on the pole clustering methods have been discussed in [7], [8], [56]–[64].

In this article, a new system reduction scheme is proposed which also overcomes the limitations of the pole clustering scheme [37]. The proposed method can be applied for the reduction of large-scale non-minimum, minimum phase systems and the systems having non-strictly and strictly proposer transfer. A generalized pole clustering algorithm is used for the determination of the denominator and the time moment matching method is used for obtaining the numerator of the reduced model. By using the time moment matching technique, the proposed algorithm ensures the matching of the steady state responses of the actual and reduced systems. Due to that, the gain adjustment factor is not required in the proposed method as it requires in the pole clustering method [37]. By using the generalized pole clustering method, almost all dominant poles of the original system are retained in the reduced model due that the transient response of the reduced model is approximately matching with the transient response of the original system. Therefore, the tuning factor is also not required in the proposed method for the matching of the transient responses. The proposed technique is simple, precise and guarantees the stability of the reduced system if the given complex system is stable. The remaining article is structured as follows. In Section 2, the problem statement of model diminution is presented. The basic algorithms of the proposed model diminution method are explained in Section 3. The application of the proposed method for the designing of the controller is explained in Section 4. The numerical simulations of the

proposed method are given in Section 5. The conclusion and future works of the proposed method are drawn in Section 6.

2. PROBLEM FORMULATION

Let the transfer function of the large-scale single input single output linear dynamic plant be in the form of

$$G_p(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{r-1} d_i s^i}{\sum_{i=0}^{r} e_i s^i} = \frac{d_0+d_1 s+\cdots+d_{n-1}s^{n-1}}{e_0+e_1 s+e_2 s^2+\cdots+e_n s^n} \quad (1)$$

where $d_0, d_1 \ldots d_{n-1}$ and $e_0, e_1 \ldots e_n$ are the known scalar constants of the actual complete order system. The main objective of the model reduction is to compute the unknown parameters of r^{th} order $(r < n)$ reduced system having the transfer function in the form of

$$R_r(s) = \frac{Q_r(s)}{P_r(s)} = \frac{\sum_{i=0}^{r-1} q_i s^i}{\sum_{i=0}^{r} p_i s^i} = \frac{q_0+q_1 s+q_2 s^2+\cdots+q_{r-1}s^{r-1}}{p_0+p_1 s+p_2 s^2+\cdots+p_{r-1}s^{r-1}+p_r s^r} \quad (2)$$

where $q_0, q_1 \ldots q_{r-1}$ and $p_0, p_1 \ldots p_r$ are the unknown scalar constants of the desired reduced system. Consider the transfer matrix of n^{th} order large scale multi-input multi output LTI system with u inputs and v outputs as follows

$$[G_p(s)] = \frac{1}{D(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & \cdots & a_{1u}(s) \\ a_{21}(s) & a_{22}(s) & \cdots & a_{2u}(s) \\ \vdots & \vdots & \vdots & \vdots \\ a_{v1}(s) & a_{v2}(s) & \cdots & a_{vu}(s) \end{bmatrix} \quad (3)$$

$$= [g_{ij}(s)]_{v\times u} \quad (4)$$

where $i = 1, 2, 3, \ldots, v;\ j = 1, 2, 3, \ldots, u$. Hence $g_{ij}(s)$ can be expressed as

$$g_{ij}(s) = \frac{a_{ij}(s)}{D(s)} \quad (5)$$

The equivalent transfer matrix of r^{th} order estimated plant $[R_r(s)]$ with u inputs and v outputs is synthesized as follows

$$[R_r(s)] = \frac{1}{P_r(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & \cdots & b_{1u}(s) \\ b_{21}(s) & b_{22}(s) & \cdots & b_{2u}(s) \\ \vdots & \vdots & \vdots & \vdots \\ b_{v1}(s) & b_{v2}(s) & \cdots & b_{vu}(s) \end{bmatrix} \quad (6)$$

$$= [r_{ij}(s)]_{v\times u} \quad (7)$$

where $i = 1, 2, 3, \ldots, v;\ j = 1, 2, 3, \ldots, u$. Therefore $r_{ij}(s)$ can be written as

$$r_{ij}(s) = \frac{b_{ij}(s)}{P_r(s)} \quad (8)$$

where $g_{ij}(s)$ and $r_{ij}(s)$ are the numerous components of the complete dimensional and the lower dimensional transfer function matrices correspondingly. The objective of the article is to realize the r^{th} order reduced systems in the form of Equations (2) and (6) from the Equations (1) and (3) respectively and which preserve the fundamental properties of the original model.

3. PROPOSED MODEL REDUCTION METHOD

For obtaining the reduced order model, the proposed method has been explained by describing the following two steps. The proposed method for the determination of the denominator polynomial is based on the finding of clusters and cluster centers. The following steps are used for clustering the poles in the s-plane.

3.1 Process for evaluating the denominator polynomial of the simplified model

The time and frequency responses of any dynamical plants are affected by the position, density (numbers of poles lie within a specific region) and the number of poles of the plant. In most of the model diminution methods, the poles far away from the vertical axis of the s-plane are ignored [35], [48], [54], [65]–[67]. In this proposed process, the effects of all poles of the actual plant are considered in the evaluation of the approximated plant. For the r^{th} order simplified model, all the poles of the actual plant are grouped into the "r" clusters. The poles of the cluster reflect their impact in the cluster center of that group. The poles in a particular group are included based on their dominance i.e. relative distance from the origin of the s-plane. To find the clusters (groups) of the poles, the following steps are used.

1) Complex and real poles are grouped individually.
2) The left half and right half of s-plane poles are grouped independently.
3) Poles lying at the origin and vertical axis of the s-plane are preserved directly in the lower dimensional system.

Clustering of real poles

Let us assume the higher dimensional plant of Equation (1) in the pole-zero form as given follows

$$G_p(s) = \frac{N(s)}{D(s)}$$

where,

$$N(s) = (s+z_1)(s+z_2)\dots(s+z_{r-1})(s+z_r)(s+z_{r+1})\dots(s+z_{2r-1})(s+z_{2r})(s+z_{2r+1})\dots(s+z_{n-1}) \quad (9)$$

and

$$D(s) = (s+p_1)(s+p_2)\dots(s+p_{r-1})(s+p_r)(s+p_{r+1})\dots(s+p_{2r-1})(s+p_{2r})(s+p_{2r+1})\dots(s+p_n) \quad (10)$$

Let the order of the lower order model is r $(r < n)$ and for rth order simplified model, r number of groups (clusters) of the poles are formed. The left half of s-plane poles are organized in the ascending sequence as

$$-p_1, -p_2, \dots, -p_j, \dots, -p_r, \dots, -p_n \forall\ |p_j| < |p_{j+1}| \quad (11)$$

The first pole is located in the cluster-1, the second pole is located in the cluster-2 and the r^{th} pole is located in the cluster$-r$. In the same way, $(r+1)^{th}$ pole is located in the cluster-1, $(r+2)^{th}$ pole is located in the cluster-2 and $2r^{th}$ pole is kept in the cluster$-r$ and this process is stopped after the allocation of the last pole. The main benefit of the placing of the poles in this way is that each cluster will have starting poles of Equation (11). The starting poles of Equation (11) are dominating poles compared to the ending poles of Equation (11). In this way, "r" dominating poles of the higher dimensional plant are allocated in all "r" clusters. Hence, each cluster keeps the dominating pole of the original system as compared to the present algorithms [37], [56]–[64] which keeps all dominating poles in the first cluster. After the allocation of the poles in the groups, the cluster centers of groups are attained as

$$c_{1p} = -\left\{\left(\left[\frac{1}{|p_1|^X}\right] + \left[\frac{1}{|p_{r+1}|^X}\right] + \left[\frac{1}{|p_{2r+1}|^X}\right] + \cdots \left[\frac{1}{|p_k|^X}\right]\right)/k\right\}^{-(1/X)} \quad (12)$$

$$c_{2p} = -\left\{\left(\left[\frac{1}{|p_2|^X}\right] + \left[\frac{1}{|p_{r+2}|^X}\right] + \left[\frac{1}{|p_{2r+2}|^X}\right] + \cdots \left[\frac{1}{|p_l|^X}\right]\right)/l\right\}^{-(1/X)} \quad (13)$$

… … … … …

$$c_{rp} = -\left\{\left(\left[\frac{1}{|p_r|^X}\right] + \left[\frac{1}{|p_{2r}|^X}\right] + \left[\frac{1}{|p_{3r}|^X}\right] + \cdots \left[\frac{1}{|p_m|^X}\right]\right)/m\right\}^{-(1/X)} \quad (14)$$

where, $c_{1p}, c_{2p}, \ldots, c_{rp}$ are the cluster centers of the clusters and X is the order of the root. The value of X can be taken as a natural number and it depends upon the requirement of the accuracy of the lower dimensional plant. And k, l and m are the number of poles allocated in the first group, second cluster and r^{th} group respectively. Depending upon the number of poles in the large-scale plants, the values of k, l and m may have an equal or different value. For the particular value of the order of the root (X), the cluster centers given in Equations (12-14) are dependent upon the poles which are nearer to the origin of the s-plane. As the value of X increases from more than one, the cluster centers are going nearer to the dominant poles. Because the inverse of the root of any smaller magnitude quantity is converted into a larger magnitude quantity as compared to the inverse of the root of a larger magnitude quantity. Hence, it is summarized that each cluster center is dependent upon the dominant pole and the value of the cluster center is nearer to the dominant pole of that cluster. It is also obvious from Equations (12-14) that when X is equal to one, the proposed algorithm of clustering is the same as the standard pole clustering technique given in [37]. After finding the cluster centers, the denominator coefficients of the lower dimensional model are attained as

$$D_r(s) = (s - c_{1p})(s - c_{2p}) \ldots (s - c_{rp}) \quad (15)$$

The cluster centers of real poles existing in the right half of the s-plane are determined by changing the negative sign with the positive sign of the cluster centers in the Equations (12-14).

Clustering of complex poles

For the rth order reduced model, $(r/2)$ pair of clusters of complex poles should be constructed. Consider the denominator of the nth order original model having complex poles only as

$$D(s) = (s + a_{1p} \pm jb_{1p})(s + a_{2p} \pm jb_{2p}) \ldots (s + a_{rp/2} \pm jb_{rp/2}) \ldots (s + a_{np/2} \pm jb_{np/2}) \quad (16)$$

On the basis of dominance of the poles existing in the left half of the s-plane are arranged in ascending order and allocated in $(r/2)$ pair of groups. The group centers of are achieved as

$$A_{1p} \pm jB_{1p} = -\left\{\left(\left[\frac{1}{|a_{1p}|^X}\right] + \left[\frac{1}{|a_{(r+1)p}|^X}\right] + \cdots + \left[\frac{1}{|a_{kp}|^X}\right]\right)/k\right\}^{-(1/X)} \pm$$

$$\left\{\left(\left[\frac{1}{|b_{1p}|^X}\right] + \left[\frac{1}{|b_{(r+1)p}|^X}\right] + \cdots + \left[\frac{1}{|b_{kp}|^X}\right]\right)/k\right\}^{-(1/X)} \quad (17)$$

$$A_{2p} \pm jB_{2p} = \left\{\left(\left[\frac{1}{|a_{2p}|^X}\right] + \left[\frac{1}{|a_{(r+2)p}|^X}\right] + \cdots + \left[\frac{1}{|a_{lp}|^X}\right]\right)/l\right\}^{-(1/X)} \pm$$

$$\left\{\left(\left[\frac{1}{|b_{2p}|^X}\right] + \left[\frac{1}{|b_{(r+2)p}|^X}\right] + \cdots + \left[\frac{1}{|b_{lp}|^X}\right]\right)/l\right\}^{-(1/X)} \quad (18)$$

…. …. …

$$A_{rp/2} \pm jB_{rp/2} = -\left\{\left(\left[\frac{1}{|a_{rp/2}|^X}\right] + \left[\frac{1}{|a_{rp}|^X}\right] + \cdots + \left[\frac{1}{|a_{mp}|^X}\right]\right)/m\right\}^{-(1/X)} \pm$$

$$\left\{\left(\left[\frac{1}{|b_{rp/2}|^X}\right]+\left[\frac{1}{|b_{rp}|^X}\right]+\cdots+\left[\frac{1}{|b_{mp}|^X}\right]\right)/m\right\}^{-(1/X)} \quad (19)$$

For the complex poles, $A_{ip/2} \pm jB_{ip/2}\,(i = 1, 2, \ldots r)$ are the group centers. The numbers of poles allocated in the first group, second cluster and in rth group are represented as $k, l, and\ \ m$ respectively. The order of the root is represented as X, its value is dependent upon the required accuracy of the lower dimensional plant. By applying the group centers of the complex poles, the simplified denominator polynomial is computed as

$$D_r(s) = (s - A_{1p} \pm jB_{1p})(s - A_{2p} \pm jB_{2p}) \ldots (s - A_{rp/2} \pm jB_{rp/2}) \quad (20)$$

The cluster centers the right half of s-plane complex poles are determined by replacing the negative sign with the positive sign in the Equations (17-19).

Clustering of the real and complex poles

The cluster centers are determined in the same way as the Equations (12-14) are used for the real poles and Equations (17-19) are used for the complex poles. The number of clusters made for the real poles and the number of clusters required for the complex poles is finalized on the basis of the order of the reduced model. For the r^{th} order reduced system, assume α clusters are made for the real poles and β clusters are then made for the complex poles, the proposed denominator polynomial of the simplified model is determined as

$$D_r(s) = \prod_{i=1, j=\alpha+1}^{\alpha,\beta}(s - c_{ip})\,(s - A_{jp} \pm\ jB_{jp}) \quad | \quad \alpha + \beta = r \quad (21)$$

3.2 Procedure for computing the numerator coefficients of the simplified model

The time moment matching method is a standard model diminution scheme for the simplification of the higher dimensional plants [39], [68]–[72]. It assures the conservation of initial few time moments of the higher dimensional plant in the reduced dimensional plant but it does not ensure the stability of the lower dimensional plant even though the higher dimensional plant is stable [68], [73]. To overcome the instability problem of the time moment matching algorithm, the denominator polynomial of the reduced model is achieved by using the generalized pole clustering method as discussed in Section 3.1. To preserve the features of the Moment matching algorithm method in the proposed process, the numerator of the proposed reduced system is achieved by the Moment matching algorithm method. The power-series expansion of $G(s)$ of Equation (1) around $s = 0$ is

$$G_p(s) = \frac{\sum_{i=0}^{r-1} d_i s^i}{\sum_{i=0}^{r} e_i s^i} = c_0 + c_1 s + c_2 s^2 + \cdots + c_r s^r + \cdots \quad (22)$$

The parameters $\{c_j : j = 1, 2, \ldots, \infty\}$ are the time moments of the original complete order model $G(s)$ and it is determined by applying the moment generating algorithm given in [33], [74], [75]. The Equation (22) is obtained by using the Taylor series expansion of $G(s)$ about $s = 0$ of Equation (1) as follows

$$c_0 = d_0$$
$$c_i = \frac{1}{e_0}\left[d_i - \sum_{j=1}^{i} e_i c_{i-j}\right],\ \ i > 0 \quad (23)$$
$$e_i = 0, i > n - 1$$

The numerator coefficients of the reduced system can be attained by comparing the Equations (2) with (22) as follows

$$R_r(s) = \frac{Q_r(s)}{P_r(s)} = \frac{\sum_{i=0}^{r-1} q_i s^i}{\sum_{i=0}^{r} p_i s^i} = \frac{q_0 + q_1 s + q_2 s^2 + \cdots + q_{r-1} s^{r-1}}{p_0 + p_1 s + p_2 s^2 + \cdots + p_{r-1} s^{r-1} + p_r s^r} = c_0 + c_1 s + c_2 s^2 + \cdots + c_r s^r \quad (24)$$

After cross-multiplication of Equation (23), the coefficients of the various power of "s" are compared and it gives the following "r" numbers of equations.

$$\begin{aligned} q_0 &= c_0 p_0 \\ q_1 &= c_0 p_1 + c_1 p_0 \\ &\vdots \\ q_{r-1} &= c_0 p_{r-1} + c_1 p_{r-2} + \cdots + c_{r-1} p_0 \end{aligned} \quad (25)$$

The proposed numerator polynomial $Q_r(s)$ is obtained by using initial "r" numbers of time moments of the original system. Therefore it promises the conservation of first "r" time moments in the reduced model [7], [8], [33], [62], [68], [73], [76].

The most remarkable fact about the proposed method is that the proposed denominator polynomial evaluated from this method contains the effect of all the poles regardless of all former methods, which designs a denominator polynomial with just by using dominant poles only. Another most interesting fact about the proposed method is that the reduced denominator polynomial contains all dominant poles irrespective of the reduced denominator polynomial obtained by several other techniques based on pole clustering methods containing only one dominant pole of the original system. So, these make the proposed technique the best suitable model order reduction method.

4. DESIGN OF CONTROLLER

The controller design, analysis and simulation of the higher dimensional plants are challenging works. As the order of the plant is increased, the complexity and controller design cost are proportionally increased [7], [8]. This problem can be compensated if a "good" simplified order model is attainable for the complicated system and the controller is designed by applying the simplified system. For sensing the state variables, several sensors are required in the higher dimensional plant for the creation of feedback controllers. To avoid the use of many sensors, series controllers are desirable in the place of feedback controllers.

For obtaining the desirable behavior of a dynamic system, a reference plant ($M(s)$) is constructed based on the given specifications and the closed loop performance of the controlled plant with unity feedback is equivalent to the behavior of the reference plant. In [77]–[79], the procedures for finding the reference system from the specified data are discussed. Consider a controller, that gives the anticipated closed loop response and defined by the following transfer function as [80], [81]

$$G_c(s) = \frac{K(1+K_1 s)}{s(1+K_2 s)} \quad (26)$$

For the controller designed by applying lower dimensional plant, an open loop reference model ($\widetilde{M}(s)$) is required and it can be achieved from the closed loop reference model ($M(s)$) as

$$\widetilde{M}(s) = \frac{M(s)}{1-M(s)} \quad (27)$$

The behaviour of the open loop-controlled system is assumed to be similar to the behaviour of the open loop reference model. Hence, the unknown parameters of the controller are computed as

$$G_c(s) G_p(s) = \widetilde{M}(s) \quad (28)$$

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)} = \frac{\sum_{i=0}^{2} e_i s^i}{s} \quad (29)$$

where e_i $(i = 0, 1, 2)$ are the power series coefficients about $s = 0$, and the moment generating method given in [33] is used for the determination of these coefficients. And $G_p(s)$ is the transfer function of the higher dimensional plants. The mathematical computation and simulation time of the controller designed

for the higher order systems can be reduced by replacing the actual plant $G_p(s)$ with an equivalent reduced model in the Equation (27). The unknown constants of the controller are determined by equating the Equations (25) and (28) as

$$\frac{K(1+K_1 s)}{s(1+K_2 s)} = \frac{e_0+e_1 s+e_2 s^2}{s} \quad (30)$$

The Padé type approximations of the controller having the anticipated configuration are attained by solving Equation (29). After determining the controller constants, the closed loop model of the higher dimensional plant is determined as

$$G_{cl}(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \quad (31)$$

5. ILLUSTRATIVE EXAMPLES

The performance of the proposed method is compared with the other existing reduction schemes by computing the different performance error indices such as ISE, RISE, IAE and ITAE. The performance error indices given in [61], [63], [82], [83] are computed between the transient parts of the actual plant and the lower dimensional plants and described as

$$\text{ISE} = \int_0^\infty [y(t) - y_r(t)]^2 dt \quad (32)$$

$$\text{RISE} = \int_0^\infty [y(t) - y_r(t)]^2 dt \,/\, \int_0^\infty [\hat{y}(t)]^2 dt \quad (33)$$

$$\text{IAE} = \int_0^\infty |y(t) - y_r(t)| dt \quad (34)$$

$$\text{ITAE} = \int_0^\infty t|y(t) - y_r(t)| dt \quad (35)$$

where $y(t)$ and $y_r(t)$ are the time responses of the large-scale plant and the lower order model respectively. The impulse response of the original model is $\hat{y}(t)$. The performance error indices are computed for the various lower dimensional systems determined by the proposed algorithm and some other standard system diminution schemes given in the literature. The less value of the error indices for the reduced model indicates that the reduced model is a better approximant of the original system.

Example 1: Let us assume a ninth-order standard plant discuss by many researchers [63], [84]–[87]

$$G(s) = \frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700} \quad (36)$$

The denominator equation of the given plant is written as

$$D(s) = s^9+9s^8 + 66s^7 + 294s^6 + 1029s^5 + 2541s^4 + 4684s^3 + 5856s^2 + 4620s + 1700 \quad (37)$$

Poles: $-1, -1-1j, -1+1j, -1-2j, -1+2j,$
$-1-3j, -1+3j, -1-4j, -1+4j.$

This system has one real pole; it will be retained in the reduced model. For the third order computed plant, the complex poles are grouped into one cluster and cluster center is obtained by using Equations (17)-(19) as

$$A_z \pm jB_z = -\left\{\left(\left[\frac{1}{1^X}\right] + \left[\frac{1}{1^X}\right] + \left[\frac{1}{1^X}\right] + \left[\frac{1}{1^X}\right]\right)/4\right\}^{-(1/X)} \pm \left\{\left(\left[\frac{1}{1^X}\right] + \left[\frac{1}{2^X}\right] + \left[\frac{1}{3^X}\right] + \left[\frac{1}{4^X}\right]\right)/4\right\}^{-(1/X)} \quad (38)$$

By applying the cluster centers, the denominator polynomial of the lower dimensional plant is attained as

$$D_3(s) = (s+1)(s + A_z \pm jB_z) \quad (39)$$

Taking numerous values of X, the various denominator equations of the reduced model will be attained. The numerator coefficients are calculated by applying the Time moment matching method as explained in Section 3.2. The lower dimensional plant attained by the proposed method with $X = 5$, is

$$R_3(s) = \frac{-0.3614s^2 - 0.9193s + 2.7164}{s^3 + 3s^2 + 4.7164s + 2.7164} \quad (40)$$

In Figure 1, the step responses of the higher order plant and lower dimensional plants determined by the proposed algorithm as well as some existing processes are compared. It is plainly noticed that the lower order plant attained by the given algorithm offers an exact approximation to the higher dimensional plant. In Table 1, the performance error indices of the various reduction methods are shown. It is found that the performance error indices of the given scheme are having the lowest indices as equated to some classical techniques [34], [35], [48], [66], [67], recently proposed methods [3], [83], [88], [89] and optimization methods [58], [84]. Table 2 also showing the efficiency and usefulness of the presented scheme to the other popular reduced order modeling algorithms present in the literature. Therefore, according to the graphical representation and error indices table, the proposed technique is the best suited to obtain the reduced order approximation of the large-scale systems.

Table 1. Comparison of performance error indices of the proposed method with some other existing system diminution techniques

Reduction method	Lower order system	ISE	RISE	IAE	ITAE
Hankel norm approx. technique [67]	$\frac{0.5263s^2 - 1.286s + 2}{s^3 + 2.039s^2 + 4.402s +}$	13	2.9465	116.81	6027.1
Padé approximation and differentiation technique [88]	$\frac{2.933s^2 + 33.05s +}{3.372s^3 + 29.51s^2 + 93.}$	9.2321	1.9621	14.894	20.9372
Differentiation technique [90]	$\frac{2.933s^2 + 33.05s +}{3.372s^3 + 29.51s^2 + 93.}$	9.2321	1.9621	14.894	20.9372
Balanced truncation method [66], Schur decomposition method [89]	$\frac{0.1405s^2 - 0.8492s + 1}{s^3 + 1.575s^2 + 3.523s +}$	8.5453	1.8161	90.295	4777.3
Balanced truncation and factor division algorithm [55], improved balanced truncation method [91]	$\frac{1.674s^2 - 0.0393s + 1}{s^3 + 1.575s^2 + 3.523s +}$	4.3331	0.9209	14.073	36.9938
Routh Hurwitz method [34]	$\frac{259.8s^2 + 864s +}{913.9s^3 + 1500s^2 + 302}$	3.9217	0.8335	10.864	21.6615
Response matching technique [85]	$\frac{0.2945s^2 - 2.202s + 2.}{s^3 + 2.501s^2 + 4.77s + 2}$	1.6057	0.3413	9.3606	28.6362
Routh Hurwitz and Padé approx. [46], [92], Routh Hurwitz and factor division algorithm [93], Improved Routh Hurwitz [83]	$\frac{-751.695s^2 - 504s -}{913.9s^3 + 1500s^2 + 302}$	1.4852	0.3156	8.8486	26.2855
Truncation methodology [94]	$\frac{291s^2 + 1093s + 1}{4684s^3 + 5856s^2 + 462}$	0.7825	0.1663	7.5642	35.289

Mihailov stability and Padé approximation method [48]	$\frac{335.3018s^2 + 674s}{3099s^2s^3 + 5031s^2 + 42}$	0.7078	0.1504	6.4704	24.228
Mihailov stability and improved Padé approximation method [3]	$\frac{674s + 1700}{3099s^2s^3 + 5031s^2 + 42}$	0.6032	0.1282	6.0996	25.411
Stability equation [35]	$\frac{285s^2 + 1093s + 1}{3048s^3 + 5031s^2 + 462(}$	0.581	0.1235	4.9845	13.641
Stability equation and big bang big crunch method [84]	$\frac{0.0789s^2 + 0.3142s + 0.}{s^3 + 1.3s^2 + 1.34s + 0.4}$	0.4732	0.1006	4.9434	17.1018
Modified balanced truncation method [95]	$\frac{0.1405s^2 - 0.8492s + 1}{s^3 + 1.575s^2 + 3.523s +}$	0.385	0.818	4.7721	20.2788
Pole clustering and Padé approx. method [57], Proposed technique ($\boldsymbol{X = 1}$)	$\frac{0.21s^2 - 3.0365s + 4.68}{s^3 + 3s^2 + 6.6864s + 4.6}$	0.3281	0.0697	3.3275	7.0460
Modified pole clustering method and Genetic Algorithm [58]	$\frac{-0.264s^2 + 0.483s + 0}{s^3 + 2.195s^2 + 2.046s +}$	0.2916	0.0620	3.5788	10.5086
Routh-Padé approximations [50], Routh approximation and factor division [1], improved Routh approximation [53]	$\frac{-0.264s^2 + 0.4822s +}{s^3 + 2.1913s^2 + 2.0384s}$	0.2886	0.0613	3.5857	10.6276
Modified pole clustering method and Padé approx. [96], modified pole clustering and factor division method [64], improved pole clustering method [59]	$\frac{-0.2739s^2 + 0.4879s +}{s^3 + 2.195s^2 + 2.046s +}$	0.2836	0.0603	3.5516	10.4551
Stability equation and Padé approximation [47], stability equation and factor division method [52]	$\frac{285s^2 + 1093s + 1}{3048s^3 + 5031s^2 + 462(}$	0.2672	0.0568	3.8768	14.513
Proposed technique ($\boldsymbol{X = 3}$)	$\frac{-0.2039s^2 - 1.5031s +}{s^3 + 3s^2 + 5.2596s + 3}$	0.1639	0.0360	2.3143	4.530
Proposed technique ($\boldsymbol{X = 5}$)	$\frac{-0.3614s^2 - 0.9193s +}{s^3 + 3s^2 + 4.7164s + 2}$	0.1639	0.0348	2.2137	4.0155

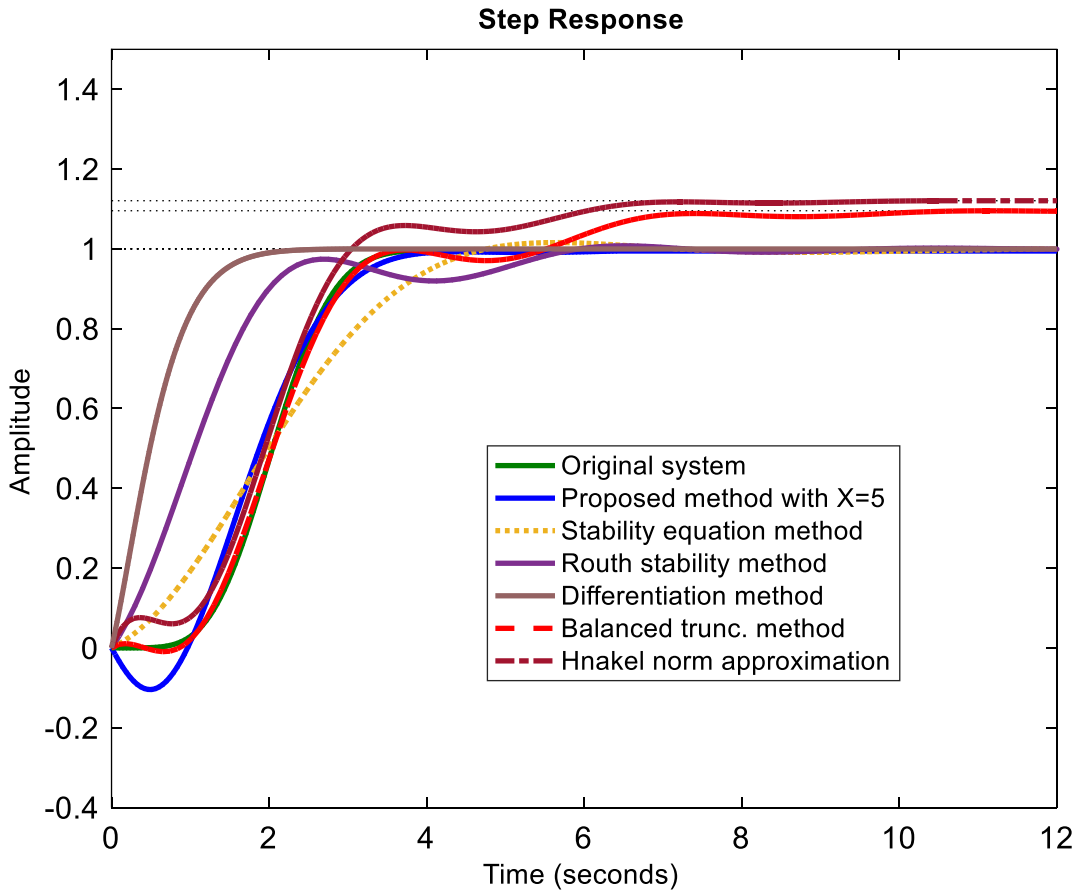

Figure 1. Comparison of step response for full and lower order plants

Example 2: Consider a sixth-order multivariable plant [2], [56], [64], [97] with two-inputs and two-outputs illustrated by the following transfer matrix as

$$[G_p(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (41)$$

$$= \frac{1}{D(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

where,

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and

$$A_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$A_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$A_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$A_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

For achieving the denominator polynomial of the second order reduced plant, the two clusters are made.

Cluster-1 : $-1, -3, -10$
Cluster-2 : $-2, -5, -20.$

The centers of these clusters are determined by applying (12)-(14). The characteristic equation of the proposed simplified model is attained as

$$D_2(s) = (s - c_{1p})(s - c_{2p}) \quad (42)$$

For the different values of X, the various denominator equations of the lower dimensional plant will be attained. The numerator coefficients are evaluated by using the Time moment matching method as described in Section 3.2. The lower dimensional system determined by the proposed scheme with $X = 50$ is

$$[R_2(s)] = \frac{\begin{bmatrix} 1.1858s+2.0898 & 0.8505s+0.8359 \\ 0.5406s+1.0449 & 1.6734s+2.0898 \end{bmatrix}}{s^2+3.0666s+2.0898} \quad (43)$$

Table 2: Comparison of various system reduction schemes in terms of ISE

Reduction technique	Reduced model	$r_{11}(s)$	$r_{12}(s)$	$r_{21}(s)$	$r_{22}(s)$
Parmar Mukherjee and Prasad [51]	$\frac{\begin{bmatrix} 6.0429s+8.4707 & 3.9419s+3.3883 \\ 2.8097s+4.2354 & 8.0195s+8.4707 \end{bmatrix}}{s^2+13.6666s+8.4707}$	0.225	0.0682	0.0613	0.6780
Sikander and Prasad [52]	$\frac{\begin{bmatrix} 0.7938s+0.6181 & 0.4273s+0.2472 \\ 0.37952s+0.309 & 0.93382s+0.6181 \end{bmatrix}}{s^2+1.34952s+0.6181}$	0.1672	0.0958	0.0312	0.2004
Narwal and Prasad [97]	$\frac{\begin{bmatrix} 0.8930s+0.6181 & 0.4517s+0.2472 \\ 0.4314s+0.3091 & 1.0579s+0.6181 \end{bmatrix}}{s^2+1.34952s+0.6181}$	0.1615	0.0897	0.0296	251.3574
Parmar, Prasad and Mukherjee [98]	$\frac{\begin{bmatrix} 0.8503s+0.6171 & 0.4617s+0.2466 \\ 0.4093s+0.3086 & 0.9977s+0.6171 \end{bmatrix}}{s^2+1.34952s+0.6181}$	0.1471	0.0884	0.0258	0.1598
Prajapati and Prasad [53]	$\frac{\begin{bmatrix} 0.9098s+0.7091 & 0.4916s+0.2836 \\ 0.4373s+0.3545 & 1.0753s+0.7091 \end{bmatrix}}{s^2+1.548s+0.7091}$	0.0765	0.0595	0.0115	0.0808
Proposed method ($X = 10$)	$\frac{\begin{bmatrix} 1.106s+2.4914 & 0.8909s+0.9966 \\ 0.4907s+1.2457 & 1.6874s+2.4914 \end{bmatrix}}{s^2+3.3483s+2.4914}$	0.0157	0.0003	0.0024	0.0381
Vishwakarma and Prasad [58]	$\frac{\begin{bmatrix} 1.1816s+3.6508 & 1.0466s+1.4603 \\ 0.4982s+1.8254 & 1.6911s+3.6508 \end{bmatrix}}{s^2+4.3374s+3.6508}$	0.0151	0.0078	0.0030	0.0469
Narwal and Prasad [61]	$\frac{\begin{bmatrix} 1.3276s+3.0962 & 1.0447s+1.2444 \\ 0.6116s+1.5480 & 1.7815s+3.0960 \end{bmatrix}}{s^2+4.0965s+3.0965}$	0.0093	0.0040	0.0008	247.8492
Proposed method ($X = 50$)	$\frac{\begin{bmatrix} 1.1858s+2.0898 & 0.8505s+0.8359 \\ 0.5406s+1.0449 & 1.6734s+2.0898 \end{bmatrix}}{s^2+3.0666s+2.0898}$	0.0089	0.0002	0.0008	0.0377

The time response comparison of the reduced order system obtained by the proposed method with the original system and the reduced model obtained by some other existing methods is shown in Figure 2. From this plot, the response of the proposed reduced model is completely matched with the response of the original system. For quantitative comparison, the proposed technique is also compared with other model reduction techniques in terms of ISE values as tabulated in Table 2. From Table 2, when X is 10, ISE is not least error but if $X = 50$ then the proposed method gives the least error when compared with some other existing schemes. So, all the performance parameters taken for the comparison are giving the best values for this method, which shows that the proposed method is best among the techniques discussed in the error indices table.

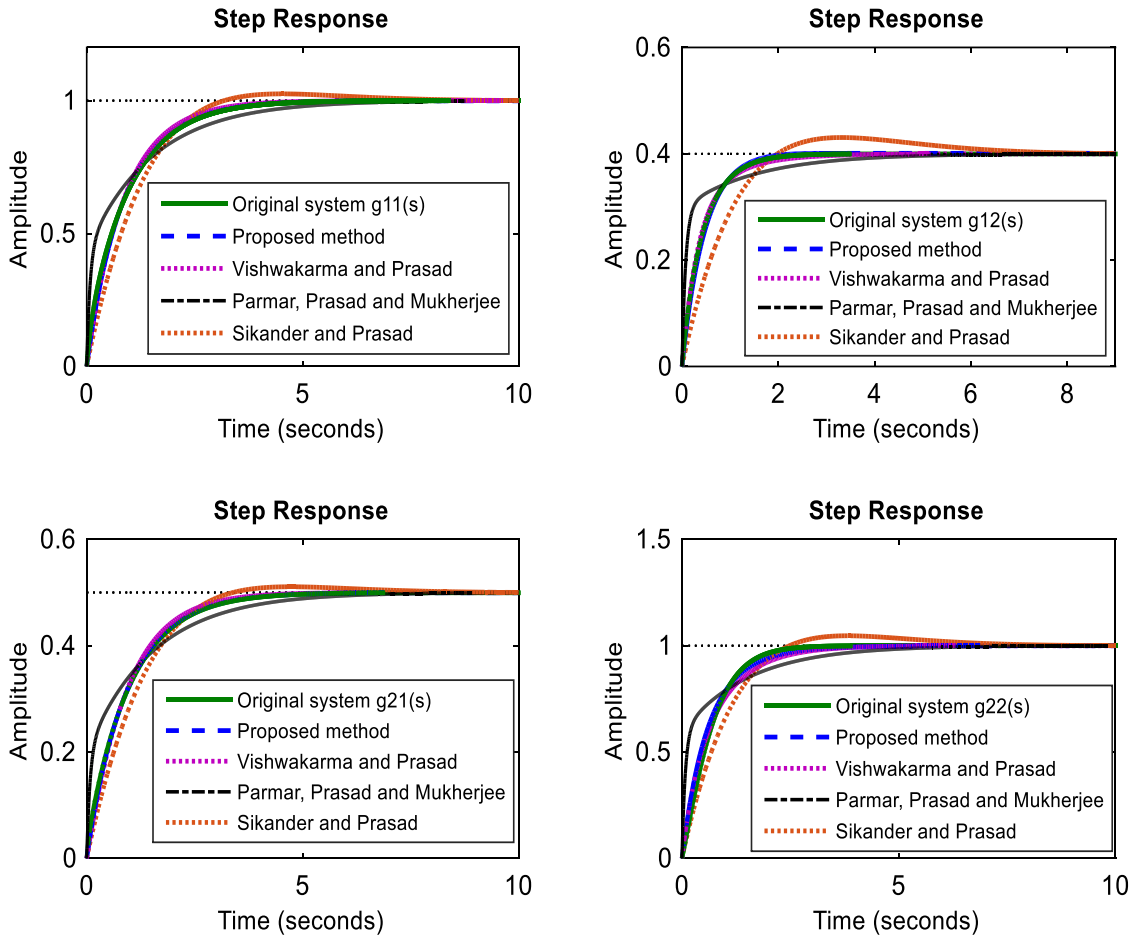

Figure 2: Comparison of step responses of the higher order plant and lower dimensional plants for example 3

Example 3: Consider a sixth order transfer function of the helicopter engine including a fuel controller model discussed in [99] as

$$G(s) = \frac{248.05s^4+1483.3s^3+91931s^2+468730s+634950}{s^6+26.24s^5+1363.1s^4+26803s^3+326900s^2+859170s+528050} \quad (44)$$

The input of plant is speed demand and the output is propeller speed and the critically damped reference model given in [99] is

$$M(s) = \frac{4}{s^2+4s+4} \quad (45)$$

From the given reference model, the open loop reference model is determined as

$$\widetilde{M}(s) = \frac{M(s)}{1-M(s)} = \frac{4}{s(s+4)} \quad (46)$$

The parameters of the controller determined by the actual system are as

$$G_c(s) = \frac{\widetilde{M}(s)}{G(s)} = \frac{e_0 + e_1 s + e_2 s^2}{s} = \frac{K(1+K_1 s)}{s(1+K_2 s)}$$

$$= \frac{0.8316+0.5313s-0.2841s^2}{s} \quad (47)$$

Hence,

$$K = 0.8316, K_1 = 1.1735, K_2 = 0.5347 \quad (48)$$

The closed loop plant with compensator determined by the original plant is given as:

$$R_{cl}(s) = \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \quad (49)$$

The lower dimensional model determined by the proposed technique with $X = 50$ is given as follows

$$R_2(s) = \frac{0.6357s+1.48}{s^2+3.042s+2.056} \quad (50)$$

By using the proposed lower order plant of Equation (61), the compensator is attained as

$$G_{cr}(s) = \frac{\widetilde{M}(s)}{R_r(s)} = \frac{e_0 + e_1 s + e_2 s^2}{s} = \frac{K(1+K_1 s)}{s(1+K_2 s)}$$

$$= \frac{0.8316+0.5313s-0.2841s^2}{s} \quad (51)$$

Hence,

$$K = 0.8316, K_1 = 1.1735, K_2 = 0.5347 \quad (52)$$

The closed loop transfer with compensator determined by the proposed reduced plant is obtained as:

$$R_{cl}(s) = \frac{G(s)G_{cr}(s)}{1+G(s)G_{cr}(s)} \quad (53)$$

From Equations (59) and (63), it is obvious that the parameters of the compensator determined by using the actual plant are identical to the parameters of the compensator attained by the reduced model of Equation (61). But the design of a compensator from a higher order plant is tough work as compared to the compensator design from an equivalent lower order plant. Hence, the proposed reduced dimensional model can be applied for the design of the compensator in place of the higher dimensional plant.

The comparison of time response of the closed loop actual plant with compensator is shown in Figure 3. The compensators are attained by using the higher and lower dimensional plants. The simulation result demonstrates that the proposed compensator performs well under both steady state and transient behavior. The time domain specifications of the closed loop plants with compensators are shown in Table 3. From this table, it can be observed that the specifications of the closed loop model with the compensator obtained by the approximant are similar as that of the closed loop plant with the compensator designed by the complex plant and these specifications are also matching with the reference plant specifications. Hence, the proposed technique can be applied for the compensator design for the fulfillment of the essential behavior of the real time plants. Overall, it can be summarized that the values of time domain specification mostly reduce which indicates the proposed method is highly preferable.

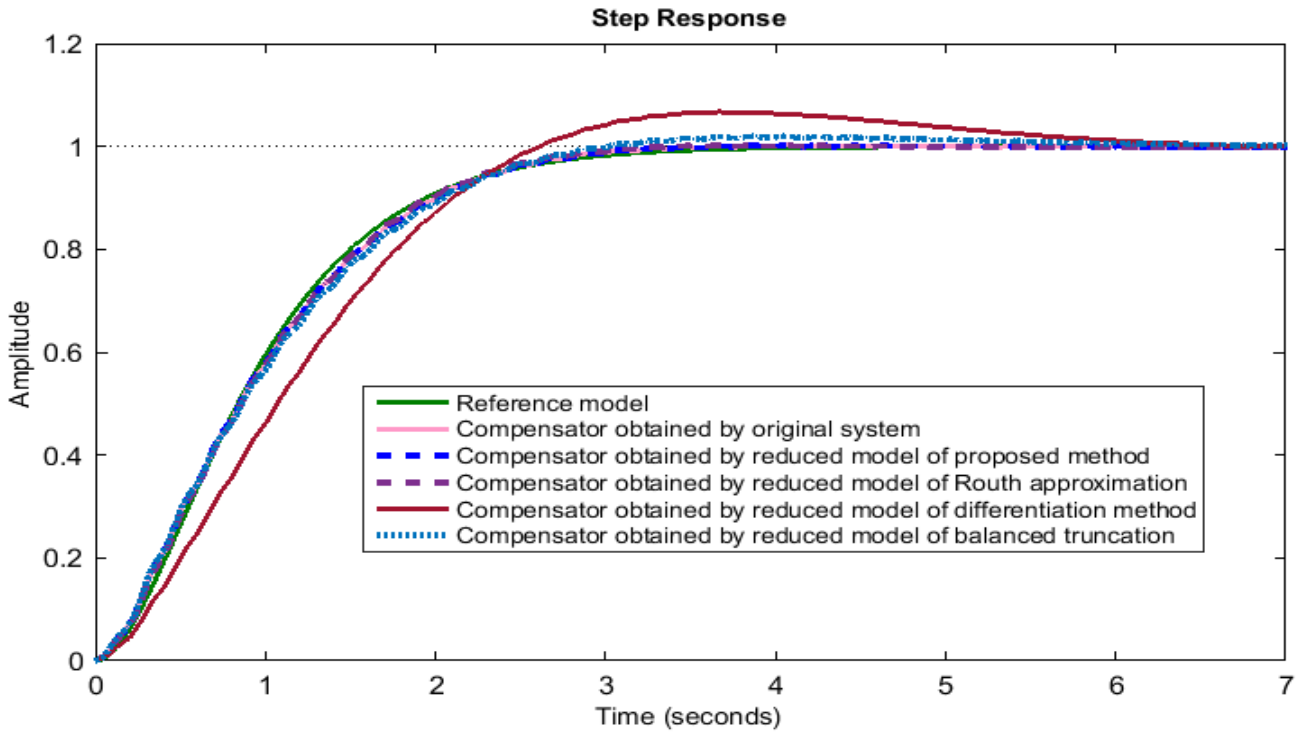

Figure 3: Comparison of time responses of closed loop plants and reference model

Table 3: Comparison of closed loop plants in terms of time domain specifications

Reduction scheme	Reduced order plant	Compensator (K, K_1, K_2)	Rise time	Settling time	Peak overshoot	Peak time
---	Reference system	---	1.679	2.917	0.9991	4.690
---	Original plant	0.8316, 1.1735, 0.5347	1.7628	2.8107	1.0026	4.0502
Proposed technique (X=50)	$\frac{0.6357s + 1.48}{s^2 + 3.042s + 2.056}$	0.8316, 1.1735, 0.5347	1.7628	2.8107	1.0026	4.0502
Balanced truncation [66], Schur decomposition method [100]	$\frac{3.917s^2 + 122.7s + 1431}{s^3 + 2.156s^2 + 1019s + 1234}$	0.8623, 0.8715, 0.3815	1.7996	4.071	1.0202	4.0673
Stability equation method [35]	$\frac{9.0184s^2 + 46.873s + 63.495}{2.593s^3 + 33.003s^2 + 85.917s + 52.8}$	0.8316, 1.16, 0.5211	1.7706	2.8223	1.0026	4.2493
Stability equation and	$\frac{9.5695s^2 + 46.873s + 63.495}{2.593s^3 + 33.003s^2 + 85.917s + 52.8}$	0.8316, 1.1735,	1.7628	2.8107	1.0026	4.0502

factor division [52], Stability equation and Padé approximation [47]		0.5347				
Routh approximation [101]	$\frac{4.6367s^2 + 20.3172s + 27.522}{s^3 + 14.0706s^2 + 37.2422s + 22.889}$	0.8317, 1.2174, 0.5786	1.7329	2.6802	1.0030	3.8650
Improved Routh approximation [53]	$\frac{3.8651s^2 + 20.3182s + 27.5227}{s^3 + 14.0706s^2 + 37.2422s + 22.889}$	0.8316, 1.1735, 0.5347	1.7628	2.8106	1.0026	4.0502
Differentiation method [90]	$\frac{18.39s^2 + 281.2s + 761.9}{1.6082s^3 + 78.46s^2 + 515.5s + 633.7}$	0.8317, 0.7754, 0.581	1.7931	5.6076	1.0674	3.6679
Differentiation and Padé approximation [88]	$\frac{283.98s^2 - 57.431s + 761.9}{1.6082s^3 + 78.46s^2 + 515.5s + 633.7}$	0.8316, 1.1735, 0.581	1.7628	2.8107	1.0026	4.0503
Padé approximation [33]	$\frac{3.48s^2 + 18.52s + 27.37}{s^3 + 12.44s^2 + 35.64s + 22.76}$	0.8316, 1.1744, 0.5352	1.7621	2.8112	1.0026	4.0558
Balanced realization and factor division method [55]	$\frac{3.917s^2 + 122.7s + 1431}{s^3 + 2.156s^2 + 1019s + 1234}$	0.8316, 1.1735, 0.5347	1.7628	2.8106	1.0026	4.0502
Routh Hurwitz technique [34]	$\frac{13.55s^2 + 399.20s + 634.95}{4.21s^3 + 227.73s^2 + 818.61s + 528.(}$	0.8316, 1.1739, 0.5023	1.7783	2.9819	1.0001	4.8363
Routh Hurwitz and factor division [93]	$\frac{16.035s^2 + 419.96s + 634.95}{4.21s^3 + 227.73s^2 + 818.61s + 528.(}$	0.8316, 1.1735, 0.5347	1.7628	2.8106	1.0026	4.0502
Modified balanced truncation [95]	$\frac{3.917s^2 + 122.7s + 1480.8}{s^3 + 2.156s^2 + 1019s + 1234}$	0.8316, 0.8695, 0.3764	1.8473	2.8117	1.0185	4.0481

Hankel norm method [67]	$\frac{4.498s^2 + 119.4s + 1845}{s^3 + 2.527s^2 + 1018s + 1608}$	0.8715, 0.6866, 0.3683	1.781 4	5.2981	1.0452	3.666 4

6. CONCLUSION

This paper proposed a technique for the reduction of complexity of linear dynamical systems, based on the Time moment matching method and generalized pole clustering technique. This algorithm guarantees the retention of stability, dominant poles and first few time moments of the complete order plant in the lower order plant. It is shown in Figures 1-2 that the proposed model reduction approach produces reduced models that accurately approximate the static and dynamic behavior of the higher dimensional plant. Tables 1-2 indicate that the proposed method produced the reduced models which give the least error indices and comparable to the other existing popular and recently proposed model reduction methods. Furthermore, the computed reduced order plant is used for the design of the controller by using the simple moment matching algorithm. This controller is used for the controlling of the original system. From Table 3, the controller designed by applying the lower dimensional plant is giving approximately the same time domain specification as the controller designed by the original system. The proposed method can also be extended for the design of the controller for the multi-variable high dimensional systems.

Data Availability

All data generated or analysed during this study are included in this article.

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