A Framework For Adaptive E-Learning Module Recommendation Using Intuitionistic Fuzzy Sets And Euclidean Distance

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Abstract:
Intuitionistic Fuzzy Set which extends classical fuzzy sets by incorporating both membership and non-membership degrees along with hesitation, provide a richer way to model uncertainty. This paper introduces an innovative framework for intelligent E-learning recommendation systems by integrating intuitionistic fuzzy sets (IFS), Euclidean distance metrics, and pattern recognition techniques. The proposed model effectively captures both learner's confidence (membership) and hesitation (non-membership) levels, providing detailed representations of individual learning profiles. The minimal Euclidean distance identifies the most similar student profiles enabling precise personalized recommendations and data driven instructional decisions.

Keywords: Intuitionistic fuzzy sets, Euclidean distance, E-learning, pattern recognition, personalized recommendation.

1. Introduction:

Fuzzy sets were introduced independently by Lotfi A. Zadeh in 1965, a professor of Electrical engineering and computer science at the University of California, Berkeley, published the seminar paper "Fuzzy sets." In this paper he introduced the concept of fuzzy sets as an extension of classical set theory, where elements have degrees of membership ranging from 0 to1. In the late 1970's Zadeh [15] further developed fuzzy logic and fuzzy relations leading to Fuzzy logic systems (FLS) that aimed to mimic human reasoning. This system made use of IF-THEN rules and could handle vague concepts effectively. Notably in 1980's Japanese companies began to adopt fuzzy logic in products like Air conditioners, washing machines and Cameras highlighting its capabilities in real world applications. How ever in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1minus the membership degree because there may be some hesitation degree .This leads Atanassov [1] in1983 to define a new type of Fuzzy set called Intuitionistic Fuzzy Set (IFS) which consider the parameters a degree of membership and non- membership of all elements of the set and also a degree of hesitation or uncertainty associated with each element Thus an Intuitionistic Fuzzy Set (IFS) helps to represent this uncertainity through three components as follows membership function, non-membership function and hesitation margin $\mu_A(x)$, $\gamma_A(x)$ and $\pi_A(x)$ respectively. IFS have been effectively applied to model hesitation in multi-criteria decisionmaking processes by Atanassov [3] 1999, pattern recognition (Atanssov 2012),Control systems (Bustince et al. 2013) , Medical diagnosis (Deschrijver and Kerri 2003), Classification and Clustering (Bustince et al 2013) So it is clear to say that IFS is the best tool to solve the real life problems .In this paper we use Intuitionistic Fuzzy sets and the normalized Euclidean distance [14], method to calculate the distance specifically between each student profile and Module profile. E learning has transformed modern education by enabling flexible, student-centered learning experiences beyond the limitations of traditional classrooms. The unprecedented shift to online education during the COVID- 19 pandemic underscored the urgent need for intelligent, adaptive systems that can personalize learning pathways to match each student's unique abilities and needs. As a result of small Euclidean distance indicates a strong match between the student's profile and the module's requirements. Therefore, the system should prioritize recommending this module, as it is likely to significantly enhance learning outcomes.

2. Fundamentals of Fuzzy sets:

Definition 2.1 [5]: A set is a well-defined collection of distinct objects, considered as s single entity. The objects in a set are called Elements or members, and a set is usually denoted by capital letters.

Example 2.1.1:

The set of mathematics topics studied in high school

A= {Algebra, Calculus, Geometry, Statistics, Trigonometry}

Definition 2.2 [15]:

Let X be a non - empty set. A Fuzzy set 'A' drawn from X is defined as

$A= \{\langle x, \mu_A(x) \rangle: x\in X\}$ where $\mu_A(x):X \rightarrow [0,1]$ is the membership function of the fuzzy set 'A'.

Example 2.1.2:

For the purpose of measuring student confidence level in an E-learning consider, the student's confidence scores be X= {1,2,3,4,5}

Confident learners= {(1,0.0), (2,0.2), (3,0.5), (4,0.8), (5,1.0)}

In this fuzzy set rating themselves at level 3 has a membership degree of 0.5, indicates Moderate confidence.

A rating of 5 corresponds to a membership degree of 1.0, indicates the student is Fully confident with no uncertainty.

Lower scores like 1 have a membership degree of 0.0 (μ= 0), indicates No confidence.

Definition 2.3 [2]:

Let X be a non – empty set. An intuitionistic fuzzy set A in X is an object having the form $A= \{\langle x, \mu_A(x), \gamma_A(x) \rangle: X \rightarrow [0,1]\}$ which defines respectively the degree of membership and non- membership of the element $x\in X$ to the set A, which is the subset of X, for every element $x\in X$, $0\le \mu_A(x)+ \gamma_A(x)\le 1$. From fuzzy set theory, if the membership degree of an element x is $\mu(x)$, if the non -membership degree of an element x is $1- \mu(x)$

$\pi_A(x)=1- \mu_A(x)- \gamma_A(x)$ is called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is degree of indeterminacy (uncertain or hesitant) of $x \in X$ to the intuitionistic fuzzy set A and $\pi_A(x) \in [0,1]$

It means $\pi_A(x):X \rightarrow [0,1]$, $x\in X$ where $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to intuitionistic fuzzy set A or not.

Example 2.1.3:

Consider the set of possible student learning activity levels in the E-learning platform X= {1,2,3,4,5}

For each activity level we find membership (μ), non-membership (γ) and hesitation (π) as follows:

Activity level	membership (μ)	non-membership (γ)	hesitation (π) $\pi=1-\mu-\gamma$
1(low level)	0.0	0.9	0.1
2	0.2	0.7	0.1
3	0.5	0.4	0.1
4	0.8	0.1	0.1
5(very high)	1.0	0.0	0.0

Active students= {(1,0.0,0.9), (2,0.2,0.7), (3,0.5,0.4), (4,0.8,0.1), (5,1.0,0.0)}
This approach allows us to capture not only how active (membership) or inactive
Students (non- membership) but also the uncertainty(hesitation) about their participation.

Definition 2.4 [10]:

An L-fuzzy subset A of G is said to be an intuitionistic L-fuzzy sub l-group (ILFS l G) of G if for any x, y ∈G

i) $\mu_A(xy^{-1}) \geq \mu_A(xy)$, $\gamma_A(xy^{-1}) \leq \gamma_A(xy)$,

ii) $\mu_A(x \vee y) \geq \mu_A(x) \wedge \mu_A(y)$, $\gamma_A(x \wedge y) \leq \gamma_A(x) \vee \gamma_A(y)$,

iii) $\mu_A(x \wedge y) \geq \mu_A(x) \wedge \mu_A(y)$, $\gamma_A(x \vee y) \leq \gamma_A(x) \vee \gamma_A(y)$ ∀ x, y ∈G

Definition 2.5 [9]:

A fuzzy subset A of G is said to be a anti fuzzy group of G, if for all x, y ∈G

i) $A(xy) \leq \max\{A(x), A(y)\}$

ii) $A(x^{-1}) = A(x)$

3. Basic Relations and operations on intuitionistic fuzzy sets:

1. $A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ ∀x∈ X-Inclusion
2. $A = B \leftrightarrow \mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ ∀x∈ X-Equality
3. $A^c = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$-Complement
4. $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$- Union
5. $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$- Intersection
6. $A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)\rangle : x \in X\}$-Addition
7. $A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x) - \gamma_A(x)\gamma_B(x)\rangle : x \in X\}$-Multiplication
8. $A - B = \{\langle x, \min(\mu_A(x), \gamma_B(x)), \max(\gamma_A(x), \mu_B(x))\rangle : x \in X\}$-Difference
9. $A \Delta B = \{\langle x, \max[\min(\mu_A(x), \gamma_B(x), \min(\mu_B(x), \gamma_A(x)], \min[\max(\gamma_A(x), \mu_B(x), \max(\gamma_B(x), \mu_A(x)]\rangle : x \in X\}$-Symmetric difference
10. $A \times B = \{\langle \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)\rangle : x \in X\}$-Cartesian product

From the basic operations, we deduced the following relations:

1. $A \times B = B \times A$
2. $(A \times B) \times C = A \times (B \times C)$
3. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
4. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
5. $A \times (B \oplus C) = (A \times B) \oplus (A \times C)$
6. $A \times (B \otimes C) = (A \times B) \otimes (A \times C)$

Algebra laws in intuitionistic fuzzy sets:
Let A, B and C are intuitionistic fuzzy sets in X, then,
1. $(A^{c})^{c} = A$- Complementary Law
2. (i) A ∪ A = A (ii) A ∩ A= A- Idempotent Laws
3. (i) A ∪ B= B ∪ A (ii) A ∩ B = B ∩ A
(iii) A ⊕ B = B ⊕ A
(iv) A⊗B = B ⊗ A - Commutative Laws
4.(A U B) U C= A U (B U C) (ii) A ∩ B) ∩ C) = A ∩ (B ∩ C)
(iii) A ⊕ (B⊕ C) = (A⊕ B) ⊕ C
(iv) A⊗ (B⊗ C) = (A ⊗B) ⊗ C – Associative laws
5. (i) AU (B∩C) = (A U B) ∩ (A UC) (ii) A ∩ (B U C) = (A ∩B) U (A∩ C)
(iii) A⊕ (B U C) = (A⊕ B) U (A ⊕ C) (iv) A ⊕ (B ∩ C) = (A⊕ B) ∩ (A ⊕ C)
(v) A ⊗ (B U C) = (A ⊗ B) U (A⊗ C)
(vi) A ⊗ (B ∩ C) = (A ⊗ B) ∩ (A⊗ C)- Distributive Laws
6. (i) $(A \cup B)^{c} = A^{c} \cap B^{c}$ (ii) $(A \cap B)^{c} = A^{c} \cup B^{c}$
(iii) $(A \oplus B)^{c} = A^{c} \otimes B^{c}$ (iv) $(A \otimes B)^{c} = A^{c} \oplus B^{c}$ –De Morgan's Laws
7. (i) A ∩ (AUB) = A (ii) A U (A∩ B) = A
8.(i) $\Phi^{c} = X$ ii) $X^{c} = \Phi$ (iii) A U Φ= A (iv) A ∩ Φ= Φ v) $A \cap A^{c} = \Phi$
9) (i) AU X=X (ii) $A \cup A^{c} = X$ (iii) A ∩ X=A- Absorption Laws

4.Theoretical Foundation for Intuitionistic Fuzzy Set Operations:
In our proposed E-learning framework, we model student knowledge and learning objectives as IFS. To ensure rigorous analysis and manipulation of these sets, we adopt fundamental theorems established in the literature on IFS.

Theorem 1:
Let A and B be two Intuitionistic fuzzy sets defined on a non-empty universe X.
Then $X= A \cup (A \cap B^{c})$ then X=A
Proof:
Let $A= \{\langle x, \mu_A(x), \gamma_A(x) \rangle: x \in X\}$ and $B= \{\langle x, \mu_B(x), \gamma_B(x) \rangle: x \in X\}$ then
$B^{c} = \{\langle x, \gamma_B(x), \mu_B(x) \rangle: x \in X\}$ then $A \cap B^{c} = \{\langle x, \min(\mu_A(x), \gamma_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle: x \in X\}$
Then for each x ∈ X:
For membership-function:
$\mu_X(x)= \max(\mu_A(x), \min(\mu_A(x), \gamma_B(x)))$
Let $a= \mu_A(x)$, $b= \gamma_B(x)$ Then
Max (a, min (a, b)) = a [because min (a, b) ≤ a]
$\mu_A(x)$, $b= \gamma_B(x)$
$\mu_X(x)= \mu_A(x)$
For Non membership function:
$\gamma_X(x)= \min(\gamma_A(x), \max(\gamma_A(x), \mu_B(x)))$
Let $a= \gamma_A(x)$, $b= \mu_B(x)$
Then, min (a, max (a, b)) = a [because max (a, b) ≥ a]
$\gamma_X(x)= \gamma_A(x)$
Hence for every x ∈ X, $X= \langle x, \mu_X(x), \gamma_X(x) \rangle = \langle x, \mu_A(x), \gamma_A(x) \rangle \rightarrow X= A$

Theorem 2:
Let A and B be two Intuitionistic fuzzy sets defined on a non-empty universe X.
Then $(A^c \cup B^c) = (A \cap B)^c$
Proof:
Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$
$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))\}$
$A \cap B = \{\langle x, \min((\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))\}$
$\mu_{A^c \cup B}(x) = \max(\gamma_A(x), \mu_B(x))$
$\gamma_{A^c \cup B}(x) = \min(\mu_A(x), \gamma_B(x))$
$(A^c \cup B)^c = \{\langle x, \min((\mu_A(x), \gamma_B(x)), \max((\gamma_A(x), \mu_B(x))\}$
$\mu_{A \cap B^c}(x) = \min((\mu_A(x), \gamma_B(x))$
$\gamma_{A \cap B^c}(x) = \max(\gamma_A(x), \mu_B(x))$
It is clear that $(A^c \cup B^c) = (A \cap B)^c$ for all $x \in X$
Theorem 3: Let A and B be two Intuitionistic fuzzy sets defined on a non-empty set X.
Then $A \cap (B^c \cup A^c) = A - B$
Proof:
Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$
$\mu_{B^c \cup A^c}(x) = \max(\gamma_B(x), \gamma_A(x))$
$\gamma_{B^c \cup A^c}(x) = \min(\mu_B(x), \mu_A(x))$
Now intersect with A
$\mu_{LHS}(x) = \min(\mu_A(x), \max(\gamma_B(x), \gamma_A(x))$
$\gamma_{LHS}(x) = \max(\gamma_A(x), \min(\mu_B(x), \mu_A(x))$
$\mu_{RHS}(x) = \min(\mu_A(x), \gamma_B(x))$
$\gamma_{RHS}(x) = \max(\gamma_A(x), \mu_B(x))$
$\min(\mu_A(x), \max(\gamma_B(x), \gamma_A(x))) = \min(\mu_A(x), \gamma_B(x))$
Similarly, $\max(\gamma_A(x), \min(\mu_B(x), \mu_A(x))) = \max(\gamma_A(x), \mu_B(x))$
Because if $\mu_B(x) \geq \gamma_A(x)$ then $\min(\mu_B(x), \mu_A(x)) \leq \mu_B(x)$
Thus, $\max(\gamma_A(x), \min(\mu_B(x), (\mu_A(x))) = \max(\gamma_A(x), \mu_B(x))$
Since both the membership and non- membership functions of LHS and RHS are equal for all $x \in X$. We conclude $A \cap (B^c \cup A^c) = A - B$
Theorem 4:
Let A and B be two Intuitionistic fuzzy sets defined on a non-empty set X.
Then $A^c \cap (A \cup B) = A^c \cap B$
Proof:
Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$ then
$A^c = \{\langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X\}$
$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma\gamma_B(x))\}$
$\mu_{LHS}(x) = \min(\gamma_A(x), \max(\mu_A(x), \mu_B(x)))$
$\mu_{RHS}(x) = \min(\gamma_A(x), \mu_B(x))$
$\min(\gamma_A(x), \max(\mu_A(x), \mu_B(x)) = \min(\gamma_A(x), \mu_B(x))$
If $\mu_A(x) \leq \mu_B(x)$ then $\max(\mu_A(x), \mu_B(x)) = \mu_B(x)$
$\Rightarrow \min(\gamma_A(x), \mu_B(x)) = \min(\gamma_A(x), \max(\mu_A(x), \mu_B(x))$
$\max(\mu_A(x), \min(\gamma_A(x), \gamma_B(x)) = \max(\mu_A(x), \gamma_B(x))\}$

If $\mu_A(x) \leq \mu_B(x)$ and $\mu_A(x) \leq \gamma_B(x)$ then $A^c \cap (A \cup B) = A^c \cap B$

Theorem 5:

Let A and B be two Intuitionistic fuzzy sets defined on a non-empty universe X.

Then A Δ B= if and only if A= B

Proof:

Let A= $\{\langle x, \mu_A(x), \gamma_A(x) \rangle: x \in X\}$ and B= $\{\langle x, \mu_B(x), \gamma_B(x) \rangle: x \in X\}$ then

A Δ B= (A - B) U (B - A)

A - B= $\{\langle x, \min(\mu_A(x), \gamma_A(x)), \max(\gamma_A(x), \mu_B(x) \rangle: x \in X\}$

Suppose A Δ B= Φ ⇒A - B= Φ and B - A= Φ

$\min(\mu_A(x), \gamma_B(x)) = 0$ and $\min(\mu_B(x), \gamma_A(x)) = 0$

⇒ $\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ for all $x \in X$

Hence A=B

Conversely,

Suppose A=B then $\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ for all $x \in X$

⇒ A-B= $\{\langle x, \min(\mu_A(x), \gamma_B(x), \max(\gamma_A(x), \mu_B(x) \rangle\}$

= $\{\langle x, \min(\mu_A(x), \gamma_A(x), \max(\gamma_A(x), \mu_A(x)) \rangle\}$

By the condition $\mu_A(x) + \gamma_A(x) \leq 1$

We know that at least one of the membership or non -membership degrees must be small enough such that the result becomes,

⇒ $\min(\mu_A(x), \gamma_A(x)) = 0$ and $\max(\gamma_A(x), \mu_A(x) \leq 1$

Thus A - B= Φ Similarly B-A= Φ

⇒ A Δ B= Φ if and only if A= B

Definition 5.1:

Let X be non-empty set. Intuitionistic fuzzy sets A, B, C x∈ X. The distance measures d between intuitionistic fuzzy sets A and B is a mapping d: X × X→ [0,1]; if d (A, B) satisfies the following axioms:

a) $0 \leq d(A, B) \leq 1$
b) d (A, B) if and only if A=B
c) d (A, B) = d (B, A)
d) d (A, C) + d (B, C) ≥d (A, B)
e) if A⊆B⊆C, then d (A, C) ≥d (A, B) and d (A, C) ≥ d (B, C)

Distance measures describe the difference between intuitionistic fuzzy sets and can be considered as the dual concept of similarity measures. Various distance measures for intuitionistic fuzzy sets have been proposed [12] & [13]

Definition 5.2:

Let A and B be two intuitionistic fuzzy sets defined in X and given by A= $\{\langle x, \mu_A(x), \gamma_A(x), \pi_A(x) \rangle | x \in X\}$, B= $\{\langle x, \mu_B(x), \gamma_B(x), \pi_B(x) \rangle | x \in X\}$ where X= $x_1, x_2, x_3, \ldots, x_n$, = 1, 2, …., n. Based on the geometric interpretation of intuitionistic fuzzy sets [7,8,9] introduced the following four distance measures to quantify the distance between A and B.

1.The Hamming Distance:

$d_H(A, B) = \frac{1}{2}\sum_{i=1}^{n}(| \mu_A(x_i) - \mu_B(x_i) | + | \gamma_A(x_i) - \gamma_B(x_i) | + | \pi_A(x_i) - \pi_B(x_i) |)$

2.The Euclidean Distance:

$d_E(A, B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\gamma_A(x_i) - \gamma_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))]^2}$

3.The Normalized Hamming Distance:

$d_{n\text{-}H}(A, B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i))| + |\gamma_A(x_i) - \gamma_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$

4.The Normalized Euclidean Distance:

$d_{n\text{-}E}(A, B) = \sqrt{\left(\frac{1}{2n}\right)\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\gamma_A(x_i) - \gamma_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$

Where $\mu_A(x_i)$ and $\mu_B(x_i)$ are the membership degrees of x_i in sets A and B respectively, $\gamma_A(x_i)$ and $\gamma_B(x_i)$ are the non-membership degrees of x_i in sets A and B respectively and $\pi_A(x_i)$ and $\pi_B(x_i)$ represents the degree of hesitation or uncertainty of x_i and n is the number of elements in the universe of discourse X.

Example 1:

In an E-learning course, two groups of students A and B were assessed on three key skills. Let x_1, x_2 and x_3 represents Time management, Technical Proficiency, Participation in discussions respectively. Let μ_A= {0.7,0.8,0.6},γ_A = {0.2,0.1,0.3} and μ_B= {0.6,0.9,0.5}, γ_B = {0.3,0.05,0.4} represents performance of group A and group B as intuitionistic fuzzy sets.

Using the above four distance measures we calculate the distance between two groups A and B as follows:

1. Using Hamming Distance between group A and group B is given by $d_H(A, B) = \frac{0.55}{6} = 0.0917$
2. The Euclidean Distance between group A and group B is given by $d_E(A, B) = \sqrt{0.00875} = 0.0935$
3. The Normalized Hamming Distance between group A and group B is given by $d_{n\text{-}H}(A, B) = \frac{0.55}{3} = 0.1833$
4. The Normalized Euclidean Distance between group A and group B is given $d_{n\text{-}E}(A, B) = \sqrt{0.0175} = 0.1323$

Among the four Distances, Hamming and Euclidean show very close values, indicating minimal differences between the group A and B. Normalized distances highlight individual skill variation more strongly. Euclidean distance best reflects overall performance similarity in E-learning. It balances membership and non- membership differences without exaggeration. Thus, Euclidean distance is the most suitable metric for comparing student groups.

6. Modeling Pattern recognition in E-Learning Using Intuitionistic Fuzzy Sets [11]:

A set of patterns is given in intuitionistic fuzzy sets and another unknown pattern called Sample is given in intuitionistic fuzzy sets. Both the sets of the pattern and that of the sample are within the same feature space 'n'. To find the distance between any of the patterns and the sample. Let each pattern A_j= {⟨x_i, $\mu_A(x_i)$, $\gamma_A(x_i)$, $\pi_A(x_i)$⟩: $x_i \in X$ where i=1,2…, n and

A_j = {A_1, A_2, A_3, …, A_m} foe each m ∈ N also there is a sample to be recognized that is

B= {⟨x_i, $\mu_B(x_i)$, $\gamma_B(x_i)$, $\pi_B(x_i)$ ⟩: $x_i \in X$} where i=1,2, …, n

Example:

In an E-Learning environment, we want to classify an unknown student's readiness based on Intuitionistic fuzzy sets describing five features F= {F1, F2, F3, F4, F5} It means F= {Internal Access quality, Device availability, Time management skill, Self Motivations, Online communication skills}. Let the students be

$\{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$ and eight pattern be represented by Intuitionistic Fuzzy Sets in X= $\{x_1, x_2, x_3, x_4, x_5\}$.
S_1= {(0.8,0.1,0.1), (0.7,0.2,0.1), (0.6,0.2,0.2), (0.7,0.1,0.2), (0.6,0.3,0.1)}
S_2= {(0.9,0.05,0.05), (0.6,0.2,0.2), (0.7,0.2,0.1), (0.6,0.3,0.1), (0.7,0.2,0.1)}
S_3= {(0.7, 0.1,0.2), (0.8,0.1,0.1), (0.6,0.1,0.3), (0.5,0.3,0.2), (0.6,0.2,0.2)}
S_4= {(0.6, 0.2,0.2), (0.5,0.3,0.2), (0.7,0.2,0.1), (0.4,0.4,0.2), (0.5,0.3,0.2)}
S_5= {0.9,0.1,0.0), (0.9,0.05,0.05), (0.8.0.1,0.1), (0.7,0.2,0.1), (0.8,0.1,0.1)}
S_6= {0.7,0.2,0.1), (0.6,0.3,0.1), (0.5,0.4,0.1), (0.6,0.2,0.2), (0.6,0.2,0.2)}
S_7= {0.5,0.3,0.2), (0.6,0.2,0.2), (0.6,0.3,0.1), (0.6,0.2,0.2), (0.7,0.2,0.1)}
S_8= {(0.8,0.1,0.1), (0.7,0.2,0.1), (0.7,0.1,0.2), (0.6,0.2,0.2), (0.8,0.1,0.1)}
Consider B is the new, unclassified student pattern whose characteristics are known but not yet assigned to any of the existing student classes (S_1 to S_8). Once computing its similarity to each known student pattern (S_1 to S_8) and based on the minimum distance, assign it to the closet known pattern. Let B= {(0.75, 0.15, 0.10), (0.65, 0.2, 0.15), (0.7, 0.15, 0.15), (0.6, 0.25, 0.15), (0.7, 0.2, 0.1) S, B x∈ X
Using Normalized Euclidean distance 4, We get the following results:
$d_{n-E}(S_1, B)$ =0.1265, $d_{n-E}(S_2, B)$ = 0.1000, $d_{n-E}(S_3, B)$ =0.1549, $d_{n-E}(S_4, B)$ =0.2000,
$d_{n-E}(S_5, B)$ = 0.1897, $d_{n-E}(S_6, B)$ = 0.1732, $d_{n-E}(S_7, B)$ = 0.1673, $d_{n-E}(S_8, B)$ =0.0894,
The minimum distance between S_8 and the unknown pattern B is 0.0894. The distance between S_4 and the unknown pattern B is 0.2000 is the greatest. We say that the unknown pattern B belongs to S_8 in terms of Internal Access Quality, Device availability, Time management skill, Self motivations, Online communication skills.

7. E-Learning research using Intuitionistic fuzzy sets and normalized Euclidean distance [7]:

Let there are 6 students say S= $\{S_1, S_2, S_3, S_4, S_5, S_6\}$, P= $\{P_1, P_2, P_3, P_4, P_5, P_6\}$ be the set of E-Learning readiness patterns (P1= High internal access and device quality, moderate motivation, moderate communication, P_2 = Moderate access, strong motivation, good communication, P_3= Moderate across all features with slightly lower device availability, P_4 =Balanced profile, average in all categories, P_5 = Strong self-motivation and communication skills, P_6 = High environment quality, strong device support, average time management) based on 6 features, F= $\{F_1, F_2, F_3, F_4, F_5, F_6\}$ where F_1= Internet Access quality, F_2= Device availability, F_3 =Time management skill, F_4 =Self motivations, F_5 = Online communication skills, F_6 =Internet and learning environment be a set of features. Each student and pattern are represented by an intuitionistic fuzzy set (μ, γ, π) for each feature.

Table I: E-learning readiness patterns:

Pattern	F_1	F_2	F_3	F_4	F_5	F_6
P_1	(0.90, 0.05, 0.05)	(0.85,0.10, 0.05)	(0.80, 0.15, 0.05)	(0.78, 0.12, 0.10)	(0.82, 0.10, 0.08)	(0.79, 0.15,0.06)
P_2	(0.88, 0.07, 0.05)	(0.82,0.13, 0.05)	(0.76, 0.18, 0.06)	(0.80, 0.15, 0.05)	(0.85, 0.08, 0.07)	(0.81, 0.14, 0.05)
P_3	(0.84, 0.10, 0.06)	(0.78,0.18, 0.04)	(0.75, 0.20, 0.05)	(0.76,0.17, 0.07)	(0.80, 0.15, 0.05)	(0.77, 0.18,0.05)
P_4	(0.86, 0.08, 0.06)	(0.80, 0.15, 0.05)	(0.79,0.16, 0.05)	(0.75, 0.18, 0.07)	(0.81, 0.13, 0.06)	(0.78, 0.16, 0.06)

P_5	(0.87, 0.06, 0.07)	(0.83,0.12, 0.05)	(0.78, 0.17, 0.05)	(0.77, 0.14, 0.09)	(0.83, 0.11, 0.06)	(0.80, 0.13, 0.07)
P_6	(0.89, 0.05, 0.06)	(0.84, 0.11, 0.05)	(0.77, 0.19, 0.04)	(0.79, 0.13, 0.08)	(0.84, 0.09, 0.07)	(0.82, 0.12,0.06)

Table 2: Intuitionistic fuzzy sets of Students

Student	F_1	F2	F3	F_4	F5	F6
S_1	(0.88,0.06, 0.06)	(0.81, 0.13, 0.06)	(0.77, 0.18, 0.05)	(0.74,0.18, 0.08)	(0.80, 0.12, 0.08	(0.76, 0.18, 0.06)
S_2	(0.85,0.08, 0.07)	(0.79, 0.16, 0.05)	(0.74,0.21, 0.05)	(0.75, 0.17, 0.08)	(0.79, 0.14, 0.07)	(0.75, 0.18, 0.07)
S_3	(0.86, 0.07, 0.07)	(0.80, 0.14,0.06)	(0.76, 0.19, 0.05)	(0.76, 0.16, 0.08)	(0.81, 0.13, 0.06)	(0.77, 0.17, 0.06)
S_4	(0.89, 0.05, 0.06)	(0.83,0.12, 0.05)	(0.78,0.17, 0.05)	(0.78, 0.14,0.08)	(0.83, 0.11, 0.06)	(0.79, 0.14, 0.06)
S_5	(0.87, 0.06,0.07)	(0.82, 0.13,0.05)	(0.75, 0.20, 0.05)	(0.77, 0.15, 0.08)	(0.82, 0.10, 0.08)	(0.78, 0.16, 0.06)
S_6	(0.90, 0.04, 0.06)	(0.84, 0.11, 0.05)	(0.76, 0.18, 0.06)	(0.79, 0.12, 0.09)	(0.84, 0.09, 0.07)	(0.81, 0.13, 0.06)

Table 3: Normalized Euclidean Distance Between Each Student and each Pattern

Student	P_1	P_2	P_3	P_4	P_5	P_6	Best match
S_1	0.04690	0.04899	0.04163	0.02517	0.03916	0.05260	P_4
S_2	0.06733	0.05944	0.02236	0.03559	0.05196	0.06557	P_3
S_3	0.04899	0.04282	0.02887	0.02236	0.03266	0.04761	P_4
S_4	0.02380	0.02887	0.05686	0.03606	0.01291	0.02236	P_5
S_5	0.040541	0.03109	0.04435	0.03512	0.02646	0.03317	P_5
S_6	0.02769	0.02944	0.07071	0.05447	0.02517	0.01414	P_6

This confirms that the model successfully identifies student alignment with E-learning readiness patterns using intuitionistic fuzzy sets and normalized Euclidean distance.

8. Conclusion

This research successfully presents a novel mathematical framework for adaptive E-learning module recommendation using intuitionistic fuzzy sets (IFS) and normalized Euclidean distance. By modeling each learner's readiness using intuitionistic fuzzy values capturing degrees of membership, non - membership, and hesitation, we affectively represent real-world uncertainity in student's E-learning behavior. The result shows that this solution, using intuitionistic fuzzy sets and Euclidean distance, offers a strong and easy to understand model for providing personalized learning experience. This framework offers a practical and interpretable solution for adaptive education, with potential for future expansion through real-time dynamic learning data. We extend the current model on Euclidean measure in intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set.

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