Balancing of Chemical Reaction Equations and Linear Diophantine Equations

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Abstract
The process of balancing chemical reaction equation is illustrated through representing the chemical reaction equation as a system of linear Diophantine equations and solving the same.

Keywords: Balancing chemical reaction equation , System of linear Diophantine equations, Integer solutions.

1. INTRODUCTION

Today, mathematicians have been attracted by the subject of Diophantine equations as it has plenty of interesting applications in various fields. In recent years , number theorists are attracted by the subject of chemistry as it requires integer solutions to the system of linear Diophantine equations which occurs in the process of balancing chemical reaction equation obtained by the reactions of chemical compounds and their products. In this context , one may refer [1-6].
In this paper, we consider balancing of some choices of chemical reaction equations through solving system of linear Diophantine equations.

illustration 1

Consider the chemical reaction equation

$$H_3PO_4 + Mg(OH)_2 \rightarrow Mg_3(PO_4)_2 + H_2O \qquad (1)$$

This reaction consists of four elements: Magnesium (Mg) ,Phosphorus (P), Hydrogen (H) and Oxygen (O). The above equation (1) is an unbalanced
chemical reaction as the number of atoms of each element is not equal on both sides of the reaction. That is, the reactant side (L.H.S. of (1)) and the product side
(R.H.S. of (1)) have a different number of atoms for the elements. To make (1) as the balanced chemical reaction , the procedure is illustrated below:
Given chemical reaction consists of two reactants and two products.
Let a, b be two unknown coefficients of reactants and c,d are two unknown coefficients of products. Then , consider (1) as

$$a\,(H_3PO_4) + b\,(Mg(OH)_2) \rightarrow c\,(Mg_3(PO_4)_2) + d\,(H_2O) \qquad (2)$$

By comparing the coefficients of corresponding elements on both sides of (2) , one has the following system of linear Diophantine equations:

For H: 3a+2 b=2 d (3)

For O: 4 a+2 b=8 c+d (4)

For P: a=2 c (5)

For Mg: b=3 c (6)

Using (5)&(6) in (3) ,we get

d=6 c (7)

Note that (5),(6)&(7) satisfy (4). Employing (5),(6)&(7) in (2) ,the chemical reaction equation is written as

$$2c\,(H_3PO_4) + 3c\,(Mg(OH)_2) \rightarrow c\,(Mg_3(PO_4)_2) + 6c\,(H_2O)$$

which represents balanced chemical reaction equation where c is an arbitrary non-zero positive integer.

illustration 2

Consider the unbalanced chemical reaction equation

$$Ca_3(PO_4)_2 + H_3PO_4 \rightarrow Ca\,(H_2PO_4)_2 \qquad (8)$$

This reaction consists of four elements: Calsium (Ca) ,Phosphorus (P), Hydrogen (H) and Oxygen (O). Given chemical reaction consists of two reactants and one product. Let a,b be two unknown coefficients of reactants and c the unknown coefficient of the product. Then , consider (8) as

$$a[Ca_3(PO_4)_2] + b\,(H_3PO_4) \rightarrow c\,(Ca\,(H_2PO_4)_2) \qquad (9)$$

By comparing the coefficients of corresponding elements on both sides of (9) , one has the following system of linear Diophantine equations:

For H: 3b=4 c (10)

For O: 8 a+4 b=8 c (11)

For P: 2 a+b=2 c (12)

For Ca: 3 a= c (13)

From (12) & (13),we get

b=4 a (14)

Note that (13)&(14) satisfy (10)&(11). Employing (13)&(14) in (9) ,the chemical reaction equation is written as

$$a[Ca_3(PO_4)_2] + 4a\,(H_3PO_4) \rightarrow 3a\,(Ca\,(H_2PO_4)_2)$$

which represents balanced chemical reaction equation where a is an arbitrary non-zero positive integer.

Illustration 3

Consider the unbalanced chemical reaction equation

$$KMnO_4 + H_2SO_4 + H_2O_2 \rightarrow K_2SO_4 + MnSO_4 + H_2O + O_2 \qquad (15)$$

This reaction consists of five elements: Manganese (Mn) ,Sulfur (S), Potassium(K) ,Hydrogen (H) and Oxygen (O). Given chemical reaction consists of three reactants and four products. Let a,b ,c be three unknown coefficients of reactants and d,e,f,g the unknown coefficients of the products. Then , consider (15) as

$$a(KMnO_4)+b(H_2SO_4)+c(H_2O_2)\rightarrow d(K_2SO_4)+e(MnSO_4)+f(H_2O)+g(O_2) \qquad (16)$$

By comparing the coefficients of corresponding elements on both sides of (16) , one has the following system of linear Diophantine equations:

For O: $4a+4b+2c=4d+4e+f+2g$ (17)

For Mn: $a=e$ (18)

For K: $a=2d$ (19)

For S: $b=d+e$ (20)

For H: $2b+2c=2f$ (21)

Using (18),(19)&(21) in (17) ,we have

$3b+c=2a+2g$ (22)

Multiplying (22) by 2, one has

$6b+2c=4a+4g$ (23)

Using (20) in (23) ,we get

$6d+6e+2c=4a+4g$

In view of (18)&(19) ,from the above equation ,we have

$2c=4g-5a$

Taking

$a=2t$ (24)

in the above equation, we get

$2g=5t+c$

Choosing

$c=2s-t$ (25)

we obtain

$g=2t+s$ (26)

In view of (18),(19),(20)&(21), it is seen that

$e=2t,\ d=t,\ b=3t,\ f=2s+2t$ (27)

From (16) ,the balanced chemical reaction equation is

$$2t(KMnO_4)+3t(H_2SO_4)+(2s-t)(H_2O_2)\rightarrow t(K_2SO_4)+2t(MnSO_4)+(2s+2t)(H_2O)+(2t+s)(O_2)$$

where t,s are arbitrary non-zero positive integers.

Illustration 4

Consider the unbalanced chemical reaction equation

$$KMnO_4+FeSO_4+H_2SO_4\rightarrow KHSO_4+MnSO_4+Fe_2(SO_4)_3+H_2O \qquad (28)$$

This reaction consists of six elements: Manganese (Mn) ,Sulfur (S), Potassium(K),Iron (Fe) , Hydrogen (H) and Oxygen (O). Given chemical reaction consists of three reactants and four products. Let a,b ,c be three unknown coefficients of reactants and d,e,f,g the unknown coefficients of the products. Then , consider (28) as

$$a(KMnO_4)+b(FeSO_4)+c(H_2SO_4)\rightarrow d(KHSO_4)+e(MnSO_4)+f(Fe_2(SO_4)_3)+g(H_2O)$$

(29)

By comparing the coefficients of corresponding elements on both sides of (29) , one has the following system of linear Diophantine equations:

For K: $a=d$ (30)

For Mn: $a=e$ (31)

For Fe: $b=2f$ (32)

For S: $b+c=d+e+3f$ (33)

For H: $2c=d+2g$ (34)

For O $4a+4b+4c=4d+4e+12f+g$ (35)

Multiplying (33) by 4,we get

$4b+4c=4d+4e+12f$ (36)

(35)-(36) gives $g=4a$ (37)

Using (30)&(37) in (34) ,we get

$2c=9a$ (38)

Using (30),(31) ,(32) in (33) ,we have

$f=c-2a$ (39)

Taking

$a=2t$

in (30),(31) , (37),(38),(39)& (32) ,we have

$d=e=2t, g=8t, c=9t, f=5t, b=10t$

From (29) , the balanced chemical reaction equation is

$$2t(KMnO_4)+10t(FeSO_4)+9t(H_2SO_4)\rightarrow 2t(KHSO_4)+2t(MnSO_4)+5t(Fe_2(SO_4)_3)+8t(H_2O)$$

where t is an arbitrary non-zero positive integer.

Note: In Illustrations 3 and4 ,choosing suitably the values to t & s ,the balanced chemical reaction equations in [1] are obtained.

2. Conclusion

In this paper , we have presented balancing of four choices of chemical reaction equations through solving system of linear Diophantine equations. One may search for the applications of system of linear Diophantine equations to other branches of mathematics.

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