

On MHD viscoelastic non-Newtonian nanofluid flow past a moving surface with application to chemical vapor deposition

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Abstract:

The UCM fluid model represents a viscoelastic non-Newtonian fluid that can be widely used for the fluids which possess both elastic and viscous properties, like most solid or fluid-like matters. It plays an especially important role in polymer processing, coating flows, and CVD-related modelling. This present study focuses on the significance of thermal behaviour of non-Newtonian fluid on nanofluid flow past a linear velocity stretching sheet. Similarity transformations are embedded to convert the fluid flow governing equations into ordinary differential equations and solved numerically by using shooting technique and RK45 method. It is observed that viscoelastic parameter suppresses the velocity distribution whereas it increases the temperature and concentration distribution.

Key words: MHD, non-Newtonian fluid flow, shooting method.

1. Introduction

Chemical vapor deposition processes in which polymeric or organometallic precursors are involved can be successfully modelled by an Upper-Convected Maxwell fluid over a stretching surface. The UCM model is one that contains viscoelastic memory effects due to which momentum, heat, and mass transport within the boundary layer is significantly affected. Due to elastic stresses, the concentration gradients and reaction rates at the substrate modify and, hence, deposition flux, film thickness, and uniformity control. The present modelling framework provides an insight view into the optimization of the advanced CVD process where the non-Newtonian behaviour cannot be neglected. Due to these wide range of advantages in upper convected maxwell fluid many researchers investigated boundary layer flows problem using this model. Vajravelu et al.[1] studied UCM model over at stretching sheet with chemical reaction effects. Sadeghy et al.[2] reported this model with flat plate fluid flow problem with low Reynolds number model. Hayat T et al. [3] presented series solutions to the non-Newtonian viscoelastic fluid flow past a moving surface. Later this work is extended by Hayat T and Sajid [4] with the contribution roles of magnetic field and thermal radiation and their results shows that radiative flows highly impact non-Newtonian thermal boundary layers. Kumari and Gopinath [5] studied about the importance of mixed convection effect on viscoelastic model. The authors in this study explored about significance of second grade fluid model under applied magnetic field strength and its impact on thermal energy and drag force at fluid surface interface. Their results show that temperature profile suppresses for non-Newtonian parameter whereas friction is in non-decreasing mode. Kumar and Uma [6] reported melting heat transfer effect on non-Newtonian fluid flows and compared with characteristics of Newtonian case. Sui et al.[7] examined the non Fourier heat flux model for UCM fluid flow over a moving surface. Later Subash Abel et al.[8] considered magnetohydrodynamics issues in non-Newtonian flows particularly in viscoelastic models. Time dependent UCM model was investigated by Palani et al.[9] in which chemical reaction of viscoelastic models in foams, polymer melts and coatings are focused more. Later this research work is extended by Swati Mukhopadhyay et al.[10] for explaining the behaviour of viscoelastic model in porous

media under uniform suction and blowing effects. Chemical vaporisation and deposition and the flow dynamics are usually explained in terms of the Upper, Convected Maxwell (UCM) model, which combines both viscous and elastic stresses through an upper, convected derivative of the stress tensor. This model guarantees frame, invariance and thus takes into account the stretching as well as the rotating of the fluid elements under strong thermal and shear gradients that are typical of CVD reactors. (See [11], [12], [13], [14]). In a CVD reactor with UCM fluid flow the equations of momentum, energy, and species transport are very closely intertwined. The velocity field and boundary layer structure near a heated substrate are changed by the elastic stresses which also affect heat transfer and mass transport of reactive species to the surface. Application of nanofluid is widely focused now a days due to its fast heat exchange capability. Many researchers focused on including thermophysical properties of nanoparticles in boundary layer also. Sulochana and Kumar [15] examined the importance of electromagnetic forces in boundary layer flow past a stretching sheet. Pavithra et al. [16] extended the models to tri-hybrid nanofluid flow problems with advanced neural network models. Recently hybrid nanofluids are widely used in many industrial applications for enhancing heat transfer in fluid flows over a stretching sheet. (See [17], [18], [19]). Zainal et al. [20] investigated hybrid nanofluid flow past a stretching/shrinking surface using upper convected Maxwell model. Roy and Kairi [21] considered this model for discussing Bio convection problem with chemical reaction effects. Cao Limei et al. [22] analysed UCM model using Lei group technique for understanding thermophysical properties impact on viscoelastic models. In view of the above cited references, many of the researchers discussed upper convected Maxwell fluid flow with stretching shrinking sheet, but many authors restricted their discussion to non-porous media flows. So, the main aim of this study is to explore the viscoelastic model on a porous medium with Brownian motion and thermophoresis effects.

2. Problem statement and mathematical representation of fluid flow.

Assume an incompressible viscoelastic fluid over a linear stretching sheet with multiple slip effects. Consider the stretching sheet velocity is assumed as $U_w(x) = ax$. Where a is stretching constant. Transverse magnetic field $B_0(x)$ is applied in y -axis direction Figure-1 represents physical model of the CFD problem and their boundary conditions. The non-Newtonian fluid flow direction takes place in x -axis direction and y -axis is placed such that it is perpendicular to the stretching sheet. With these assumptions and following the research works of (See [10], [23]) the governing partial differential equations for mass momentum of conservation, energy and concentration can be written as follows.

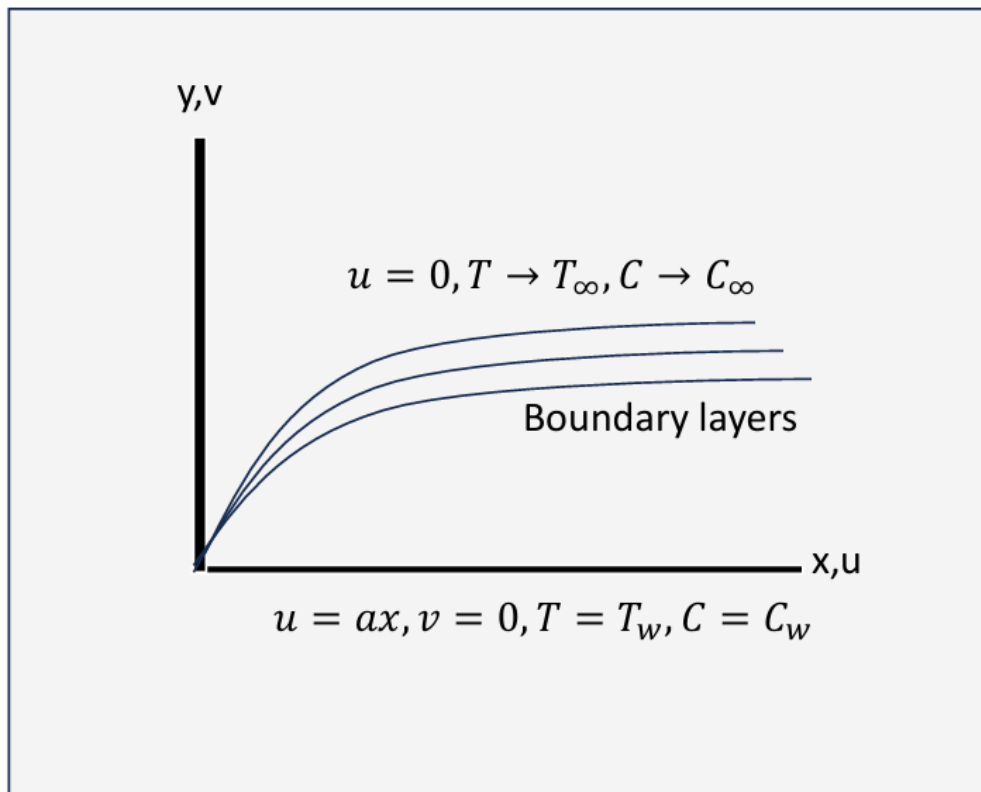


Figure 1: Physical representation of the fluid flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial x^2} - \gamma \left(2uv \frac{\partial^2 u}{\partial x \partial y} + u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} \right) + \left(\frac{\sigma B_0^2}{\rho_f} \right) u - \frac{v}{K} u \quad (2)$$

$$\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \alpha \left(\frac{\partial^2 T}{\partial y^2} \right) + \tau \left[D_B \left(\frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

Boundary conditions are assumed as follows:

$$u = ax, v = 0, T = T_w, C = C_w \text{ at } y = 0. \quad (5)$$

$$u = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ at } y = \infty \quad (6)$$

3. Similarity transformation and non-dimensionalised system

Since the governing equations (2)-(4) are highly non linear and not possible to obtain analytical solution for the same. Hence we will transform these equations into system of ordinary differential equations using the following similarity variables.

$$\eta = \sqrt{\frac{a}{\nu}} y, u = af'(\eta), v = -\sqrt{av} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \quad (7)$$

After utilising the above similarity variable, the non dimensionalised differential equations are

$$(1 - \gamma f^2) \frac{\partial^3 f}{\partial \eta^3} - \left(\frac{\partial f}{\partial \eta} \right)^2 + f \frac{\partial^2 f}{\partial \eta^2} + (M + k_p) \left(\frac{\partial f}{\partial \xi} \right) + 2\gamma f \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta^2} = 0 \quad (8)$$

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} + Pr f \frac{\partial \theta}{\partial \eta} + \frac{\partial \theta}{\partial \eta} \frac{\partial \phi}{\partial \eta} Nb + Nt \left(\frac{\partial \theta}{\partial \eta} \right)^2 = 0 \quad (9)$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + Le f \frac{\partial \phi}{\partial \eta} + \frac{Nt}{Nb} \left(\frac{\partial^2 \theta}{\partial \eta^2} \right) = 0 \quad (10)$$

And the non dimensionalised boundary conditions are

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \text{ and } f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \quad (11)$$

physical quantities can be defined as skin friction coefficient $C_f = \frac{\tau_w}{\rho u_w^2}$ and Nusselt number $Nu_x = \frac{xq_w}{k_f(T_f - T_w)}$, Sherwood number $Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}$ where $\tau_w = \mu(1 + \delta) \frac{\partial u}{\partial y}$, $q_w = -k \left[\frac{\partial T}{\partial y} \right]_{y=0} + q_r$ and $q_m = -D_B \left[\frac{\partial c}{\partial y} \right]_{y=0}$. and non dimensionalised forms of these physical quantities are :

$$C_f \sqrt{Re_x} = \left[\frac{\partial^2 F}{\partial \eta^2} \right]_{\eta=0}, \quad \frac{Nu_x}{\sqrt{Re_x}} = - \left[\frac{\partial \theta}{\partial \eta} \right]_{\eta=0} \quad \text{and} \quad \frac{Sh_x}{\sqrt{Re_x}} = - \left[\frac{\partial \phi}{\partial \eta} \right]_{\eta=0} .$$

3. Results and discussions.

This segment explores the outcomes of this investigation the governing ordinary differential equations are solved by shooting method by computing missing boundary conditions. Significance of fluid flow parameters on Momentum, energy and concentration profiles. Figure 2 shows the importance of applying transverse magnetic field on the stretching sheet boundary layer flow to stabilise the flow. It is observed that velocity profile is decreased for higher values of magnetic parameter. The main reason for this behaviour could be: magnetic field stabilizes boundary layer velocity by generating Lorentz forces that oppose fluid motion, damp velocity fluctuations, and suppress instabilities in electrically conducting flows. Figure 3 shows the impact of magnetic parameter on temperature profiles from this figure it is evident that energy boundary layer diminishes for strong magnetic fields. Figure 4 shows the role of magnetic force on volume fraction concentration profile which has upside trend.

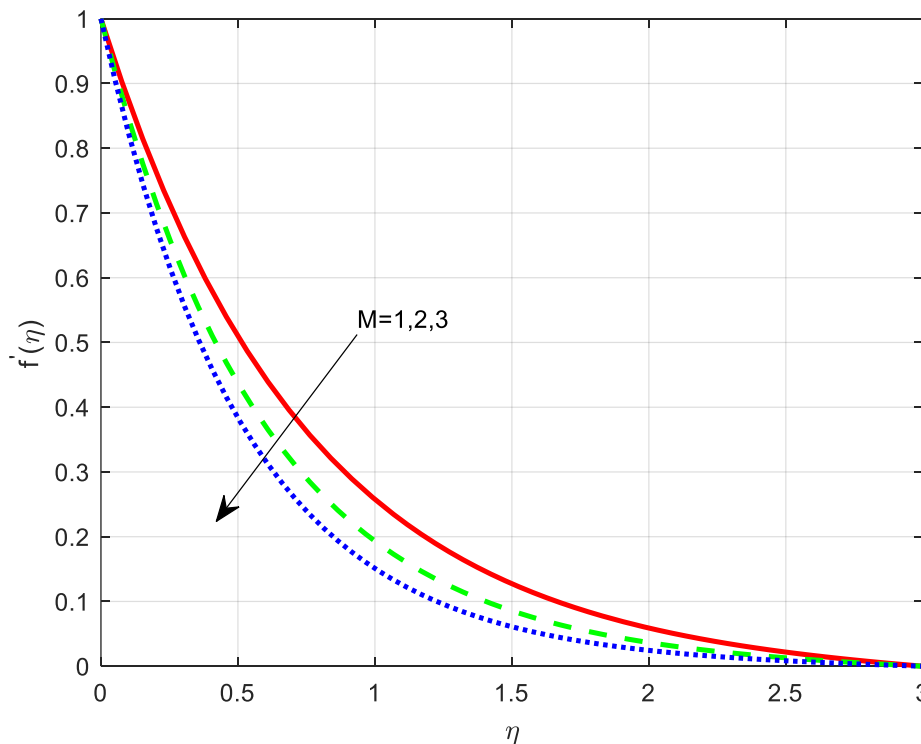


Figure 2: Impact of Lorentz force on velocity.

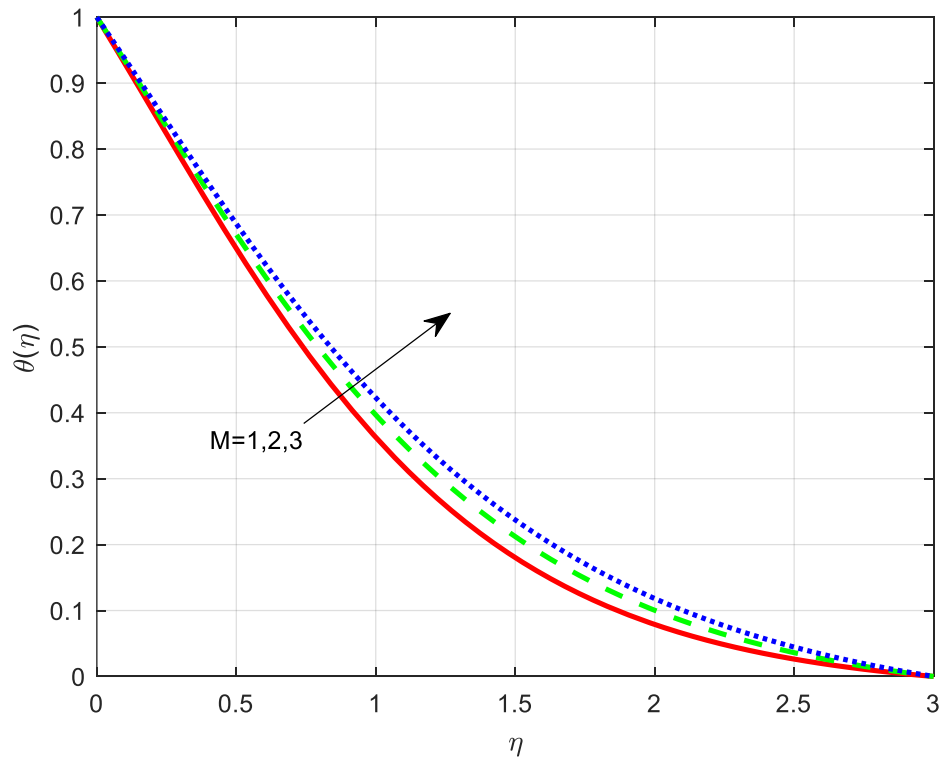


Figure 3: Impact of Lorentz force on energy profile.

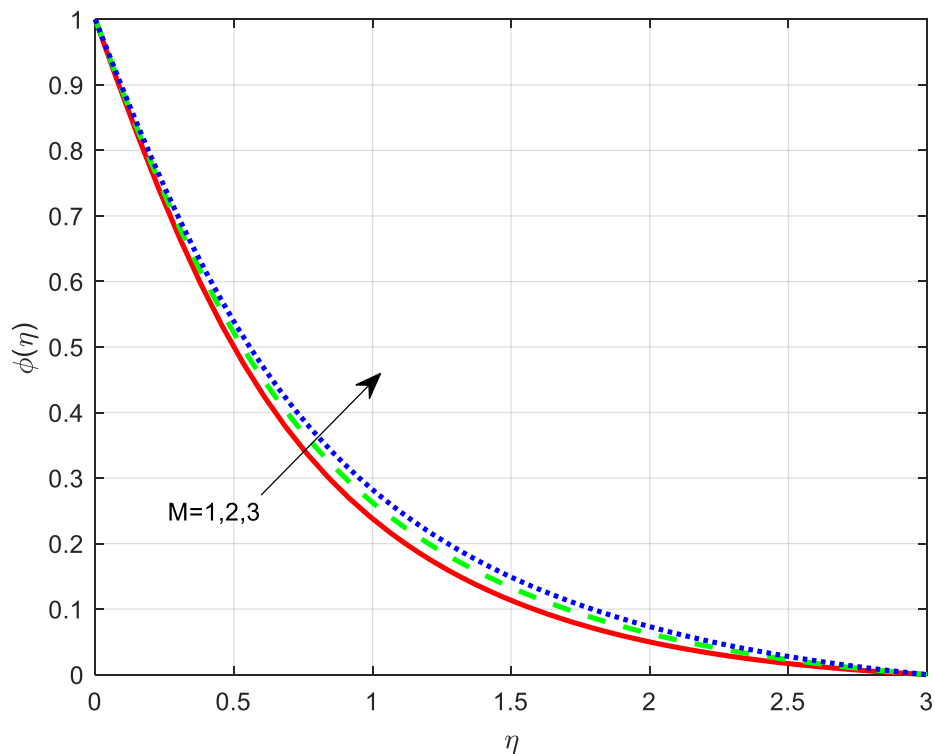


Figure 4: Impact of Lorentz force on energy profile.

The Deborah number (De), which measures the flow's elastic effects directly, is one of the most crucial parameters in Upper-Convected Maxwell (UCM) fluid flow. In UCM fluid flow, the Deborah number measures how significant elastic memory effects are in relation to the flow time scale. Significant

departures from Newtonian flow and potential stress singularities at high values result from an increase in the Deborah number, which also increases elastic stresses, normal stress differences, and non-Newtonian behaviour. Figure 5 shows the effect of visco elastic parameter (Deborah number) on momentum profile. Increasing the Deborah number in boundary-layer flows of a UCM fluid increases elastic effects, which decrease momentum diffusion and oppose deformation. In comparison to the Newtonian case, this results in a decrease in streamwise velocity within the boundary layer, a reduction in velocity gradients at the wall, and a thickening of the boundary layer.

Figure 6 reports the impact of Deborah number on energy profile. An increase in the Deborah number generally leads to a rise in fluid temperature within the boundary layer. Because the flow slows down under stronger elastic effects, fluid particles remain near the heated surface for a longer duration, allowing more heat to accumulate. When viscous dissipation is considered, elastic stresses further contribute to internal heat generation, which raises the temperature inside the boundary layer. Finally, the wall temperature gradient decreases as the Deborah number increases, leading to a reduction in the local Nusselt number. This indicates that heat transfer from the surface to the fluid becomes less efficient at higher Deborah numbers. Overall, increasing De enhances elastic effects that suppress convective heat transfer, elevate fluid temperature, and thicken the thermal boundary layer in UCM fluid flows. From Figure 7 the thickness of the concentration boundary layer increases with an increase in Deborah numbers. Due to no convection occurring in this problem due to elastic stresses, species diffusion into the free stream occurs more slowly than in a Newtonian fluid. Because of this reason too, a thicker boundary exists in comparison to a Newtonian fluid. Also, the effect of increasing Deborah numbers on increasing the wall-concentration gradient, which is related to the Sherwood number, cannot be overlooked. From the analysis above, the conclusion can be drawn that increasing Deborah numbers improves elastic resistance, reduces convective mass transport, raises the accumulation level near the surface, and extends the boundary layer in the process of UCM fluid flow.

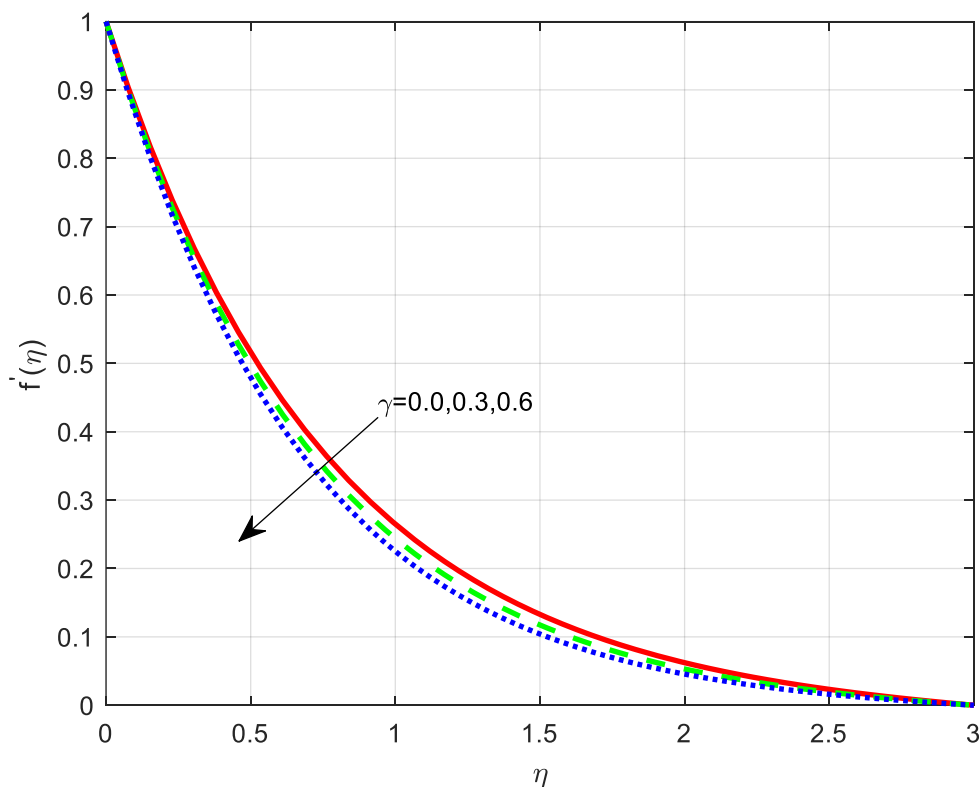


Figure 5 : Impact of visco elastic term on velocity.

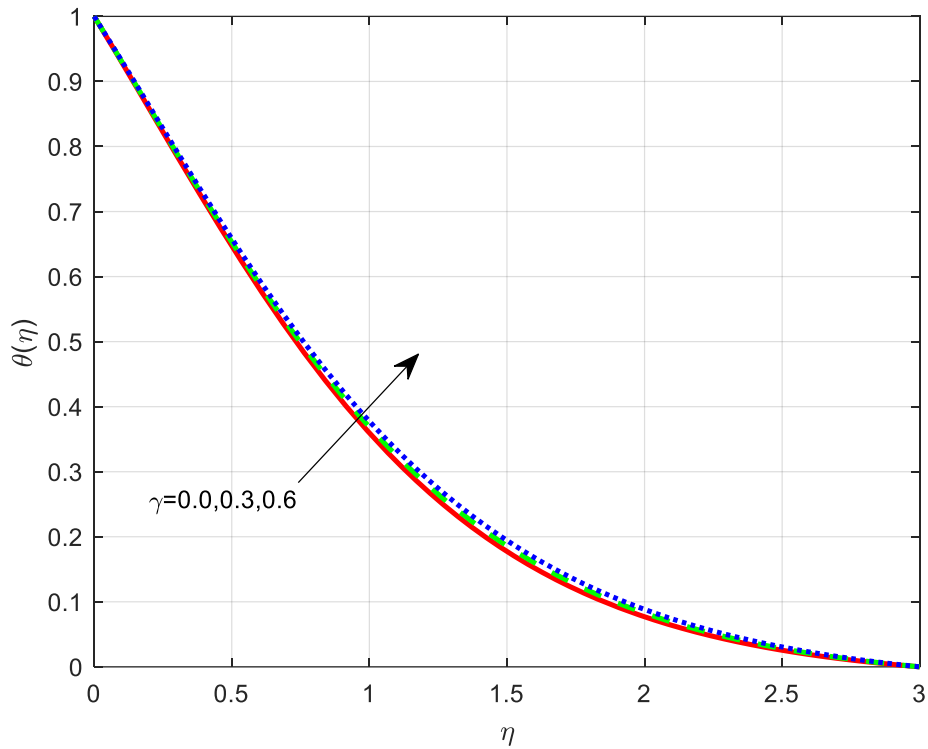


Figure 6 : Impact of visco elastic term on temperature.

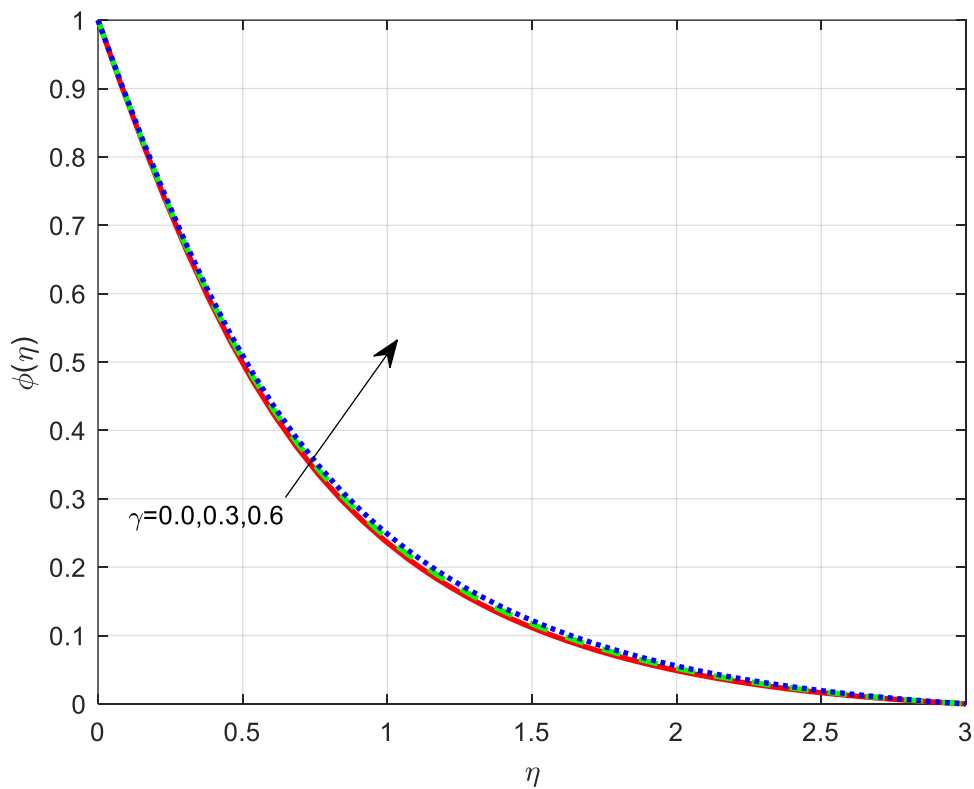


Figure 7: Impact of visco elastic term on concentration.

4. Concluding remarks.

This current numerical study examines the non-Newtonian viscoelastic fluid flow past a stretching sheet with porous media. The governing equations of the computational fluid dynamics problems are frames with the help of similarity transforms and boundary layer approximations and then solved by RKF45 method along with shooting technique for missing boundary condition. Some of the important outcomes of this study are follows:

- The magnetic parameter decreases the momentum profile due to strong Lorentz force.
- Lewis number increase the concentration profile.
- Deborah number suppresses the momentum but opposite behaviour is noticed in thermal and concentration profiles.
- Temperature and concentration profiles enhance with higher magnetic strength.

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