New Approach Formulation in Two-Level Trade-Credit Inventory Model with Reminder Cost

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Abstract
This study introduces a new approach formulation in trade credit model for deteriorating items that accounts for default risk and reminder costs, inspired by practical strategies observed in rural India. By incorporating these elements, the model aims to minimize customer defaults and improve retailer profitability. Optimization is performed using the generalized reduced gradient (GRG) method, and the results reveal that implementing reminder costs can lead to a substantial increase in overall profit.

Keywords: Two-level trade-credit, Default risk, Reminder cost, New approach

Introduction
In today's intensely competitive market landscape, trade credit has emerged as a widely adopted incentive across supply chain participants—manufacturers, distributors, and retailers—to boost purchasing capacity among downstream partners. This financing tool enhances liquidity for buyers and encourages increased order volumes. For example, trade credit is employed in nearly 80% of commercial exchanges in several European countries [1]. Traditional EOQ models, however, do not consider such delayed payment mechanisms and instead assume immediate payment upon delivery, which can strain the buyer's finances and reduce purchase potential.
To alleviate this constraint, suppliers often extend a credit duration (CP), enabling purchasers to defer payment without any upfront cost, with settlement due at the end of the credit window. If the buyer fails to pay within the agreed period, interest charges are typically imposed on the overdue amount. Meanwhile, retailers may earn returns on revenue during the credit phase, further enhancing the attractiveness of trade credit options [2].
However, the extended payment timeline introduces the risk of non-payment. Some customers exploit the system by defaulting, which poses financial challenges for sellers, particularly when the credit window is long. In rural areas of India, a common practice among shopkeepers involves employing agents to track and remind defaulters, thereby attempting to recover unpaid dues. This leads to an additional cost termed as "reminder cost," incurred to reduce default.
This research develops an EOQ model for perishable goods within a two-tier trade credit framework, where the wholesaler extends credit to the retailer, who, in turn, partially offers credit to end consumers. The model incorporates two novel components that significantly enrich the classical trade credit inventory structure:

1. **Reminder Cost as a Default-Reduction Strategy:** We introduce *reminder cost* as a proactive mechanism aimed at lowering the likelihood of customer default by encouraging timely repayment. Though effective in recovering dues, this strategy may inadvertently discourage some buyers, thereby dampen demand and affect profitability. Results reveal that retailer profit initially increases with reminder expenditure but begins to decline after reaching a threshold, indicating a clear trade-off. Moreover, the profit function exhibits a concave relationship with reminder cost, confirming the presence of an optimal investment level.
2. **Bank Loan-Based Payment Mechanism:** In addition to reminder costs, the model proposes a retailer-centric payment system inspired by contemporary bank loan practices. Under this scheme, the retailer secures a bank loan—typically at a lower interest rate than that offered by the wholesaler—to settle the supplier's dues and meet ordering expenses at the conclusion of the credit period. The loan is repaid in equal instalments using sales proceeds and interest income accrued during the cycle. By adjusting the repayment timeline, the retailer seeks to optimize profitability. Once the loan is fully repaid, any remaining earnings and accumulated interest represent the retailer's net profit.

Key features of some investigations are presented in Table 1

Table 1: Key features of some trade credit inventory models

References	Trade-credit	Default risk	Reminder cost	Formulation approach
[3]	One-level	✓	✗	Conventional
[4]	Two-level	✗	✗	Conventional
[5]	------------	✗	✗	Conventional
[6]	Two-level	✗	✗	Conventional
[7]	------------	✗	✗	Conventional
[8]	Two-level	✗	✗	Conventional
[9]	------------	✗	✗	Conventional
[10]	Two-level	✗	✗	Conventional
[11]	One-level	✗	✗	Conventional
This investigation	Two-level	✓	✓	New

NOTATIONS

C_o: Setup cost per order incurred by the retailer (including interest) in $

C_p: Unitary cost for the retailer's purchase in $

C_s: Unit-wise selling price for the retailer in $

h: Annual retailer's unit storage cost, payable at the wrap-up the business cycle in $

I_e: Retailer's annual earned interest rate in $

I_c: Interest charge of the wholesaler per year in $; with $I_c > I_e$

I_b: Bank interest rate per year in $ ($I_e < I_b < I_c$)

M: Wholesaler to the retailer CP

λ: Constant deterioration rate

Q; The quantity of orders placed by the retailer
n: Number of installments in which retailer's pays loan amount with compound interest
D(N,r_c): Annual consumption rate, function of r_c and N
a_1: Annual base demand without N and r_c
I(t): Stock level at time t
PC: Total purchasing price of retailer
HC: Total holding cost
r_c: Reminder level (decision variable)
N: Retailer's trade credit duration for the customer in years (decision variable)
T: Optimal cycle duration in years (decision variable)
wrt means with respect to
TLTC means two-level trade credit
CP means credit period

ASSUMPTIONS

The mathematical model is formulated under the assumptions stated as follows

1. This study considers a case with one dedicated wholesaler and one dedicated retailer, centered around a perishable item. Owing to its perishability, the model is examined within a one-time period setting.
2. The rate of deterioration is constant.
3. The mathematical formulation is based on the TLTC inventory model proposed, where the wholesaler and retailer offer credit periods M and N to the retailer and customers, respectively.
4. If the buyer does not settle the payment within the allowed credit time M, the seller imposes interest on the unpaid amount until full repayment. Simultaneously, the buyer accrues interest on the income received from selling the goods.
5. Offering credit to buyers introduces the possibility of non-payment by some. To mitigate this risk, the seller uses measures like hiring staff to follow up with defaulters. This leads to an extra expense, referred to as the reminder cost (RC), which is represented as:
$$RC = \rho r_c^2$$
where $r_c$is the reminder level and ρ is a cost coefficient. Clearly, if $r_c = 0$, then RC = 0.
6. The reminder strategy reduces the number of defaulters. As defaults increase with longer credit periods, the default risk is modeled as a function of r_c and N, given by:
$$d = d(N, r_c) = 1 - e^{(\beta N - a_1 r_c)}$$
where $a_1 \geq 0$ is a constant and $\beta N - a_1 r_c > 0$ to ensure $d(N, r_c) > 0$. This expression shows that default risk increases with N and decreases with r_c.
7. Some customers may find the reminder mechanism undesirable, leading to a negative impact on demand. Therefore, demand D (r_c, N) is modeled as a linear function of r_c and N:
$$D = D(r_c, N) = a_1(1 + a_2 N - \alpha r_c)$$
where a_1, a_2, and α are positive constants.
8. In the proposed framework, all sales proceeds are initially funneled into an interest-bearing account. To manage upfront expenditures—such as ordering, operational, and miscellaneous costs—the retailer relies on short-term local financing. At a specified time MMM, the accumulated funds, along with the earned interest, are used to partially settle payments to the wholesaler and repay the local

borrowings. Given that bank loans typically offer more favorable interest rates compared to wholesalers or informal lenders, the retailer then secures a bank loan to cover any remaining obligations. This loan is repaid in fixed installments under compound interest terms, utilizing both the ongoing sales income and the interest accrued in the account. The repayment duration is optimized to achieve maximum profitability. After clearing the loan, the leftover funds and accumulated interest represent the retailer's net earnings. This systematic repayment strategy effectively reduces financing costs and leverages earned interest to enhance overall profit.

MATHEMATICAL FORMULATIONS

A wholesaler supplies a perishable item to a retailer, granting a trade CP M for payment settlement. If the retailer delays payment beyond this duration, the wholesaler levies interest on the remaining balance at a rate I_c. The retailer, upon acquiring the product, extends a CP N to end consumers. The length of this period influences customer purchase behaviour.

Yet, some customers misuse the prolonged payment window and fail to pay. The proportion of such defaults tends to rise with an increase in the retailer's credit duration N. To curb these defaults, the retailer assigns staff to follow up with credit customers, incurring a cost modelled as r_c^2, where r_c reflects the intensity of reminder efforts. Increasing r_c effectively lowers the number of defaulters.

However, frequent reminders may be viewed as bothersome by some buyers, discouraging them from future purchases and thereby suppressing demand. Thus, although reminders reduce the likelihood of payment defaults, they may simultaneously have an adverse impact on customer demand. Considering all these dynamics, we proceed with the following model.

The stock level of EOQ model at time t is $\frac{dI(t)}{dt} = -D(r_c, N) - \lambda I(t)$, subject to $I(T) = 0$ and $I\ (0)\ = Q$. On solving this, we have $I(t) = \frac{D(r_c,N)}{\lambda}(e^{\lambda(T-t)} - 1)$ and total order quantity $Q = \frac{D(r_c,N)}{\lambda}(e^{\lambda T} - 1)$. Therefore, total purchasing price $PC = CpQ$ and total holding cost $HC = \frac{hD(r_c,N)}{\lambda^2}(e^{\lambda T} - \lambda T - 1)$.

Based on numerical value of $N, M\ and\ T$. There are two potential cases arises $T + N \geq M\ and\ T + N \leq M$ for retailer's profit under new approach. Here, we shall formulate the mathematical model under the case of $T + N \geq M$ and for the other case, formulation can be obtained similarly

In this setting, the retailer uses the sales proceeds and the interest earned during the period $[0, M - N]$ to pay the wholesaler at time M. To settle the remaining dues—such as purchasing and additional operational expenses—at time M, the retailer opts for a bank loan since the bank offers a lower interest rate than the wholesaler. The borrowed sum, along with accrued compound interest, is repaid in equal installments, ideally by a chosen time t' where $M \leq t' \leq N + T$, to optimize profit. The total earnings from product sales in $[0, M - N]$ and the interest accrued on that revenue from $N\ to\ M$ is given as

$$SR_1 = C_s D(1-d)(M-N)\left[1 + I_e \frac{M-N}{2}\right]$$

After paying SR_1 at M, remaining dues is $DA_1\ (say)\ = PC + OC - SR_1$. To cover this amount, the retailer secures a bank loan at a compound interest rate I_b and repays it over the period up to time t' (to be determined), through n equal payments. Each of these payments, referred to as the Equal Payment Installment (EPI), (cf. http://www.paisabazaar.com/emi-calculator) is given by

$$EPI = DA_1 \frac{I'_b(I'_b + 1)^n}{(I'_b + 1)^n - 1}, \qquad where\ \ I'_b = \frac{I'_b(t' - M)}{n}$$

Sum of revenue from $M - N\ to\ t' - N$ earned interest on it

$$SR_2 = C_s D(1-d)(t'-M)\left[1 + I_e \frac{t'-M}{2n}\right]$$

Now t' satisfied the following equation

$$n \times EPI = SR_2$$

Sum of the sales revenue from $t' - N\ to\ T$ and earned interest on it from $t'\ to\ N+T$

$$SR_3 = C_s D(1-d)(N+T-t')\left[1 + I_e \frac{N+T-t'}{2n}\right]$$

Hence total profit amount

$$TPN = SR_4 - HC - RC$$

NUMERICAL EXPERIMENT

Let's consider an inventory model with following parameter's: $a_1 = 3799, a_2 = 1.99, C_p = 1, C_s = 1.49, C_o = 246, I_e = 8.1\%, I_b = 13.3\%, I_c = 17.5\%, \beta = 0.3, M = 0.25, \alpha = 0.0146, \alpha_1 = 0.03, h = 0.2, \lambda = 10.1\%\ and\ \rho = 195.$

Optimizing TPN using LINGO 21.0 software, we obtain the optimal solution is $T^* = 0.506, t' = 0.51. r_c^* = 0.259, N^* = 0.117\ and\ TPN^* = \$\ 1216.29$. The optimal profit without reminder level i.e., $r_c = 0$ is $TPN^* = \$\ 1192.98$, mins profit is decreasing.

Now, the optimal profit with and without reminder level with different number of installments

Number of installments	**2**	**3**	**4**	**5**	**6**	**7**
Profit with reminder level	1229.986	1230.991	1232.691	1233.894	1234.693	1236.027
Profit without reminder level	1207.895	1212.444	1213.296	1214.877	1215.185	1215.041

It is clear from the table; profit is more with reminder level and also increasing with installments increases within time.

NATURE OF THE PROFIT FUNCTION

Here the Hessian matrix of the profit function in wrt all decision variable is

$$\begin{bmatrix} \frac{\partial^2 TPN}{\partial N^2} & \frac{\partial^2 TPN}{\partial N \partial T} & \frac{\partial^2 TPN}{\partial N \partial r_c} \\ \frac{\partial^2 TPN}{\partial N \partial T} & \frac{\partial^2 TPN}{\partial N^2} & \frac{\partial^2 TPN}{\partial T \partial r_c} \\ \frac{\partial^2 TPN}{\partial N \partial r_c} & \frac{\partial^2 TPN}{\partial T \partial r_c} & \frac{\partial^2 TPN}{\partial r_c^2} \end{bmatrix} = \begin{bmatrix} -8723.18 & 3452.43 & 299.60 \\ 3452.43 & -4500.07 & 407.36 \\ 299.60 & 407.36 & -763.13 \end{bmatrix}$$

It is clear from the Hessian matrix, $\frac{\partial^2 TPN}{\partial N^2}, \frac{\partial^2 TPN}{\partial N^2}\ and\ \frac{\partial^2 TPN}{\partial r_c^2}$ all are negative. Hence the profit function is concave wrt single decision variable $N, T\ and\ r_c$.

Again $\begin{vmatrix} -8723.18 & 3452.43 \\ 3452.43 & 3452.43 \end{vmatrix} = 2.73 \times 10^7 > 0$ and $\begin{vmatrix} -8723.18 & 3452.43 & 299.60 \\ 3452.43 & -4500.07 & 407.36 \\ 299.60 & 407.36 & -763.13 \end{vmatrix} =$ $-3.63 \times 10^{10} < 0$,. Hence the profit function is concave in all decisions variable simultaneously.

CONCLUSION

This section presents the mathematical structure of an EOQ model operating under a two-tier trade credit arrangement. A major drawback of trade credit lies in the possibility of customer default—where buyers may fail to settle their dues once the credit window closes. To address this issue, retailers often implement reminder mechanisms aimed at encouraging timely repayments. Although these reminders incur extra costs, they contribute positively to profit by lowering default rates. This study introduces, for the first time, the concept of a *reminder level* within an inventory framework to explicitly model and manage repayment behaviour. In addition, a new solution technique is developed to solve the model efficiently. Computational analysis reveals a compelling insight: when the number of repayment instalments increases within a fixed credit horizon, the retailer's profit improves—a noteworthy result for trade-credit-based inventory systems.

FUTURE EXTENSION

Integrating reminder costs into trade credit inventory systems marks a meaningful progression in addressing payment risk across diverse modeling frameworks. Regardless of the structure—be it EOQ, EPQ, or two-warehouse models under full or partial credit terms—this concept can be universally adopted. While trade credit remains a popular method for stimulating demand and providing financial relief to buyers, the threat of customer default continues to be a critical concern for suppliers and retailers alike. The introduction of reminder costs serves as a strategic intervention to promote timely payments, thereby reducing exposure to financial losses. Though it may lead to a modest decline in demand due to its enforcement nature, the net benefit in mitigating defaults validates its inclusion as a valuable element in trade-credit-driven inventory models.

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