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Reliability Analysis of a Sixteen State Non-Repairable System Using Markov Modelling and Laplace Transform Technique

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Abstract:
Reliability analysis is an essential aspect of assessing the performance and operational lifetime of engineering systems. This paper presents the reliability analysis of a sixteen-state non-repairable system using a continuous-time Markov modelling approach. The system consists of four independent components that are subject to failure and are assumed to be non-repairable during the mission period. A comprehensive state transition diagram is developed to represent all possible operating and failed states, and the corresponding Kolmogorov forward differential equations re formulated. The Laplace transform technique is employed to obtain the state probability functions in both the Laplace and time domains. Based on these state probabilities, the system reliability function and the Mean Time to Failure (MTTF) are derived analytically. Numerical investigations are carried out by varying the component failure rates while keeping the remaining parameters fixed to study their influence on system reliability and MTTF. The results demonstrate that an increase in component failure rates leads to a significant reduction in both system reliability and expected operating life time. The proposed methodology provides an effective analytical framework for evaluating the reliability characteristic of complex multi-state non-repairable systems and can be extended to a wide range of engineering applications.

Keywords: Reliability Analysis, Markov Modelling, Non-Repairable System, Kolmogorov Forward Equations, Laplace transform, Mean Time to Failure (MTTF), Multi-State System.

1. Introduction

Reliability analysis has become an indispensable tool in the design, evaluation, and maintenance of engineering systems. The increasing complexity of modern industrial systems demands reliable analytical techniques to assess system performance, predict failures, and improve operational efficiency. Reliability studies enable engineers and decision-makers to estimate the probability that a system performs its intended function without failure for a specified period under stated operating conditions. Such analyses are essential in the design of manufacturing systems, communication networks, power generation systems, transportation systems, and other safety-critical engineering applications.

Markov modelling is one of the most effective stochastic techniques for analyzing systems whose state transitions are governed by probabilistic failure mechanisms. Continuous-time Markov processes provide a systematic framework for representing the dynamic behaviour of multi- component systems through a

finite number of operating and failed states. B constructing an appropriate state transition diagram and formulating the corresponding Kolmogorov forward differential equations, the transient behaviour of the system can be analyzed accurately. The Laplace transform technique further facilitates the analytical solution of these differential equations, enabling the determination of state probability functions, system reliability, and other important reliability measures.

Non-repairable systems constitute n important class of engineering systems in which failed components are not restored to operation during the mission period. In such systems, components are not restored to operation during the mission component failures accumulate over time, leading to a progressive degradation of system performance until complete system failure occurs. The evaluation of reliability and Mean Time to Failure (MTTF) for non-repairable systems is therefore essential for determining system lifetime, planning replacement strategies, and improving overall system design.

In this paper, a sixteen-state non-repairable system consisting of four independent components is analyzed using a continuous-time Markov modelling approach. A comprehensive state transition diagram is developed to represent all possible operating and failed states of the system. Based on the state transition structure, the Kolmogorov forward differential equations are formulated and solved using the Laplace transform technique. The resulting state probability functions re employed to derived the system reliability function and the Mean Time to Failure (MTTF). Numerical investigations are carried out to examine the influence of component failure rates on system reliability and expected system lifetime. The proposed methodology provides an effective analytical framework for evaluating the reliability characteristics of complex multi-state non-repairable systems and offers useful insights for reliability assessment and system performance evaluation.

2. Literature Review

Reliability engineering has become an important research area because of the increasing complexity of engineering systems and the need to improve their operational performance and safety. Various analytical techniques, including reliability block diagrams, fault tree analysis, stochastic processes, and continuous-time Markov models, have been developed to evaluate system reliability under different operating conditions [1-3]. Continuous-time Markov models are particularly suitable for systems with multiple operational and failure states because they provide a systematic framework for modelling stochastic state transitions and evaluating transient reliability measures.

Trivedi and Selvamuthu presented a comprehensive overview of Markov modelling in reliability analysis and demonstrated how continuous-time Markov chain can be used to evaluate the reliability of complex systems with multiple operational states [2]. Trivedi and Babbio further extended these concepts by presenting state-space reliability models and analytical methods for reliability and availability evaluation of engineering and communication systems [3].

James Li investigated the reliability of parallel redundant systems with different failure and repair rates using Markov modelling and compared the performance of active redundancy and standby redundancy systems [4,5]. El-Damcese and Temraz analysed parallel repairable systems with different failure modes using stochastic technique, while Cherian et al, investigated the reliability of standby systems

incorporating repair mechanisms [6,7]. These studies demonstrated that Markov models provide an effective analytical framework for evaluating the reliability characteristics of engineering systems with multiple operating states.

Garima Chopra applied the Gumbel-Hougaard family copula to analyse the reliability of two dissimilar-unit parallel systems and demonstrated the usefulness of advanced probabilistic techniques for reliability assessment [8]. El-Damcese, Abbas, and El-Ghamry further extended reliability analysis to three-element parallel systems with fuzzy failure and repair rates, illustrating the applicability of stochastic modelling under uncertainty [9].

Saritha, Tirumala Devi, and Sumathi Uma Maheswari presented a Markov-based methodology for evaluating the reliability of non-repairable systems and the availability of repairable systems. They formulated the Kolmogorov forward differential equation and solved them using the Laplace transform technique to obtain state probabilities, reliability, availability, Mean time to failure (MTTF), and Mean Time Between Failures (MTBF). Their numerical investigation demonstrated that increasing failure rates decreases reliability and MTTF, whereas increasing repair rates improves the steady-state availability of repairable systems [10].

Lindemann, Malhotra, and Trivedi compared several numerical methods for evaluating continuous-time Markov reliability models and demonstrated efficient computational approaches for large state-space systems [11]. Reibman and Trivedi also investigated transient analysis techniques for Markov and Markov reward models, providing efficient numerical methods for evaluating reliability measures in stochastic systems [12].

Motivated by these studies, the present work develops a sixteen state non-repairable Markov model for a four-component engineering system. Unlike the smaller models reported in earlier studies, the proposed model considers a larger state space representing all feasible operational and failure configurations. The corresponding Kolmogorov forward differential equations are formulated and solved using the Laplace transform technique to obtain the state probability functions. Based on these analytical solutions, the system reliability function and Mean Time to Failure (MTTF) are evaluated, followed by a numerical investigation of the influence of component failure rates on overall system performance.

3. Assumptions

The following assumptions are considered for the development of the proposed sixteen-state non-repairable Markov model:

1. The system consists of independent components, each of which is subject to failure.
2. Initially, all four components are in the perfect operating condition, and the system starts from the fully operational state.
3. The system is non-repairable; therefore, failed components are not repaired or replaced during the mission period.
4. The failure of each component follows an independent exponential distribution with constant failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$.

5. The component failure rates are assumed to be statistically independent, and the failure of one component does not alter the failure rate of the remaining operational components.
6. At any instant of time, only one component failure can occur, and simultaneous failures of two or more components are neglected.
7. The system behaviour satisfies the Markov property, i.e., the future state of the system depends only on its present state and is independent of the previous history.
8. State transitions occur only due to component failures, and each transition follows the state transition diagram of the proposed Markov model.
9. The complete behaviour of the system is represented by sixteen mutually exclusive and collectively exhaustive states, consisting of operational, degraded, and completely failed states.
10. The final state represents the complete system failure (absorbing state), and once the system enters this state, no further state transitions are possible.
11. The initial probability conditions are $P_1(0)=1,\ P_i(0)=0,\ i=2,3,.....,16.$
12. The Kolmogorov forward differential equations are employed to describe the evolution of the state probabilities, and the Laplace transform technique is used to obtain the analytical solutions.

4. Notations

The following notations are used throughout this paper.

S. No.	Notation	Description
1	$S_i,\ i=1,2,3,.....,16$	State i of the proposed sixteen-state Markov models
2	$P_i(t)$	Probability that the system is in state S_i at time t
3	$P_i(s)$	Laplace transform of the state probability $P_i(t)$
4	$P_i(0)$	Initial probability of state S_i
5	λ_1	Failure rate of Component 1
6	λ_2	Failure rate of Component 2
7	λ_3	Failure rate of Component 3
8	λ_4	Failure rate of Component 4
9	t	Mission time
10	s	Laplace transform parameter
11	R(t)	Reliability function of the system at time t
12	MTTF	Mean Time to Failure of the system
13	L	Laplace transform operator
14	L^{-1}	Inverse Laplace transform operator
15	$\frac{dP_i(t)}{dt}$	Rate of change of the state probability with respect to time

5. System Description

The proposed system consists of four independent components, denoted by $C_1, C_2, C_3,$ and C_4. Initially, all the components are in good working condition, and the system begins operation from State 1, which represents the fully operational state.

The behaviour of the system is modelled using a continuous-time Markov process consisting of 16 states. Each state corresponds to a unique combination of working and failed components, as illustrated in the state transition diagram. The transitions between states occur due to the failure of individual components with constant failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$.

From State 1, the system can move to State 2, State 3, State 4, or State 5 depending on whether Component $C_1, C_2, C_3,$ or C_4 fails first, respectively. These states represent single-component failure conditions while the remaining components continue to function.

As additional component failures occur, the system enters the degraded states S6-S15, where two or three components have failed. The transition between these states is governed by the corresponding component failure rates shown in the transition are unidirectional and no backward movement between states is possible.

The state transition diagram forms the basis for developing the Kolmogorov forward differential equations. The Laplace transform technique is then employed to obtain the stat probability functions. These probabilities are subsequently use to evaluate the system reliability and the Mean Time to Failure (MTTF).

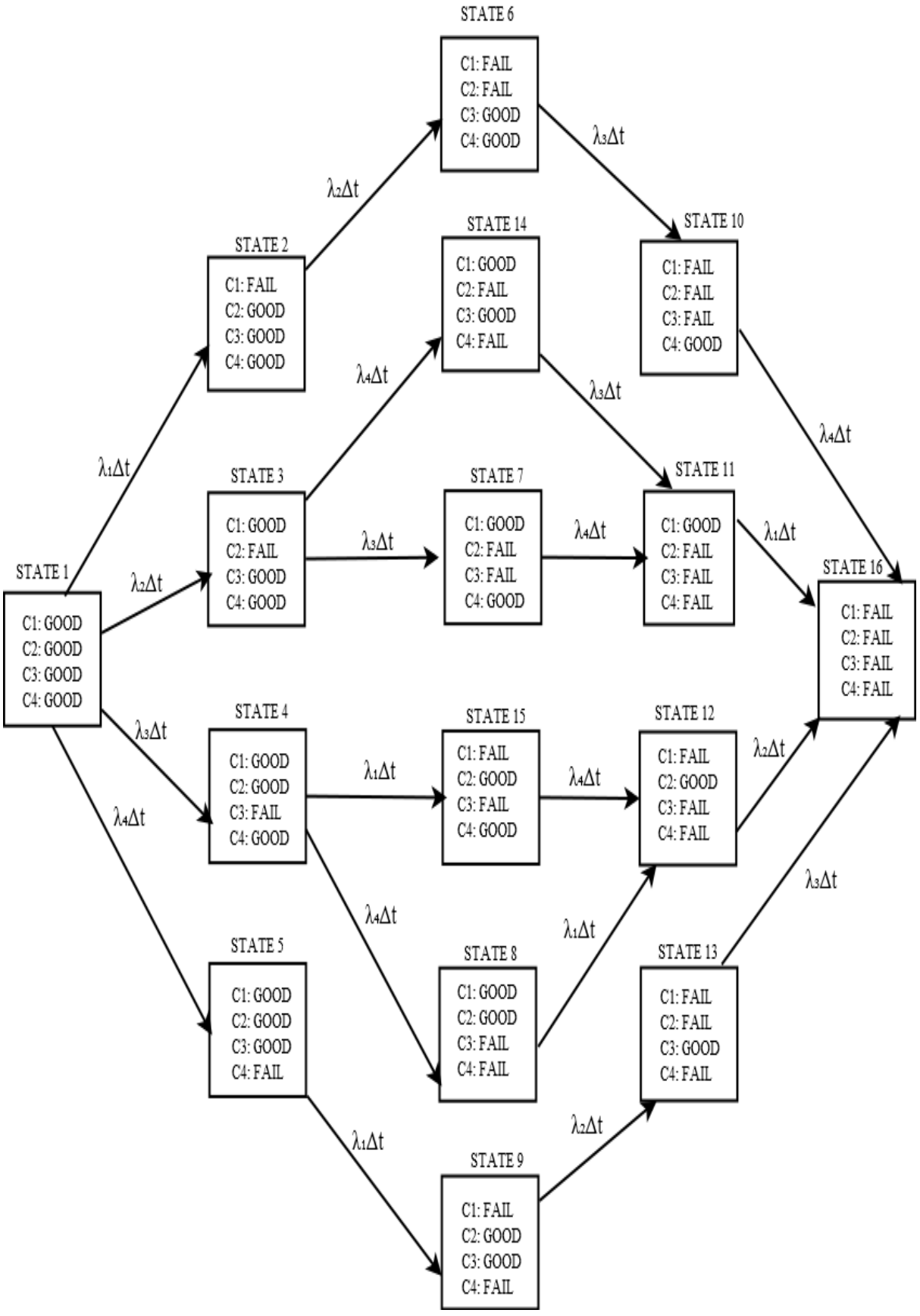


Fig 1. State Transition Diagram of the proposed Sixteen-State Non-Repairable Markov Model

6.Evaluation of Reliability of the Model

For the state 1,

$$P_1(t+\Delta t)=\left[1-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\Delta t\right]P_1(t)=P_1(t)-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(t)\Delta t$$

$$\Rightarrow P_1(t+\Delta t)-P_1(t)=-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(t)\Delta t$$

$$\Rightarrow \frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}=-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\lim_{\Delta t\to 0}\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}=-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(t)\Rightarrow \frac{dP_1(t)}{dt}=-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(t)$$

Solving the above equation by using the Laplace transformations, and taking assumption that the system good t initial time $t=0$, hence $P_1(0)=1,\ P_i(0)=0,$ for $i=1,2,3,.....,16$.

$$sP_1(s)-1=-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(s)\Rightarrow sP_1(s)+(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_1(s)=1$$

$$\Rightarrow P_1(s)(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4)=1\Rightarrow P_1(s)=\frac{1}{(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4)}$$

Taking inverse Laplace transform, we get

$$P_1(t)=e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}$$

For the state 2,

$$P_2(t+\Delta t)=\lambda_1P_1(t)\Delta t+(1-\lambda_2\Delta t)P_2(t)\Rightarrow P_2(t+\Delta t)=\lambda_1P_1(t)\Delta t+P_2(t)-\lambda_2P_2(t)\Delta t$$

$$\Rightarrow P_2(t+\Delta t)-P_2(t)=\lambda_1P_1(t)\Delta t-\lambda_2P_2(t)\Delta t\Rightarrow P_2(t+\Delta t)-P_2(t)=\left(\lambda_1P_1(t)-\lambda_2P_2(t)\right)\Delta t$$

$$\Rightarrow \frac{P_2(t+\Delta t)-P_2(t)}{\Delta t}=\lambda_1P_1(t)-\lambda_2P_2(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_2(t+\Delta t)-P_2(t)}{\Delta t}=\lambda_1P_1(t)-\lambda_2P_2(t)\Rightarrow \frac{dP_2(t)}{dt}=\lambda_1P_1(t)-\lambda_2P_2(t)$$

Solving the above equation by using the Laplace transformations and $P_2(0)=0$

$$sP_2(s)=\lambda_1P_1(s)-\lambda_2P_2(s)\Rightarrow(s+\lambda_2)P_2(s)=\lambda_1P_1(s)\Rightarrow(s+\lambda_2)P_2(s)=\frac{\lambda_1}{s+\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

$$P_2(s)=\frac{\lambda_1}{(s+\lambda_2)(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4)}$$

Taking inverse Laplace transform, we get

$$P_2(t)=\frac{\lambda_1}{(\lambda_1+\lambda_3+\lambda_4)}\left[e^{-t(\lambda_1+2\lambda_2+\lambda_3+\lambda_4)}\left\{e^{t(\lambda_1+\lambda_2+\lambda_3+\lambda_4)}-e^{t\lambda_2}\right\}\right]$$

For the state 3,

$$P_3(t+\Delta t)=\lambda_2P_1(t)\Delta t+(1-\lambda_3\Delta t-\lambda_4\Delta t)P_3(t)\Rightarrow P_3(t+\Delta t)=\left[\lambda_2P_1(t)-\lambda_3P_3(t)-\lambda_4P_3(t)\right]\Delta t+P_3(t)$$

$$\Rightarrow P_3(t+\Delta t)-P_3(t)=\left[\lambda_2 P_1(t)-\lambda_3 P_3(t)-\lambda_4 P_3(t)\right]\Delta t \Rightarrow \frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=\lambda_2 P_1(t)-\lambda_3 P_3(t)-\lambda_4 P_3(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4)P_3(t)\Rightarrow \frac{dP_3(t)}{dt}=\lambda_2 P_1(t)-(\lambda_3+\lambda_4)P_3(t)$$

Solving the above equation by using the Laplace transformations and $P_3(0)=0$

$$\Rightarrow sP_3(s)=\lambda_2 P_1(s)-(\lambda_3+\lambda_4)P_3(s)\Rightarrow (s+\lambda_3+\lambda_4)P_3(s)=\lambda_2 P_1(s)$$

$$\Rightarrow P_3(s)=\frac{\lambda_2 P_1(s)}{s+\lambda_3+\lambda_4}\Rightarrow P_3(s)=\frac{\lambda_2}{(s+\lambda_3+\lambda_4)(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4)}$$

Taking inverse Laplace transform, we get

$$P_3(t)=\frac{\lambda_2}{\lambda_1+\lambda_2}\left[-e^{(\lambda_3+\lambda_4)t-(\lambda_1+\lambda_2+2(\lambda_3+\lambda_4))t}+e^{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t-(\lambda_1+\lambda_2+2(\lambda_3+\lambda_4))t}\right]$$

For the state 4,

$$P_4(t+\Delta t)=\lambda_3 P_1(t)\Delta t+(1-\lambda_1\Delta t-\lambda_4\Delta t)P_4(t)\Rightarrow P_4(t+\Delta t)=\left(\lambda_3 P_1(t)-\lambda_1 P_4(t)-\lambda_4 P_4(t)\right)\Delta t+P_4(t)$$

$$\Rightarrow P_4(t+\Delta t)-P_4(t)=\left(\lambda_3 P_1(t)-\lambda_1 P_4(t)-\lambda_4 P_4(t)\right)\Delta t \Rightarrow \frac{P_4(t+\Delta t)-P_4(t)}{\Delta t}=\lambda_3 P_1(t)-\lambda_1 P_4(t)-\lambda_4 P_4(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_4(t+\Delta t)-P_4(t)}{\Delta t}=\lambda_3 P_1(t)-\lambda_1 P_4(t)-\lambda_4 P_4(t)\Rightarrow \frac{dP_4(t)}{dt}=\lambda_3 P_1(t)-\lambda_1 P_4(t)-\lambda_4 P_4(t)$$

Solving the above equation by using the Laplace transformations and $P_4(0)=0$

$$\Rightarrow sP_4(s)=\lambda_3 P_1(s)-(\lambda_1+\lambda_4)P_4(s)\Rightarrow (s+\lambda_1+\lambda_4)P_4(s)=\lambda_3 P_1(s)$$

$$\Rightarrow P_4(s)=\frac{\lambda_3 P_1(s)}{s+\lambda_1+\lambda_4}\Rightarrow P_4(s)=\frac{\lambda_3}{(s+\lambda_1+\lambda_4)(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4)}$$

Taking inverse Laplace transform, we get

$$P_4(t)=\frac{\lambda_3}{\lambda_2+\lambda_3}\left[e^{-(\lambda_1+\lambda_4)t}-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}\right]$$

For the state 5,

$$P_5(t+\Delta t)=\lambda_4 P_1(t)\Delta t+(1-\lambda_1\Delta t)P_5(t)\Rightarrow P_5(t+\Delta t)=\left(\lambda_4 P_1(t)-\lambda_1 P_5(t)\right)\Delta t+P_5(t)$$

$$\Rightarrow P_5(t+\Delta t)-P_5(t)=\left(\lambda_4 P_1(t)-\lambda_1 P_5(t)\right)\Delta t \Rightarrow \frac{P_5(t+\Delta t)-P_5(t)}{\Delta t}=\lambda_4 P_1(t)-\lambda_1 P_5(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_5(t+\Delta t)-P_5(t)}{\Delta t}=\lambda_4 P_1(t)-\lambda_1 P_5(t)\Rightarrow \frac{dP_5(t)}{dt}=\lambda_4 P_1(t)-\lambda_1 P_5(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0)=0$

$$\Rightarrow sP_5(s)=\lambda_4 P_1(s)-\lambda_1 P_5(s)\Rightarrow (s+\lambda_1)P_5(s)=\lambda_4 P_1(s)\Rightarrow P_5(s)=\frac{\lambda_4}{s+\lambda_1}P_1(s)$$

Taking inverse Laplace transform, we get

$$P_5(t)=\frac{\lambda_4}{\lambda_2+\lambda_3+\lambda_4}\left[e^{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t-(2\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}-e^{\lambda_1 t-(2\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}\right]$$

For state 6,

$$P_6(t+\Delta t)=\lambda_2 P_2(t)\Delta t+(1-\lambda_3\Delta t)P_6(t)\Rightarrow P_6(t+\Delta t)=\left(\lambda_2 P_2(t)-\lambda_3 P_6(t)\right)\Delta t+P_6(t)$$

$$\Rightarrow P_6(t+\Delta t)-P_6(t)=\left(\lambda_2 P_2(t)-\lambda_3 P_6(t)\right)\Delta t\Rightarrow \frac{P_6(t+\Delta t)-P_6(t)}{\Delta t}=\lambda_2 P_2(t)-\lambda_3 P_6(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_6(t+\Delta t)-P_6(t)}{\Delta t}=\lambda_2 P_2(t)-\lambda_3 P_6(t)\Rightarrow \frac{dP_6(t)}{dt}=\lambda_2 P_2(t)-\lambda_3 P_6(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0)=0$

$$\Rightarrow sP_6(s)=\lambda_2 P_2(s)-\lambda_3 P_6(s)\Rightarrow (s+\lambda_3)P_6(s)=\lambda_2 P_2(s)\Rightarrow P_6(s)=\frac{\lambda_2}{s+\lambda_3}P_2(s)$$

Taking inverse Laplace transform, we get

$$P_6(t)=\frac{\lambda_1\lambda_2 e^{-\lambda_3 t}}{(\lambda_2-\lambda_3)(\lambda_1+\lambda_2+\lambda_4)}-\frac{\lambda_1\lambda_2 e^{-\lambda_2 t}}{(\lambda_2-\lambda_3)(\lambda_1+\lambda_3+\lambda_4)}+\frac{\lambda_1\lambda_2 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)}$$

For state 7,

$$P_7(t+\Delta t)=\lambda_3 P_3(t)\Delta t+(1-\lambda_4\Delta t)P_7(t)\Rightarrow P_7(t+\Delta t)=\left(\lambda_3 P_3(t)-\lambda_4 P_7(t)\right)\Delta t+P_7(t)$$

$$\Rightarrow P_7(t+\Delta t)-P_7(t)=\left(\lambda_3 P_3(t)-\lambda_4 P_7(t)\right)\Delta t\Rightarrow \frac{P_7(t+\Delta t)-P_7(t)}{\Delta t}=\lambda_3 P_3(t)-\lambda_4 P_7(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_7(t+\Delta t)-P_7(t)}{\Delta t}=\lambda_3 P_3(t)-\lambda_4 P_7(t)\Rightarrow \frac{dP_7(t)}{dt}=\lambda_3 P_3(t)-\lambda_4 P_7(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0)=0$

$$\Rightarrow sP_7(s)=\lambda_3 P_3(s)-\lambda_4 P_7(s)\Rightarrow (s+\lambda_4)P_7(s)=\lambda_3 P_3(s)\Rightarrow P_7(s)=\frac{\lambda_3}{s+\lambda_4}P_3(s)$$

Taking inverse Laplace transform, we get

$$P_7(t) = -\frac{\lambda_2 e^{-(\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)} + \frac{\lambda_2 e^{-\lambda_4 t}}{(\lambda_1+\lambda_2+\lambda_3)} + \frac{\lambda_2\lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_3)}$$

For the state 8,

$$P_8(t+\Delta t) = \lambda_4 P_4(t)\Delta t + (1-\lambda_1\Delta t)P_8(t) \Rightarrow P_8(t+\Delta t) = (\lambda_4 P_4(t) - \lambda_1 P_8(t))\Delta t + P_8(t)$$

$$\Rightarrow P_8(t+\Delta t) - P_8(t) = (\lambda_4 P_4(t) - \lambda_1 P_8(t))\Delta t \Rightarrow \frac{P_8(t+\Delta t) - P_8(t)}{\Delta t} = \lambda_4 P_4(t) - \lambda_1 P_8(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0} \frac{P_8(t+\Delta t) - P_8(t)}{\Delta t} = \lambda_4 P_4(t) - \lambda_1 P_8(t) \Rightarrow \frac{dP_8(t)}{dt} = \lambda_4 P_4(t) - \lambda_1 P_8(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0) = 0$

$$\Rightarrow sP_8(s) = \lambda_4 P_4(s) - \lambda_1 P_8(s) \Rightarrow (s+\lambda_1)P_8(s) = \lambda_4 P_4(s) \Rightarrow P_8(s) = \frac{\lambda_4}{s+\lambda_1} P_4(s)$$

Taking inverse Laplace transform, we get

$$P_8(t) = -\frac{\lambda_3 e^{-(\lambda_1+\lambda_4)t}}{(\lambda_2+\lambda_3)} + \frac{\lambda_3 e^{-\lambda_1 t}}{(\lambda_2+\lambda_3+\lambda_4)} + \frac{\lambda_3\lambda_4 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_2+\lambda_3)(\lambda_2+\lambda_3+\lambda_4)}$$

For the state 9,

$$P_9(t+\Delta t) = \lambda_1 P_5(t)\Delta t + (1-\lambda_2\Delta t)P_9(t) \Rightarrow P_9(t+\Delta t) = (\lambda_1 P_5(t) - \lambda_2 P_9(t))\Delta t + P_9(t)$$

$$\Rightarrow P_9(t+\Delta t) - P_9(t) = (\lambda_1 P_5(t) - \lambda_2 P_9(t))\Delta t \Rightarrow \frac{P_9(t+\Delta t) - P_9(t)}{\Delta t} = \lambda_1 P_5(t) - \lambda_2 P_9(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0} \frac{P_9(t+\Delta t) - P_9(t)}{\Delta t} = \lambda_1 P_5(t) - \lambda_2 P_9(t) \Rightarrow \frac{dP_9(t)}{dt} = \lambda_1 P_5(t) - \lambda_2 P_9(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0) = 0$

$$\Rightarrow sP_9(s) = \lambda_1 P_5(s) - \lambda_2 P_9(s) \Rightarrow (s+\lambda_2)P_9(s) = \lambda_1 P_5(s) \Rightarrow P_9(s) = \frac{\lambda_1}{s+\lambda_2} P_5(s)$$

Taking inverse Laplace transform, we get

$$P_9(t) = \frac{\lambda_1\lambda_4 e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)(\lambda_1+\lambda_3+\lambda_4)} - \frac{\lambda_1\lambda_4 e^{-\lambda_1 t}}{(\lambda_1-\lambda_2)(\lambda_2+\lambda_3+\lambda_4)} + \frac{\lambda_1\lambda_4 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)}$$

For the state 10,

$$P_{10}(t+\Delta t) = \lambda_3 P_6(t)\Delta t + (1-\lambda_4\Delta t)P_{10}(t) \Rightarrow P_{10}(t+\Delta t) = (\lambda_3 P_6(t) - \lambda_4 P_{10}(t))\Delta t + P_{10}(t)$$

$$\Rightarrow P_{10}(t+\Delta t) - P_{10}(t) = (\lambda_3 P_6(t) - \lambda_4 P_{10}(t))\Delta t \Rightarrow \frac{P_{10}(t+\Delta t) - P_{10}(t)}{\Delta t} = \lambda_3 P_6(t) - \lambda_4 P_{10}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_{10}(t+\Delta t)-P_{10}(t)}{\Delta t}=\lambda_3 P_6(t)-\lambda_4 P_{10}(t)\Rightarrow \frac{dP_{10}(t)}{dt}=\lambda_3 P_6(t)-\lambda_4 P_{10}(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0)=0$

$$\Rightarrow sP_{10}(s)=\lambda_3 P_6(s)-\lambda_4 P_{10}(s)\Rightarrow (s+\lambda_4)P_{10}(s)=\lambda_3 P_6(s)\Rightarrow P_{10}(s)=\frac{\lambda_3}{s+\lambda_4}P_6(s)$$

Taking inverse Laplace transform, we get

$$P_{10}(t)=\frac{\lambda_1\lambda_2\lambda_3 e^{-\lambda_4 t}}{(\lambda_2-\lambda_4)(\lambda_1+\lambda_2+\lambda_3)(\lambda_3-\lambda_4)}-\frac{\lambda_1\lambda_2\lambda_3 e^{-\lambda_3 t}}{(\lambda_3-\lambda_4)(\lambda_1+\lambda_2+\lambda_4)(\lambda_3-\lambda_2)}$$
$$+\frac{\lambda_1\lambda_2\lambda_3 e^{-\lambda_2 t}}{(\lambda_1+\lambda_3+\lambda_4)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}-\frac{\lambda_1\lambda_2\lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)}$$

For the state 11,

$$P_{11}(t+\Delta t)=(\lambda_3 P_{14}(t)+\lambda_4 P_7(t))\Delta t+(1-\lambda_1\Delta t)P_{11}(t)\Rightarrow P_{11}(t+\Delta t)=\left[\lambda_3 P_{14}(t)+\lambda_4 P_7(t)-\lambda_1 P_{11}(t)\right]\Delta t+P_{11}(t)$$
$$\Rightarrow P_{11}(t+\Delta t)-P_{11}(t)=\left[\lambda_3 P_{14}(t)+\lambda_4 P_7(t)-\lambda_1 P_{11}(t)\right]\Delta t\Rightarrow \frac{P_{11}(t+\Delta t)-P_{11}(t)}{\Delta t}=\lambda_3 P_{14}(t)+\lambda_4 P_7(t)-\lambda_1 P_{11}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_{11}(t+\Delta t)-P_{11}(t)}{\Delta t}=\lambda_3 P_{14}(t)+\lambda_4 P_7(t)-\lambda_1 P_{11}(t)\Rightarrow \frac{dP_{11}(t)}{dt}=\lambda_3 P_{14}(t)+\lambda_4 P_7(t)-\lambda_1 P_{11}(t)$$

Solving the above equation by using the Laplace transformations and $P_3(0)=0$

$$\Rightarrow sP_{11}(s)=\lambda_3 P_{14}(s)+\lambda_4 P_7(s)-\lambda_1 P_{11}(s)\Rightarrow (s+\lambda_1)P_{11}(s)=\lambda_3 P_{14}(s)+\lambda_4 P_7(s)$$

$$\Rightarrow P_{11}(s)=\frac{\lambda_3 P_{14}(s)+\lambda_4 P_7(s)}{s+\lambda_1}$$

Taking inverse Laplace transform, we get

$$P_{11}(t)=\frac{\lambda_2\lambda_4 e^{-\lambda_4 t}}{(\lambda_1-\lambda_4)(\lambda_1+\lambda_2+\lambda_3)}+\frac{\lambda_2\lambda_3 e^{-\lambda_3 t}}{(\lambda_1-\lambda_3)(\lambda_1+\lambda_2+\lambda_4)}-\frac{\lambda_2(\lambda_3+\lambda_4)e^{-(\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)(-\lambda_1+\lambda_3+\lambda_4)}$$
$$+\frac{\lambda_2\lambda_3\lambda_4(-2\lambda_1+\lambda_3+\lambda_4)e^{-\lambda_1 t}}{(\lambda_1-\lambda_3-\lambda_4)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)(\lambda_2+\lambda_3+\lambda_4)}$$
$$-\frac{\lambda_2\lambda_3\lambda_4(2\lambda_1+2\lambda_2+\lambda_3+\lambda_4)e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)}$$

For the state 12,

$$P_{12}(t+\Delta t)=(\lambda_4 P_{15}(t)+\lambda_1 P_8(t))\Delta t+(1-\lambda_2\Delta t)P_{12}(t)\Rightarrow P_{12}(t+\Delta t)=\left[\lambda_4 P_{15}(t)+\lambda_1 P_8(t)-\lambda_2 P_{12}(t)\right]\Delta t+P_{12}(t)$$

$$\Rightarrow P_{12}(t+\Delta t)-P_{12}(t)=\left[\lambda_4 P_{15}(t)+\lambda_1 P_8(t)-\lambda_2 P_{12}(t)\right]\Delta t \Rightarrow \frac{P_{12}(t+\Delta t)-P_{12}(t)}{\Delta t}=\lambda_4 P_{15}(t)+\lambda_1 P_8(t)-\lambda_2 P_{12}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_{12}(t+\Delta t)-P_{12}(t)}{\Delta t}=\lambda_4 P_{15}(t)+\lambda_1 P_8(t)-\lambda_2 P_{12}(t)\Rightarrow \frac{dP_{12}(t)}{dt}=\lambda_4 P_{15}(t)+\lambda_1 P_8(t)-\lambda_2 P_{12}(t)$$

Solving the above equation by using the Laplace transformations and $P_3(0)=0$

$$\Rightarrow sP_{12}(s)=\lambda_4 P_{15}(s)+\lambda_1 P_8(s)-\lambda_2 P_{12}(s)\Rightarrow (s+\lambda_2)P_{12}(s)=\lambda_4 P_{15}(s)+\lambda_1 P_8(s)$$

$$\Rightarrow P_{12}(s)=\frac{\lambda_4 P_{15}(s)+\lambda_1 P_8(s)}{s+\lambda_2}$$

Taking inverse Laplace transform, we get

$$P_{12}(t)=\frac{\lambda_3\lambda_4 e^{-\lambda_4 t}}{(\lambda_2-\lambda_4)(\lambda_1+\lambda_2+\lambda_3)}+\frac{\lambda_3(\lambda_1+\lambda_4)e^{-(\lambda_1+\lambda_4)t}}{(\lambda_2+\lambda_3)(\lambda_1-\lambda_2+\lambda_4)}$$
$$-\frac{\lambda_1\lambda_3\lambda_4(\lambda_1-2\lambda_2+\lambda_4)e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_4)(\lambda_1-\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)}$$
$$-\frac{\lambda_1\lambda_3 e^{-\lambda_1 t}}{(\lambda_1-\lambda_2)(\lambda_2+\lambda_3+\lambda_4)}-\frac{\lambda_1\lambda_3\lambda_4(\lambda_1+2(\lambda_2+\lambda_3)+\lambda_4)e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_3)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)}$$

For the state 13,

$$P_{13}(t+\Delta t)=\lambda_2 P_9(t)\Delta t+(1-\lambda_3\Delta t)P_{13}(t)\Rightarrow P_{13}(t+\Delta t)=\left(\lambda_2 P_9(t)-\lambda_3 P_{13}(t)\right)\Delta t+P_{13}(t)$$

$$\Rightarrow P_{13}(t+\Delta t)-P_{13}(t)=\left(\lambda_2 P_9(t)-\lambda_3 P_{13}(t)\right)\Delta t \Rightarrow \frac{P_{13}(t+\Delta t)-P_{13}(t)}{\Delta t}=\lambda_2 P_9(t)-\lambda_3 P_{13}(t)$$

Taking the limit as $\Delta t \to 0$, and the differential equation is

$$\Rightarrow \lim_{\Delta t\to 0}\frac{P_{13}(t+\Delta t)-P_{13}(t)}{\Delta t}=\lambda_2 P_9(t)-\lambda_3 P_{13}(t)\Rightarrow \frac{dP_{13}(t)}{dt}=\lambda_2 P_9(t)-\lambda_3 P_{13}(t)$$

Solving the above equation by using the Laplace transformations and $P_5(0)=0$

$$\Rightarrow sP_{13}(s)=\lambda_2 P_9(s)-\lambda_3 P_{13}(s)\Rightarrow (s+\lambda_3)P_{13}(s)=\lambda_2 P_9(s)\Rightarrow P_{13}(s)=\frac{\lambda_2}{s+\lambda_3}P_9(s)$$

Taking inverse Laplace transform, we get

$$P_{13}(t)=\frac{\lambda_1\lambda_2\lambda_4 e^{-\lambda_3 t}}{(-\lambda_1+\lambda_3)(-\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_4)}-\frac{\lambda_1\lambda_2\lambda_4 e^{-\lambda_2 t}}{(-\lambda_1+\lambda_2)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2-\lambda_3)}$$
$$+\frac{\lambda_1\lambda_2\lambda_4 e^{-\lambda_1 t}}{(\lambda_2+\lambda_3+\lambda_4)(\lambda_1-\lambda_3)(\lambda_1-\lambda_2)}-\frac{\lambda_1\lambda_2\lambda_4 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)}$$

For the state 14,

$$P_{14}(t+\Delta t)=\lambda_4 P_3(t)\Delta t+(1-\lambda_3\Delta t)P_{14}(t)\Rightarrow P_{14}(t+\Delta t)=\left[\lambda_4 P_3(t)-\lambda_3 P_{14}(t)\right]\Delta t+P_{14}(t)$$

$$\Rightarrow P_{14}(t+\Delta t)-P_{14}(t)=\left[\lambda_4 P_3(t)-\lambda_3 P_{14}(t)\right]\Delta t\Rightarrow\frac{P_{14}(t+\Delta t)-P_{14}(t)}{\Delta t}=\lambda_4 P_3(t)-\lambda_3 P_{14}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\Rightarrow\lim_{\Delta t\to 0}\frac{P_{14}(t+\Delta t)-P_{14}(t)}{\Delta t}=\lambda_4 P_3(t)-\lambda_3 P_{14}(t)\Rightarrow\frac{dP_{14}(t)}{dt}=\lambda_4 P_3(t)-\lambda_3 P_{14}(t)$$

Solving the above equation by using the Laplace transformations and $P_3(0)=0$

$$\Rightarrow sP_{14}(s)=\lambda_4 P_3(s)-\lambda_3 P_{14}(s)\Rightarrow(s+\lambda_3)P_{14}(s)=\lambda_4 P_3(s)$$

$$\Rightarrow P_{14}(s)=\frac{\lambda_4 P_3(s)}{s+\lambda_3}$$

Taking inverse Laplace transform, we get

$$P_{14}(t)=-\frac{\lambda_2 e^{-(\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)}-\frac{\lambda_2 e^{-\lambda_3 t}}{(\lambda_1+\lambda_2+\lambda_4)}+\frac{\lambda_2\lambda_4 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_4)}$$

For the state 15,

$$P_{15}(t+\Delta t)=\lambda_1 P_4(t)\Delta t+(1-\lambda_4\Delta t)P_{15}(t)\Rightarrow P_{15}(t+\Delta t)=\left[\lambda_1 P_4(t)-\lambda_4 P_{15}(t)\right]\Delta t+P_{15}(t)$$

$$\Rightarrow P_{15}(t+\Delta t)-P_{15}(t)=\left[\lambda_1 P_4(t)-\lambda_4 P_{15}(t)\right]\Delta t\Rightarrow\frac{P_{15}(t+\Delta t)-P_{15}(t)}{\Delta t}=\lambda_1 P_4(t)-\lambda_4 P_{15}(t)$$

Taking the limit as $\Delta t\to 0$, and the differential equation is

$$\Rightarrow\lim_{\Delta t\to 0}\frac{P_{15}(t+\Delta t)-P_{15}(t)}{\Delta t}=\lambda_1 P_4(t)-\lambda_4 P_{15}(t)\Rightarrow\frac{dP_{15}(t)}{dt}=\lambda_1 P_4(t)-\lambda_4 P_{15}(t)$$

Solving the above equation by using the Laplace transformations and $P_3(0)=0$

$$\Rightarrow sP_{15}(s)=\lambda_1 P_4(s)-\lambda_4 P_{15}(s)\Rightarrow(s+\lambda_4)P_{15}(s)=\lambda_1 P_4(s)$$

$$\Rightarrow P_{15}(s)=\frac{\lambda_1 P_4(s)}{s+\lambda_4}$$

Taking inverse Laplace transform, we get

$$P_{15}(t)=-\frac{\lambda_3 e^{-(\lambda_1+\lambda_4)t}}{(\lambda_2+\lambda_3)}+\frac{\lambda_3 e^{-\lambda_4 t}}{(\lambda_1+\lambda_2+\lambda_3)}+\frac{\lambda_1\lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_3)}$$

The reliability of the 4-component system, if at least one component should operate

$$R(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) + P_6(t) + P_7(t) + P_8(t) + P_9(t) + P_{10}(t) + P_{11}(t) + P_{12}(t) + P_{13}(t) + P_{14}(t) + P_{15}(t)$$

$$\begin{aligned} R(t) = {} & \frac{\lambda_2\lambda_3 e^{-(\lambda_1+\lambda_4)t}}{(\lambda_2+\lambda_3)(1-\lambda_2+\lambda_4)} + \frac{\lambda_1\lambda_2 e^{-(\lambda_1+\lambda_4)t}}{(\lambda_1+\lambda_2)(-\lambda_1+\lambda_3+\lambda_4)} \\ & + \frac{\lambda_1\lambda_3\lambda_4(\lambda_1-2\lambda_2+\lambda_4)(\lambda_1-2\lambda_2+\lambda_3+\lambda_4)}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)(\lambda_1-\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)} \\ & + \frac{\lambda_2\left(-\lambda_1\lambda_3(\lambda_1+\lambda_2+\lambda_3)+\left(\lambda_3^2+\lambda_1(\lambda_2+3\lambda_3)\right)\lambda_4-(\lambda_1+\lambda_3)\lambda_4^2\right)}{(\lambda_1+\lambda_2+\lambda_3)(-\lambda_1+\lambda_4)(-\lambda_2+\lambda_4)(-\lambda_3+\lambda_4)} \\ & + \frac{\lambda_1\lambda_2\left(\lambda_3^2-3\lambda_3\lambda_4+\lambda_4(\lambda_1+\lambda_4)+\lambda_2(-\lambda_3+\lambda_4)\right)e^{-\lambda_3 t}}{(\lambda_1-\lambda_3)(-\lambda_2+\lambda_3)(\lambda_3-\lambda_4)(\lambda_1+\lambda_2+\lambda_4)} \\ & - \frac{\lambda_1\lambda_2\lambda_3\lambda_4\left(\lambda_1^2+(2\lambda_2+\lambda_3)(2\lambda_2+\lambda_3+\lambda_4)+\lambda_1(6\lambda_2+4\lambda_3+\lambda_4)\right)e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}}{(\lambda_1+\lambda_2)(\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)} \\ & + \frac{\lambda_2\lambda_3\left(-\lambda_1^3+\lambda_4(\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)+\lambda_1^2(2\lambda_3+5\lambda_4)-\lambda_1(2\lambda_2\lambda_4+(\lambda_3+\lambda_4)(\lambda_3+4\lambda_4))\right)e^{-\lambda_1 t}}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)(\lambda_1-\lambda_3-\lambda_4)(\lambda_2+\lambda_3+\lambda_4)} \end{aligned}$$

Mean Time to Failure (MTTF) of the system

$$MTTF = \int_0^\infty R(t)\,dt$$

$$MTTF = -\frac{\lambda_2}{(\lambda_1+\lambda_2)(\lambda_3+\lambda_4)} - \frac{\lambda_3}{(\lambda_1+\lambda_4)(\lambda_2+\lambda_3)} - \frac{\lambda_1(3\lambda_2+2\lambda_3)+\lambda_2(2\lambda_2+\lambda_3)}{(\lambda_1+\lambda_2)(\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_3+\lambda_4)} + \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

The reliability function in the exponential case:

STATE	Component 1	Component 2	Component 3	Component 4	System state
1.	GOOD	GOOD	GOOD	GOOD	SUCCESS
2.	FAIL	GOOD	GOOD	GOOD	SUCCESS
3.	GOOD	FAIL	GOOD	GOOD	SUCCESS
4.	GOOD	GOOD	FAIL	GOOD	SUCCESS
5.	GOOD	GOOD	GOOD	FAIL	SUCCESS
6.	FAIL	FAIL	GOOD	GOOD	SUCCESS
7.	GOOD	FAIL	FAIL	GOOD	SUCCESS
8.	GOOD	GOOD	FAIL	FAIL	SUCCESS
9.	FAIL	GOOD	GOOD	FAIL	SUCCESS
10.	FAIL	FAIL	FAIL	GOOD	SUCCESS
11.	GOOD	FAIL	FAIL	FAIL	SUCCESS
12.	FAIL	GOOD	FAIL	FAIL	SUCCESS
13.	FAIL	FAIL	GOOD	FAIL	SUCCESS

14.	GOOD	FAIL	GOOD	FAIL	SUCCESS
15.	FAIL	GOOD	FAIL	GOOD	SUCCESS
16.	FAIL	FAIL	FAIL	FAIL	DOWN

Table 1

STATE	Component 1	Component 2	Component 3	Component 4
1.	$e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$e^{-\lambda_4 t}$
2.	$\left(1-e^{(-\lambda_1 t)}\right)$	$e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$e^{-\lambda_4 t}$
3.	$e^{-\lambda_1 t}$	$\left(1-e^{(-\lambda_2 t)}\right)$	$e^{-\lambda_3 t}$	$e^{-\lambda_4 t}$
4.	$e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$\left(1-e^{(-\lambda_3 t)}\right)$	$e^{-\lambda_4 t}$
5.	$e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$\left(1-e^{(-\lambda_4 t)}\right)$
6.	$\left(1-e^{(-\lambda_1 t)}\right)$	$\left(1-e^{(-\lambda_2 t)}\right)$	$e^{-\lambda_3 t}$	$e^{-\lambda_4 t}$
7.	$e^{-\lambda_1 t}$	$\left(1-e^{(-\lambda_2 t)}\right)$	$\left(1-e^{(-\lambda_3 t)}\right)$	$e^{-\lambda_4 t}$
8.	$e^{-\lambda_1 t}$	$e^{-\lambda_2 t}$	$\left(1-e^{(-\lambda_3 t)}\right)$	$\left(1-e^{(-\lambda_4 t)}\right)$
9.	$\left(1-e^{(-\lambda_1 t)}\right)$	$e^{-\lambda_2 t}$	$e^{-\lambda_3 t}$	$\left(1-e^{(-\lambda_4 t)}\right)$
10.	$\left(1-e^{(-\lambda_1 t)}\right)$	$\left(1-e^{(-\lambda_2 t)}\right)$	$\left(1-e^{(-\lambda_3 t)}\right)$	$e^{-\lambda_4 t}$
11.	$e^{-\lambda_1 t}$	$\left(1-e^{(-\lambda_2 t)}\right)$	$\left(1-e^{(-\lambda_3 t)}\right)$	$\left(1-e^{(-\lambda_4 t)}\right)$
12.	$\left(1-e^{(-\lambda_1 t)}\right)$	$e^{-\lambda_2 t}$	$\left(1-e^{(-\lambda_3 t)}\right)$	$\left(1-e^{(-\lambda_4 t)}\right)$
13.	$\left(1-e^{(-\lambda_1 t)}\right)$	$\left(1-e^{(-\lambda_2 t)}\right)$	$e^{-\lambda_3 t}$	$\left(1-e^{(-\lambda_4 t)}\right)$
14.	$e^{-\lambda_1 t}$	$\left(1-e^{(-\lambda_2 t)}\right)$	$e^{-\lambda_3 t}$	$\left(1-e^{(-\lambda_4 t)}\right)$
15.	$\left(1-e^{(-\lambda_1 t)}\right)$	$e^{-\lambda_2 t}$	$\left(1-e^{(-\lambda_3 t)}\right)$	$e^{-\lambda_4 t}$
16.	$\left(1-e^{(-\lambda_1 t)}\right)$	$\left(1-e^{(-\lambda_2 t)}\right)$	$\left(1-e^{(-\lambda_3 t)}\right)$	$\left(1-e^{(-\lambda_4 t)}\right)$

Table 2

From table 2 the probability of success is

$$P(Success) = P(1)+P(2)+P(3)+P(4)+P(5)+P(6)+P(7) + P(8)+P(9)+P(10)+P(11)+P(12)+P(13)+P(14)+P(15)$$

$$P(Success) = e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}\begin{pmatrix} -1+e^{\lambda_1 t}+e^{\lambda_2 t}-e^{(\lambda_1+\lambda_2)t}+e^{\lambda_3 t}-e^{(\lambda_1+\lambda_3)t}-e^{(\lambda+\lambda_3)t} \\ e^{(\lambda_1+\lambda_2+\lambda_3)t}+e^{\lambda_4 t}-e^{(\lambda_1+\lambda_4)t}-e^{(\lambda_2+\lambda_4)t}+e^{(\lambda_1+\lambda_2+\lambda_4)t} \\ -e^{(\lambda_3+\lambda_4)t}+e^{(\lambda_1+\lambda_3+\lambda_4)t}+e^{(\lambda_2+\lambda_3+\lambda_4)t} \end{pmatrix}$$

and MTTF is

$$MTTF = \int_0^\infty P(Success)dt$$

$$MTTF = \frac{1}{\lambda_1}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1+\lambda_2}+\frac{1}{\lambda_3}-\frac{1}{\lambda_1+\lambda_3}-\frac{1}{\lambda_2+\lambda_3}+\frac{1}{\lambda_1+\lambda_2+\lambda_3}+\frac{1}{\lambda_4}$$
$$-\frac{1}{\lambda_1+\lambda_4}-\frac{1}{\lambda_2+\lambda_4}+\frac{1}{\lambda_1+\lambda_2+\lambda_4}-\frac{1}{\lambda_3+\lambda_4}+\frac{1}{\lambda_1+\lambda_3+\lambda_4}$$
$$+\frac{1}{\lambda_2+\lambda_3+\lambda_4}-\frac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

If all the three components have the same failure rate, we get

$$MTTF = \frac{25}{12\lambda}$$

7. Numerical results

4-component non-repairable system with different failure rates

S.No.	λ_1	λ_2	λ_3	λ_4	t	$R(t)$	MTTF
1.	0.002	0.015	0.035	0.055	40	0.976761	578.0863
2.	0.010	0.015	0.035	0.055	40	0.900350	181.2014
3.	0.018	0.015	0.035	0.055	40	0.844865	139.3322
4.	0.026	0.015	0.035	0.055	40	0.804574	124.4100
5.	0.034	0.015	0.035	0.055	40	0.775317	117.2207
6.	0.042	0.015	0.035	0.055	40	0.754072	113.2329
7.	0.050	0.015	0.035	0.055	40	0.738645	110.8393
8.	0.058	0.015	0.035	0.055	40	0.727443	109.3337
9.	0.066	0.015	0.035	0.055	40	0.719308	108.3614

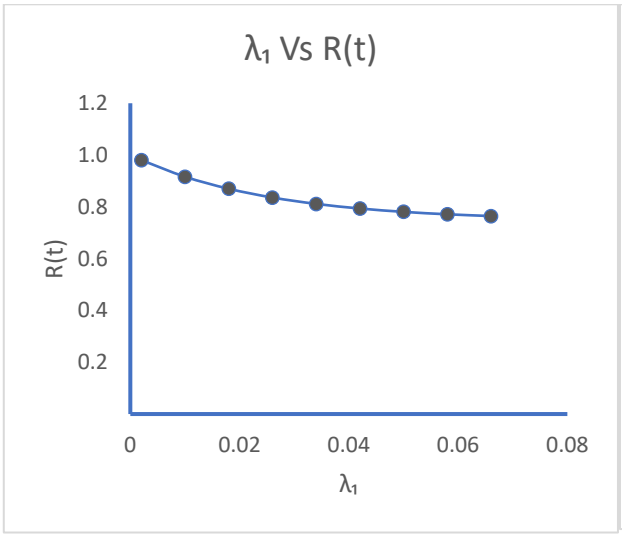
S.No.	λ_1	λ_2	λ_3	λ_4	t	$R(t)$	MTTF
1.	0.025	0.002	0.045	0.065	40	0.962447	551.9267
2.	0.025	0.010	0.045	0.065	40	0.838970	152.8419
3.	0.025	0.018	0.045	0.065	40	0.749308	109.2171
4.	0.025	0.026	0.045	0.065	40	0.684200	92.8611
5.	0.025	0.034	0.045	0.065	40	0.636921	84.4796
6.	0.025	0.042	0.045	0.065	40	0.602590	79.4852
7.	0.025	0.050	0.045	0.065	40	0.577661	76.2309
8.	0.025	0.058	0.045	0.065	40	0.559558	73.9815
9.	0.025	0.066	0.045	0.065	40	0.546413	72.3603

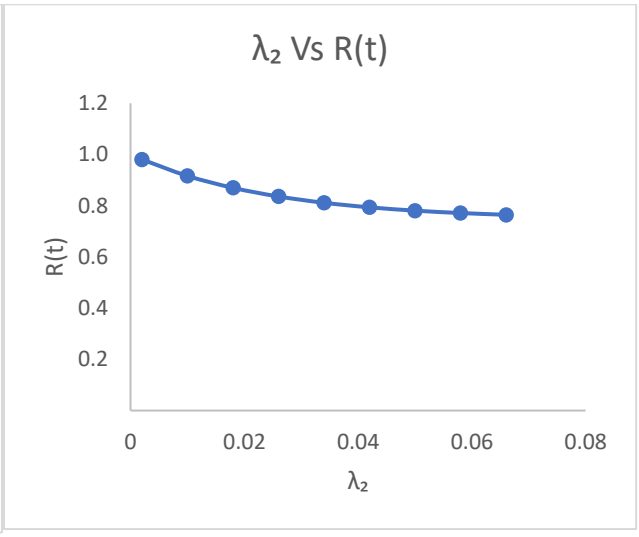
S.No.	λ_1	λ_2	λ_3	λ_4	t	$R(t)$	MTTF
1.	0.015	0.025	0.002	0.055	40	0.980502	589.104
2.	0.015	0.025	0.010	0.055	40	0.916392	191.254
3.	0.015	0.025	0.018	0.055	40	0.869838	148.549
4.	0.015	0.025	0.026	0.055	40	0.836034	132.86
5.	0.015	0.025	0.034	0.055	40	0.811486	125.062
6.	0.015	0.025	0.042	0.055	40	0.793661	120.499

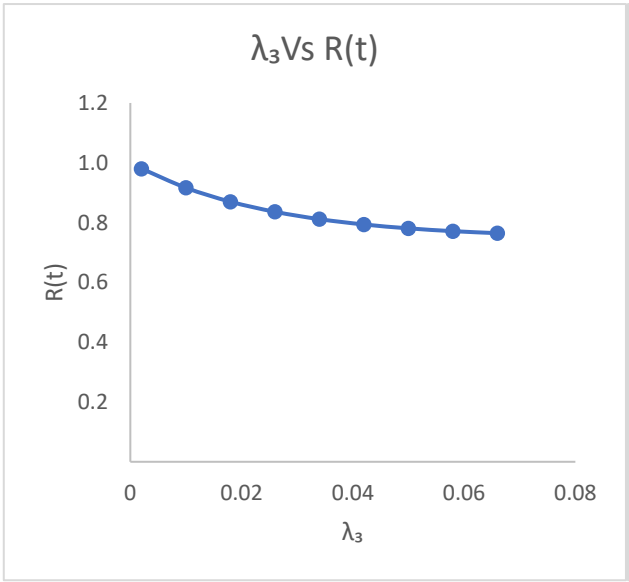
7.	0.015	0.025	0.050	0.055	40	0.780718	117.591
8.	0.015	0.025	0.058	0.055	40	0.771319	115.621
9.	0.015	0.025	0.066	0.055	40	0.764494	114.227

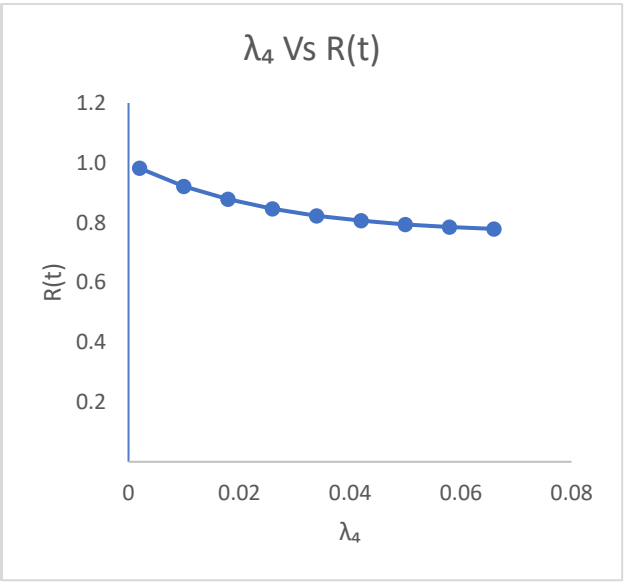
S.No.	λ_1	λ_2	λ_3	λ_4	t	$R(t)$	MTTF
1.	0.015	0.025	0.045	0.002	40	0.981697	557.866
2.	0.015	0.025	0.045	0.010	40	0.921516	173.578
3.	0.015	0.025	0.045	0.018	40	0.877816	138.226
4.	0.015	0.025	0.045	0.026	40	0.846083	127.263
5.	0.015	0.025	0.045	0.034	40	0.823040	122.714
6.	0.015	0.025	0.045	0.042	40	0.806307	120.597
7.	0.015	0.025	0.045	0.050	40	0.794157	119.589
8.	0.015	0.025	0.045	0.058	40	0.785334	119.143
9.	0.015	0.025	0.045	0.066	40	0.778927	119.002

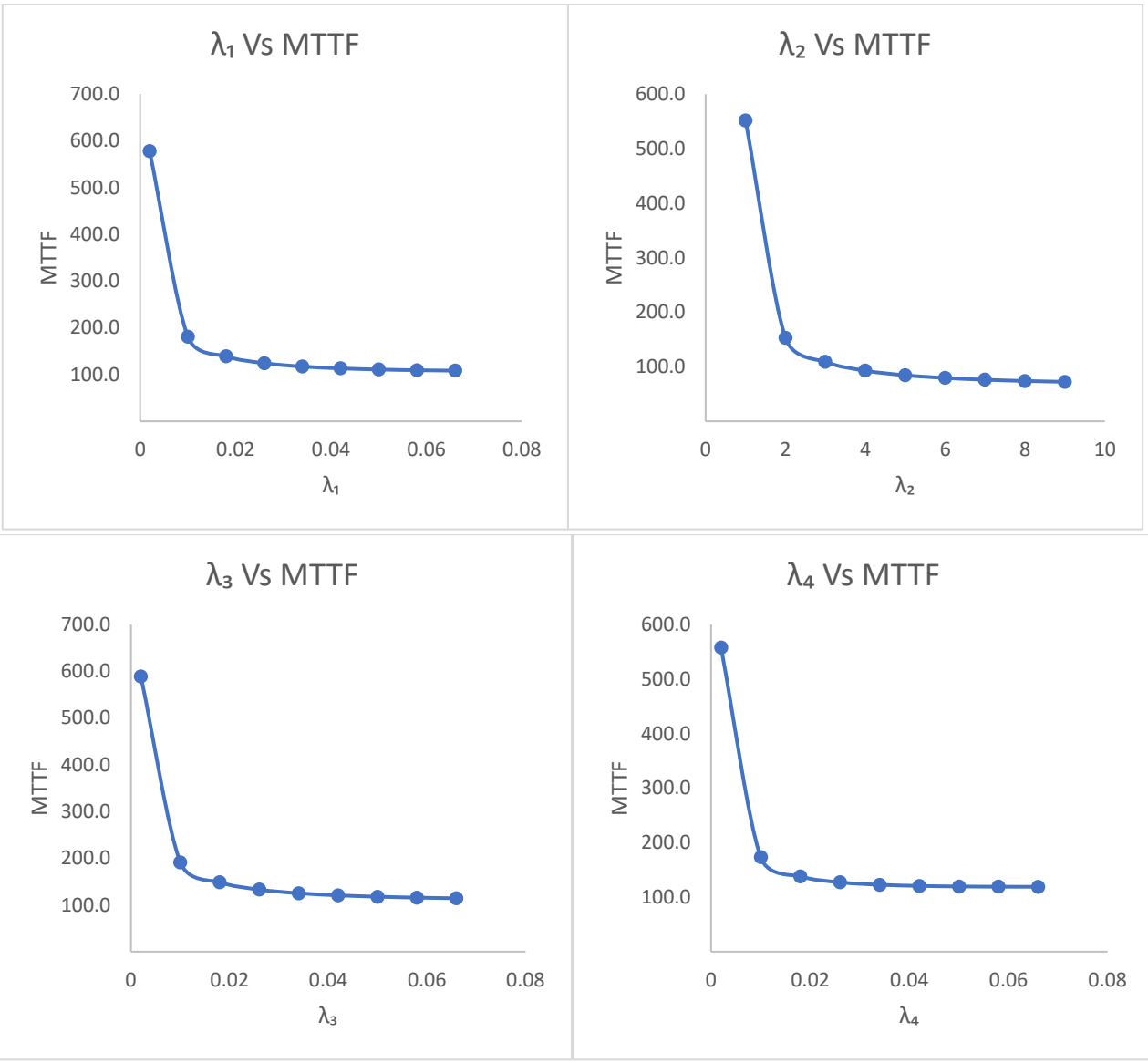
8 Graphs











9 Conclusion

The reliability behaviour of the considered non-repairable system has been investigated for different combinations of the failure rate $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ using the obtained reliability function R(t) and Mean Time to Failure (MTTF). The numerical results presented in the tables clearly indicate that the reliability of the system decreases as the corresponding component failure rate increases.

From the first set of results, when λ_1 I increased from 0.002 to 0.066 while the other failure rates are kept fixed, R(t) decreases from 0.976761 to 0.719308, and the MTTF decreases from 578.0863 to 108.3614. Similarly, variations in $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ also show a decreasing trend in both reliability and MTTF.

The results demonstrate that the system is highly sensitive to increases the likelihood of system failure and consequently reduces the expected operating lifetime of the system. The decreasing trends observed in all four sets of results are consistent with the theoretical behaviour of a non-repairable system.

Hence, it can be concluded that reducing the failure rates of the system components is essential for improving both system reliability and MTTF. The numerical results obtained provide useful information for identifying critical components and for making appropriate reliability improvement and maintenance decisions in practical engineering system.

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